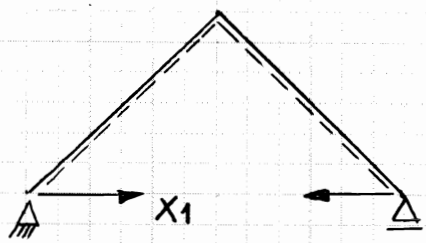
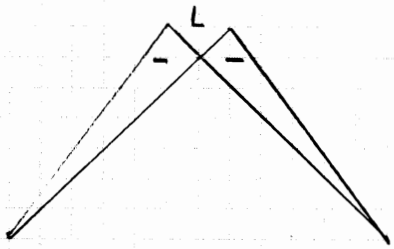
Aufgabe 1

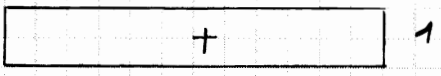
GS + ÜG



M_1 $(X_1 = 1)$



N_1 $(X_1 = 1)$



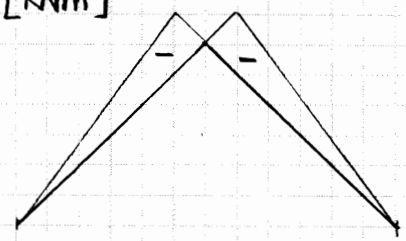
$$\delta_1 = \delta_{10} + X_1 \cdot \delta_{11}$$
$$\Delta = 0 + X_1 \cdot \delta_{11}$$

$$\delta_{11} = 2 \cdot \frac{\sqrt{2}\,L}{3} \cdot \frac{L}{EI} \cdot L + 2L \cdot \frac{1}{EA} \cdot 1 = \frac{2\sqrt{2}\,L^3}{3EI} + \frac{2L}{EA}$$

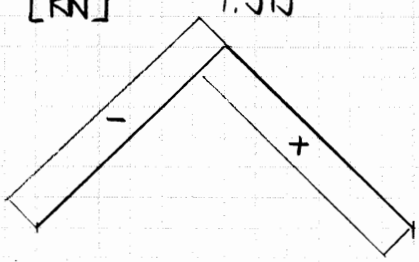
$$= \frac{2 \cdot \sqrt{2} \cdot 6000^3\,mm^3}{3 \cdot 205\,KN/mm^2 \cdot 135\,10^6\,mm^4} + \frac{2 \cdot 6000\,mm}{205\,KN/mm^2 \cdot 6000\,mm^2} = 7.368\ \frac{mm}{KN}$$

$$X_1 = \frac{\Delta}{\delta_{11}} = \frac{20\,mm}{7.368\,mm/KN} = \underline{2.714\,KN}\ \checkmark$$

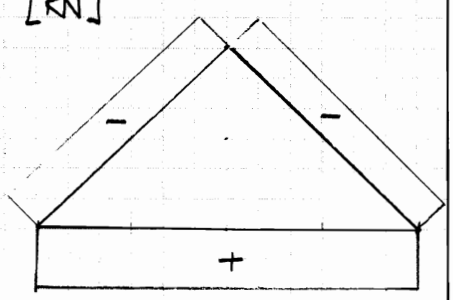
M [KNm]



V [KN]

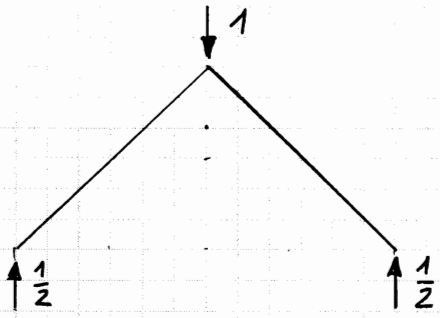


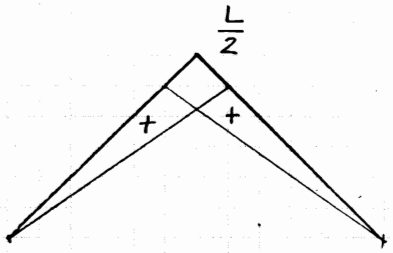
N [KN]



Vertikale Verschiebungskomponente in B

BZ 1



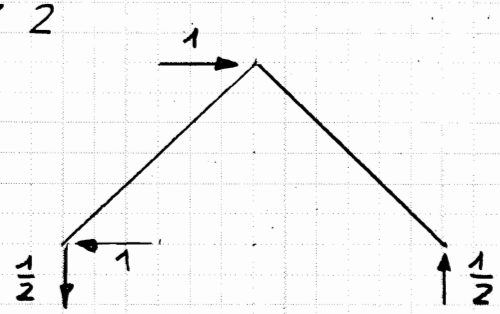


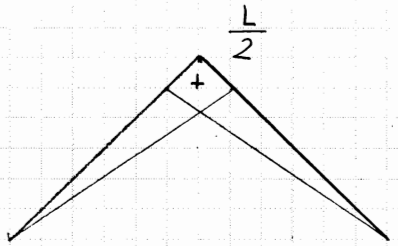
$$\delta_{B,V} = -2 \cdot \frac{\sqrt{2}\,L}{3} \cdot \frac{M(\text{Zwängung})}{EI} \cdot \frac{L}{2} = -\frac{\sqrt{2}\,L^2\,M}{3EI}$$

$$= -\frac{\sqrt{2}\cdot 6000^2\,\text{mm}^2 \cdot 16.286\,\text{kNm}}{3 \cdot 205\,\text{kN/mm}^2 \cdot 135\,10^6\,\text{mm}^4} = -9.987\,\text{mm}$$

Horizontale Verschiebungskomponente in B

BZ 2





$$\delta_{B,h} = -9.987\,\text{mm}$$

N.B Stab AC: $\varepsilon = \frac{2{,}714\,\text{kN}}{205\,\text{kN/mm}^2 \cdot 6000\,\text{mm}^2} = 0.0022\,‰$

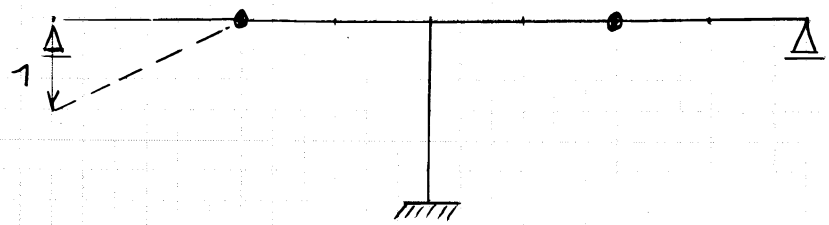
$$\Delta L = 2 \cdot 6000\,\text{mm} \cdot 0.0022\,‰ = 0.026\,\text{mm}$$

$$\delta_{C,h} = 20\,\text{mm} - 0.026\,\text{mm} = 19.974\,\text{mm}$$

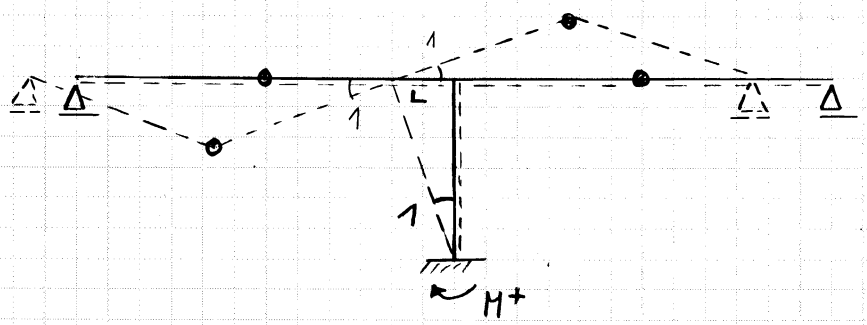
$$\delta_{B,h} = \frac{1}{2} \cdot 19.974\,\text{mm} = 9.987\,\text{mm}$$

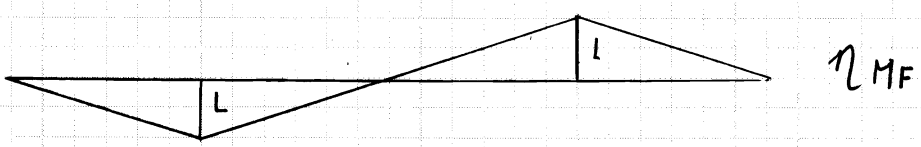
Aufgabe 2

a)

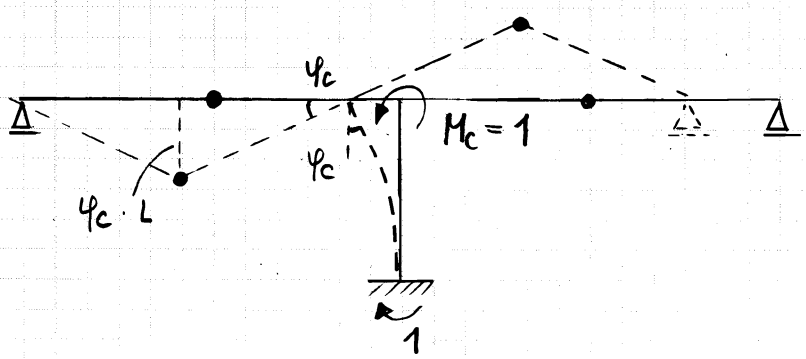


b)

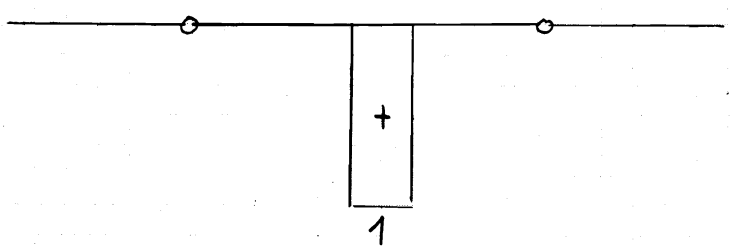




c)

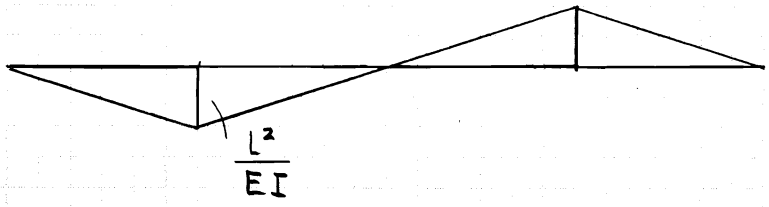


Maxwell
$M_c = 1$



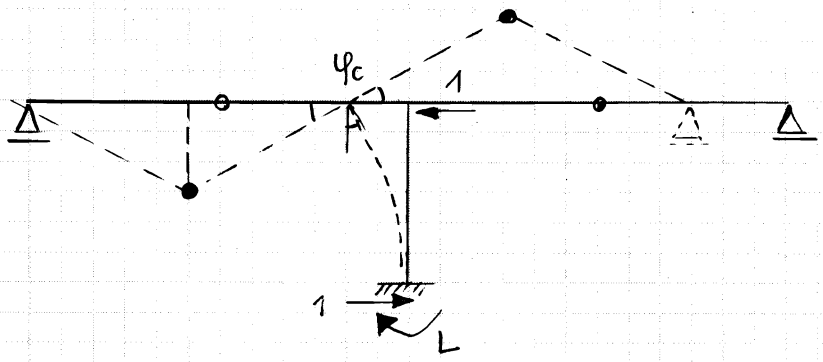
$M\ (M_c = 1)$

$$\varphi_c = L \cdot \frac{1}{EI} \cdot 1 = \frac{L}{EI}$$



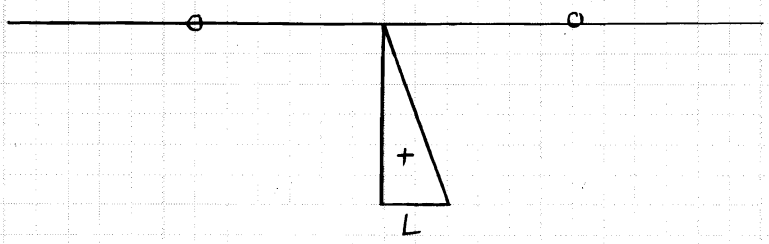
η_{φ_c} ↻ +

d)



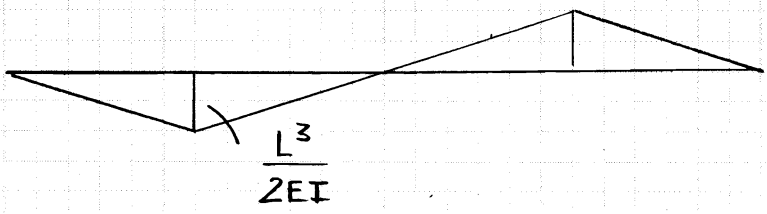
Maxwell

$H_c = 1$



$M(H_c = 1)$

$$\varphi_c = \frac{L}{2} \cdot \frac{L}{EI} \cdot 1 - \frac{L^2}{2EI}$$

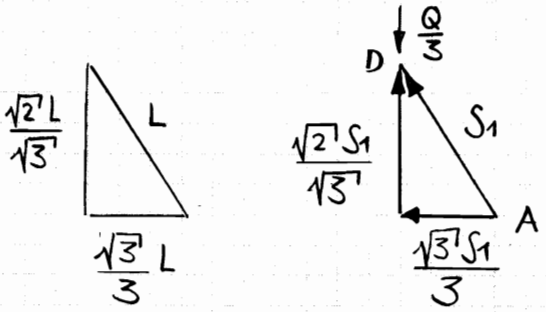


η_{u_c} ← +

Aufgabe 3

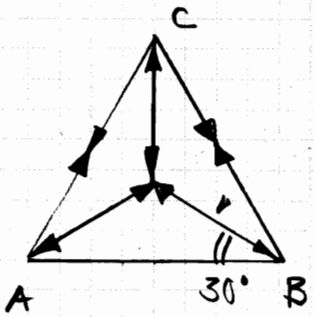
Auflagerreaktionen : $A_V = B_V = C_V = \underline{\underline{\frac{Q}{3}}}$

D : : Stabkräfte $S_1 = S_{DA} = S_{DB} = S_{DC}$



$\underline{\underline{S_1 = \frac{Q}{3} \cdot \sqrt{\frac{3}{2}}}}$ Druck ✓

ABC : Stabkräfte $S_2 = S_{AB} = S_{BC} = S_{CA}$

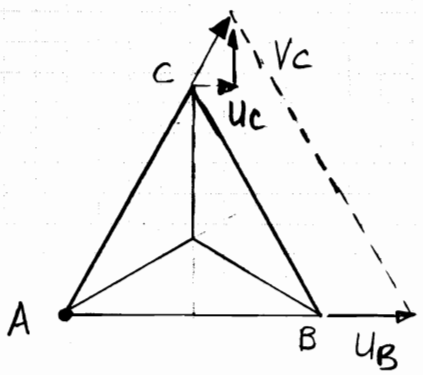


$S_{1h} = \frac{\sqrt{3}}{3} \cdot \frac{Q}{3} \cdot \sqrt{\frac{3}{2}}$

$2 \cdot S_2 \cos 30° = \frac{\sqrt{3}}{3} \cdot \frac{Q}{3} \cdot \sqrt{\frac{3}{2}}$

$S_2 = \frac{\sqrt{3}}{3} \cdot \frac{Q}{3} \cdot \sqrt{\frac{3}{2}} \cdot \frac{2}{\sqrt{3}} \cdot \frac{1}{2} = \underline{\underline{\frac{Q}{6} \cdot \sqrt{\frac{2}{3}}}}$ Zug ✓

Verschiebungen :



$u_B = \frac{S_2 \cdot L}{EA} = \underline{\underline{\frac{Q \cdot L}{6EA}\sqrt{\frac{2}{3}}}}$ ✓

$u_C = \underline{\underline{\frac{1}{2}\frac{Q \cdot L}{6EA}\sqrt{\frac{2}{3}}}}$

$v_C = \frac{\sqrt{3}}{2} \cdot \frac{Q \cdot L}{6EA}\sqrt{\frac{2}{3}}$

$= \underline{\underline{\frac{\sqrt{2}\,Q \cdot L}{12\,EA}}}$

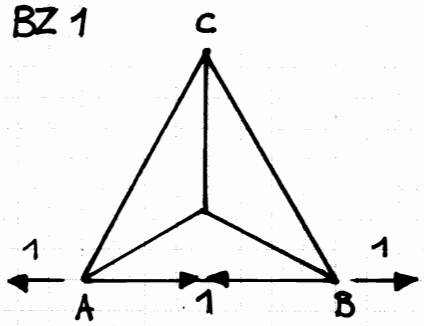
$\varepsilon(S_2) = \frac{S_2}{EA}$

$\Delta(S_2) = L \cdot \frac{S_2}{EA}$

Lösungsweg mit AG

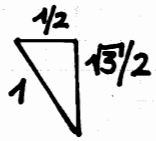
Verschiebung in B →

BZ 1

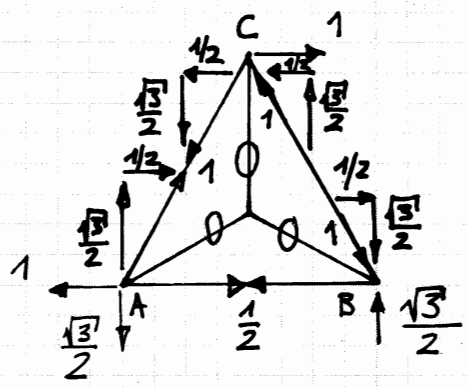


$$\delta_B = \frac{L}{EA} \cdot \frac{Q}{3\sqrt{6}} = \underline{\underline{\frac{QL}{3\sqrt{6}EA}}}$$

Verschiebung in C →



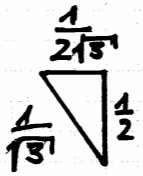
BZ 2



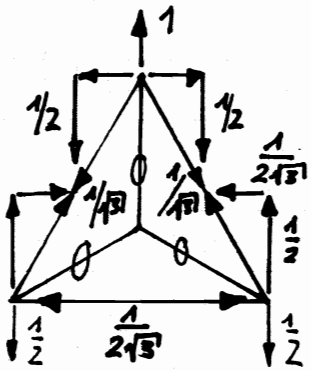
$$L \cdot \frac{\sqrt{3}}{2} \cdot 1 = B_{\updownarrow} \cdot L$$

$$\delta_C \rightarrow = \frac{L}{EA} \cdot \frac{Q}{3\sqrt{6}} \cdot \frac{1}{2} = \underline{\underline{\frac{QL}{6\sqrt{6}EA}}}$$

Verschiebung in C ↑



BZ 3



$$\delta_C \uparrow = \frac{L}{EA} \frac{Q}{3\sqrt{6}} \left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{3}} \right)$$

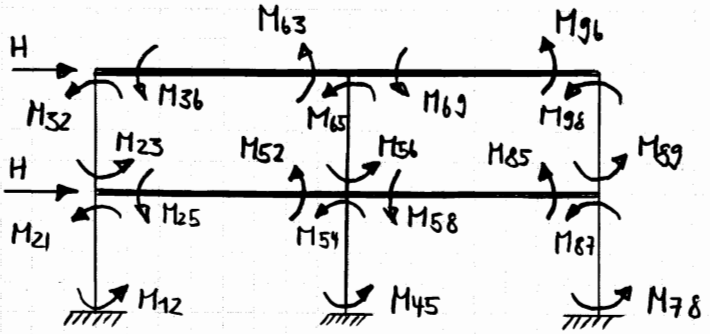
$$= \frac{QL}{3\sqrt{6}EA} \left(\frac{3}{2\sqrt{3}} \right) = \underline{\underline{\frac{\sqrt{3}QL}{6\sqrt{6}EA}}} = \underline{\underline{\frac{QL}{6\sqrt{2}EA}}}$$

Aufgabe 4a

$EI = 27675\ kNm^2$

$L = 4m$

Lösungsweg 1 : Drehwinkelverfahren



Knotenverdrehungen : keine
Stabdrehwinkel : ψ_1, ψ_2
Festeinspannm. : keine

$t_{ik} = \frac{2EI}{L}$

$S_{ik} = \frac{4EI}{L}$

Stabendmomente:

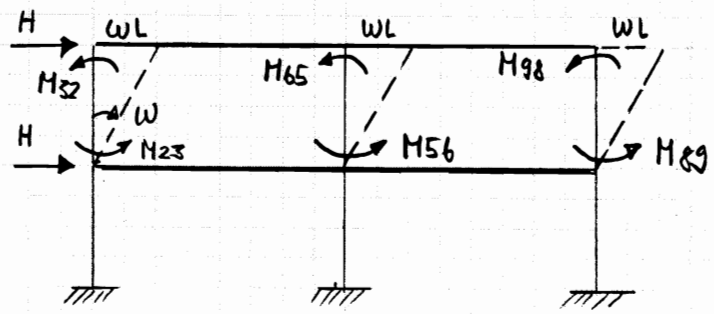
$M_{12} = M_{21} = M_{78} = M_{87} = -\frac{6EI}{L} \cdot \psi_1 = -41512.5\ kNm \cdot \psi_1$

$M_{23} = M_{32} = M_{89} = M_{98} = -\frac{6EI}{L} \cdot \psi_2 = -41512.5\ kNm \cdot \psi_2$

$M_{45} = M_{54} = -\frac{12EI}{L} \cdot \psi_1 = -83025\ kNm \cdot \psi_1$

$M_{56} = M_{65} = -\frac{12EI}{L} \psi_2 = -83025\ kNm \cdot \psi_2$

Verschiebemechanismus 1:



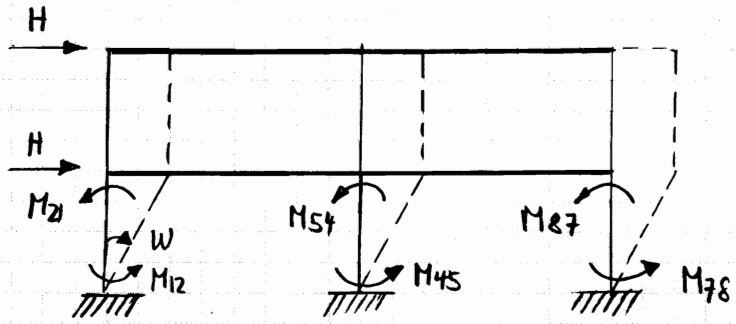
$A_i = -W \cdot (4 \cdot (-41512.5\ kNm \cdot \psi_2) + 2 \cdot (-83025\ kNm \cdot \psi_2))$

$= 332100\ kNm \cdot \psi_2 \cdot W$

$A_q = W \cdot L \cdot H = W \cdot 4m \cdot 100\ kN = 400\ kNm \cdot W$

$A_i + A_q = 0 \Rightarrow \psi_2 = -0.0012045$

Verschiebemechanismus 2:



$$A_i = -W\left(4\cdot(-41512{,}5\ \text{kNm}\cdot\psi_1)+2\cdot(-83025\ \text{kNm}\cdot\psi_1)\right)$$
$$= 332100\ \text{kNm}\cdot\psi_1\cdot W$$

$$A_q = W\cdot L\cdot 2H = W\cdot 4\text{m}\cdot 2\cdot 100\ \text{kN} = 800\ \text{kNm}\cdot W$$

$$A_i + A_q = 0 \Rightarrow \psi_1 = -0{,}0024089$$

Stabendmomente Stützen

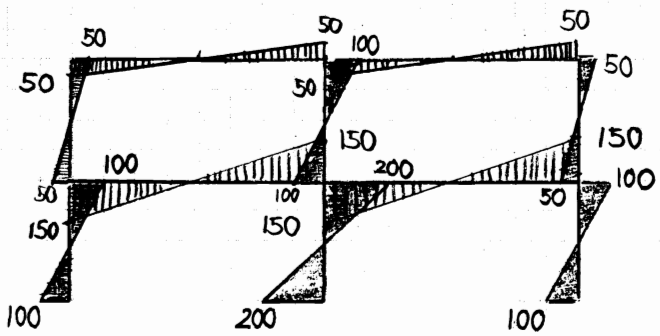
$M_{12} = M_{21} = M_{78} = M_{87} = -41512{,}5\ \text{kNm}\cdot(-0{,}0024089) = 100\ \text{kNm}$

$M_{23} = M_{32} = M_{89} = M_{98} = -41512{,}5\ \text{kNm}\cdot(-0{,}0012045) = 50\ \text{kNm}$

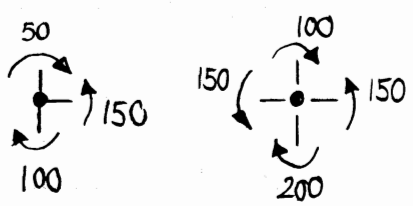
$M_{45} = M_{54} = -83025\ \text{kNm}\cdot(-0{,}0024089) = 200\ \text{kNm}$

$M_{56} = M_{65} = -83025\ \text{kNm}\cdot(-0{,}0012045) = 100\ \text{kNm}$

Riegelmomente mit Knotengleichgewicht



M [kNm]



Stockwerksverschiebungen

$u_1 = \psi_1 \cdot L = 0.0024089 \cdot 4m = 9.6mm$

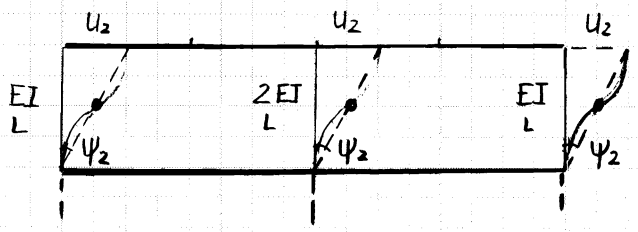
$u_2 = \psi_2 \cdot L = 0.0012045 \cdot 4m = 4.8mm$

1. Stockwerk: $u = 9.6mm$; 2. Stockwerk: $u = 14.5\ mm$

Lösungsweg 2

2. Stockwerk

U



$\varphi = 0, \quad M_{ik}^{\circ} = 0$

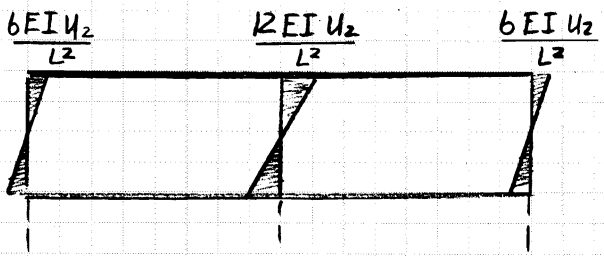
$M_{ik} = -(s_{ik} + t_{ik}) \cdot \psi_2$

$s_{ik} = \frac{4EI}{L}, \quad t_{ik} = \frac{2EI}{L}$

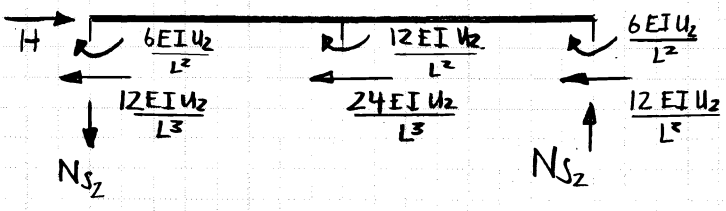
$\psi_2 = -\frac{u_2}{L}$

$M_{ik} = M_{ki} = \frac{6EIu_2}{L^2}$

M



SKD

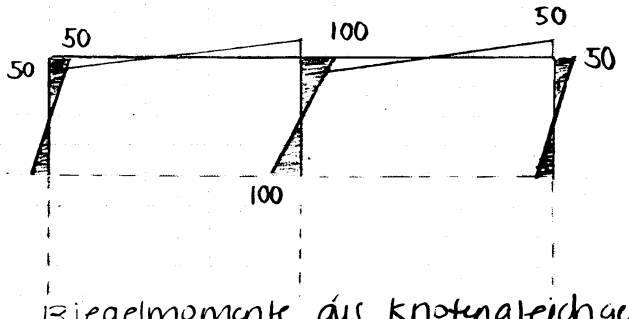


$\Sigma \rightarrow = 0 \qquad H = \frac{48EIu_2}{L^3}$

$\Sigma M = 0 \qquad N_{S_z} = \frac{6EIu_2}{L^3}$

$$u_2 = \frac{H \cdot L^3}{48EI} = \frac{100\,kN \cdot 4^3 m^3}{48 \cdot 27675\,kNm^2} = 4.8mm$$

M [kNm]



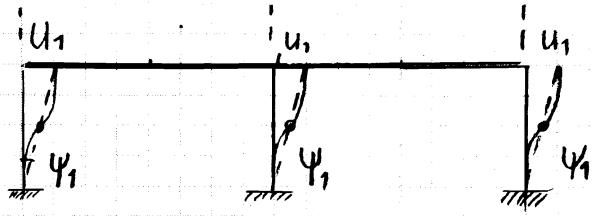
$\frac{6EIu_2}{L^2} = 50\ kNm$

$\frac{12EIu_2}{L^2} = 100\ kNm$

Riegelmomente aus Knotengleichgewicht

1. Stockwerk

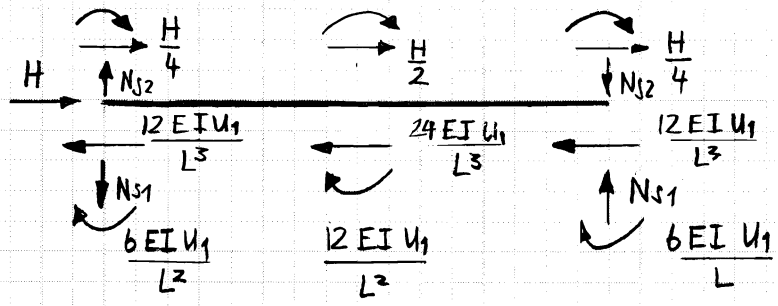
U



$M_{ik} = M_{ki} = \frac{6\,EI\,u_1}{L^2}$

analog zu Stockwerk 2

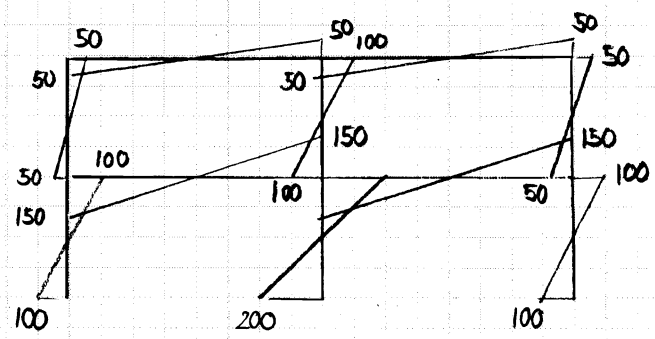
SKD



$\Sigma \rightarrow = 0 \cdot H = \frac{24\,EI\,u_1}{L^3}$

$u_1 = \frac{H \cdot L^3}{24\,EI} = \frac{100\ \text{kN}\ 4^3\,\text{m}^3}{24 \cdot 27675\ \text{kN m}^2} = 9.6\text{mm}$

M [kNm]



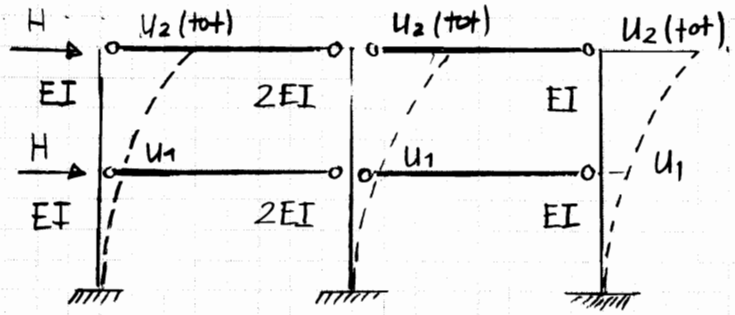
$\frac{6\,EI\,u_1}{L^2} = 100\ \text{kNm}$

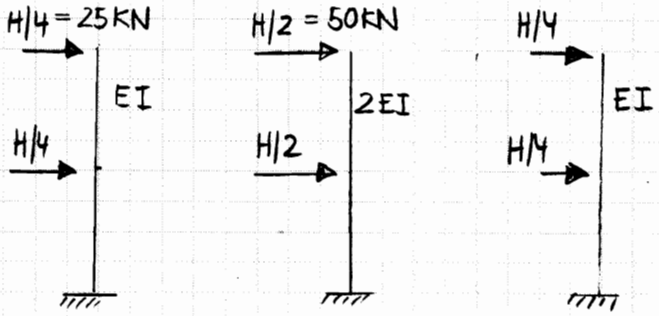
$\frac{12\,EI\,u_1}{L^2} = 200\ \text{kNm}$

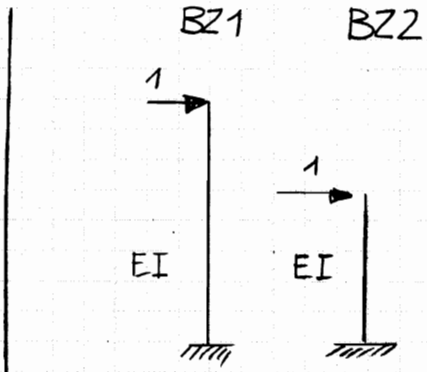
u_1 (tot) 1. Stock = u_1 = 9.6mm

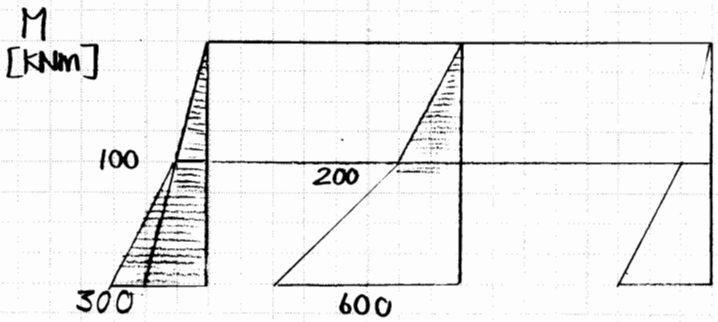
u_2 (tot) 2. Stock = $u_1 + u_2$ = 14.4mm

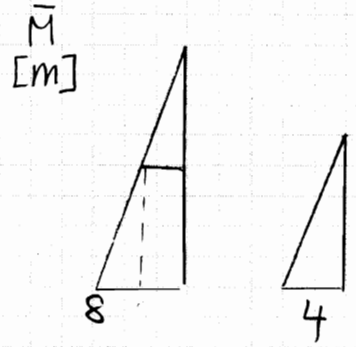
Aufgabe 4b



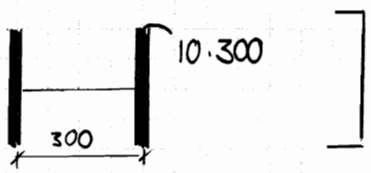








$$\sigma_{max} = \frac{300\,KNm}{135\,10^6\,mm^4} \cdot 150\,mm = 333\,N/mm^2$$



$$w_2(tot) = \frac{8\,m}{3} \cdot \frac{200\,KNm \cdot 8\,m}{27675\,KNm^2} + \frac{4m}{3} \cdot \frac{100\,KNm \cdot 4m}{27675\,KNm^2} + \frac{4m}{2} \cdot \frac{100\,KNm\ 4m}{27675\,KNm^2}$$

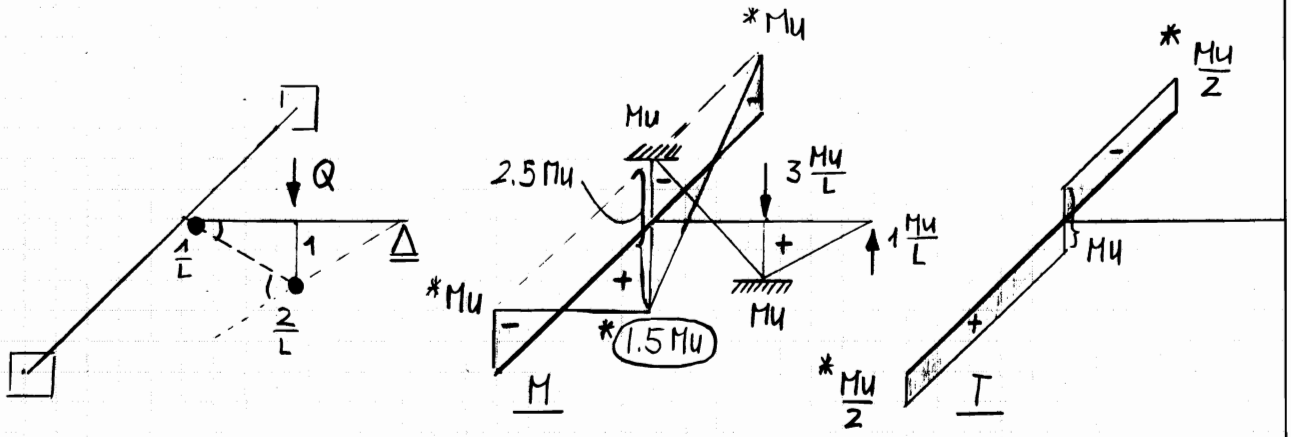
$$= 154.2\,mm + 19.3mm + 28.9mm = \underline{202\,mm}$$

$$w_1(tot) = \frac{4m}{3} \cdot \frac{200\,KNm \cdot 4m}{27675\,KNm^2} + \frac{4m}{2} \cdot \frac{100\,KNm \cdot 4m}{27675\,KNm^2}$$

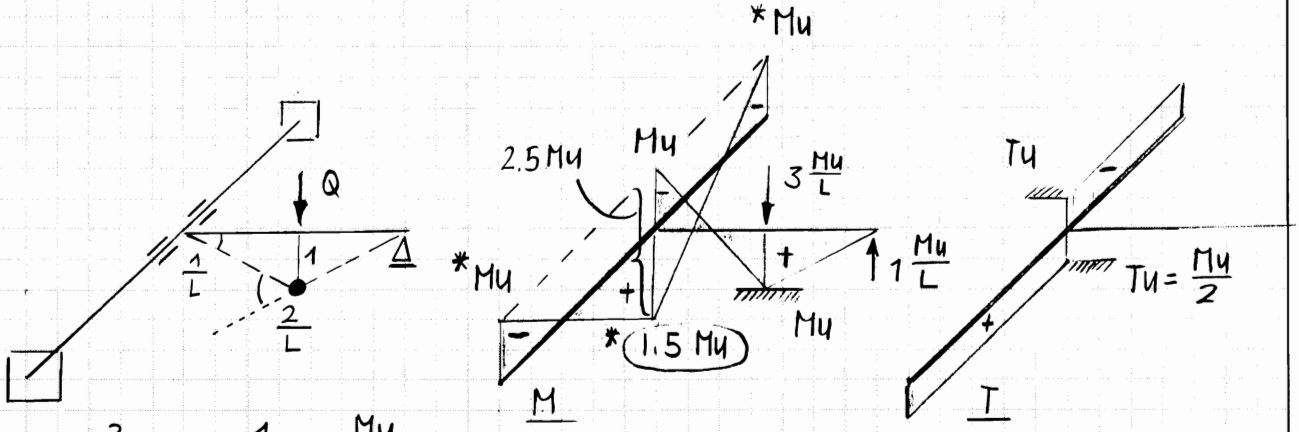
$$= 38.5\,mm + 28.9\,mm = \underline{67\,mm}$$

Aufgabe 5

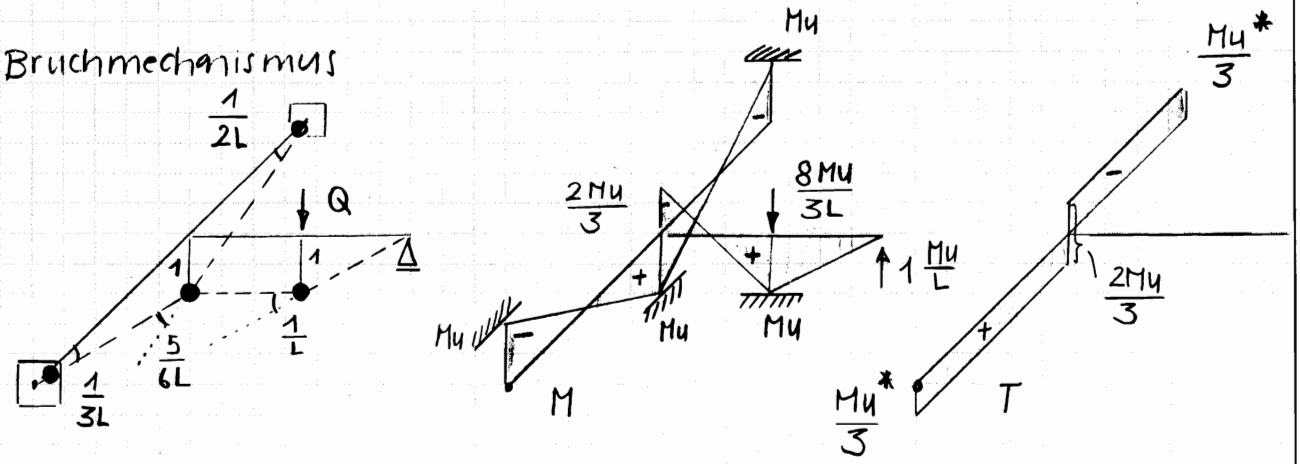
* Plast. Verteilung

① 

$$Qu = Mu\left(\frac{1}{L} + \frac{2}{L}\right) = 3\frac{Mu}{L} \qquad \frac{QL}{4} = \frac{2Mu}{L}\cdot\frac{5L}{4} = 2.5Mu$$

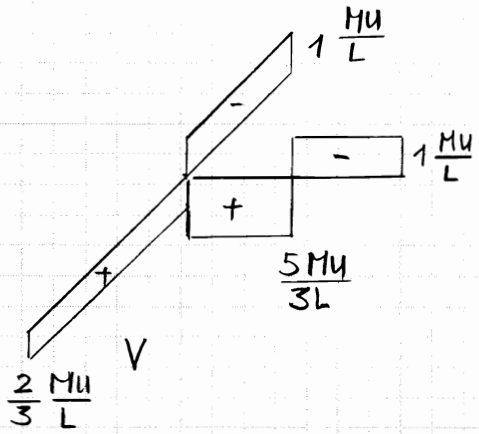
② 

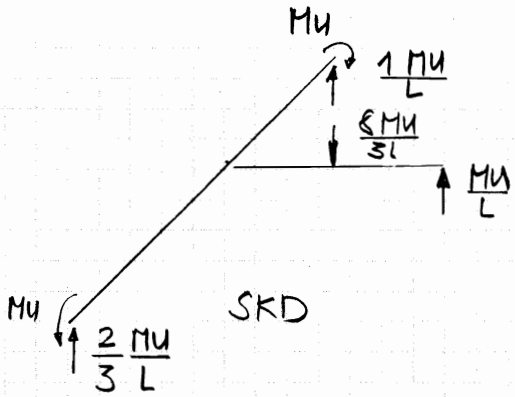
$$Qu = Mu\frac{2}{L} + 2Tu\frac{1}{L} = 3\frac{Mu}{L}$$

③ Bruchmechanismus 

$$Qu = Mu\left(\frac{1}{3L} + \frac{5}{6L} + \frac{1}{2L} + \frac{1}{L}\right) = \frac{8Mu}{3L}$$

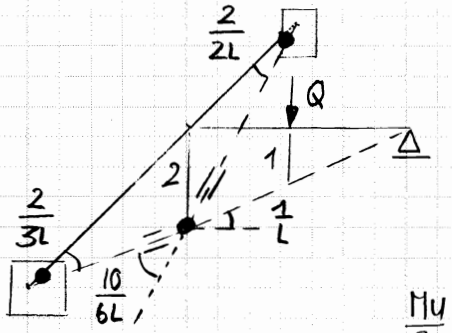
Traglast





N.B Zwei weitere Mechanismen

(4)

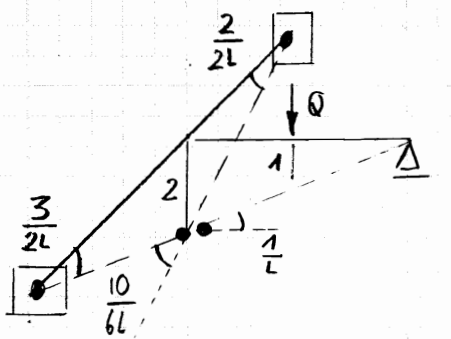


Mu/2

$$Qu = Mu\left(\frac{2}{3L} + \frac{2}{2L} + \frac{10}{6L}\right) + 2\left(\frac{Mu}{2}\right)\left(\frac{1}{L}\right)$$

$$= Mu\left(\frac{4}{6L} + \frac{6}{6L} + \frac{10}{6L} + \frac{6}{6L}\right) = \underline{\frac{13}{3}\frac{Mu}{L}}$$

(5)

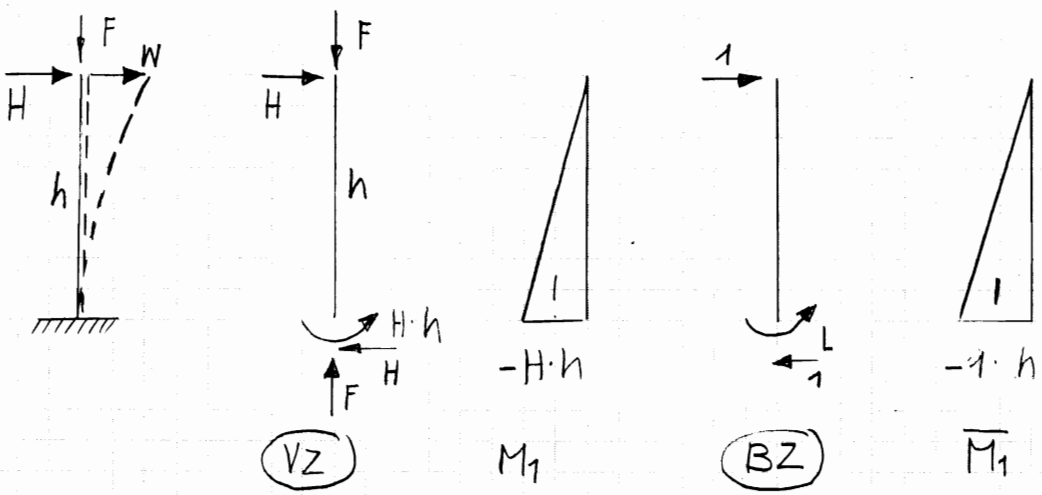


$$Qu = Mu\left(\frac{20}{6L} + \frac{1}{L}\right) = \underline{\frac{13}{3}\frac{Mu}{L}}$$

Aufgabe 6

$$EI = 205 \frac{kN}{mm^2} \cdot 270 \cdot 10^6 mm^4 = 55350 kNm^2$$

1. Ordnung $M_1, W_1(M_1)$



$$W_1 = \frac{h}{3} \cdot \frac{H \cdot h}{EI} \cdot h = \frac{H \cdot h^3}{3EI} = \frac{50 kN \cdot 6^3 m^3}{3 \cdot 55350 kNm^2} = \underline{65 mm}$$

$$M_1 \text{ (Einspannung)} = H \cdot h = 50 kN \cdot 6 m = \underline{-300 kNm}$$

2. Ordnung

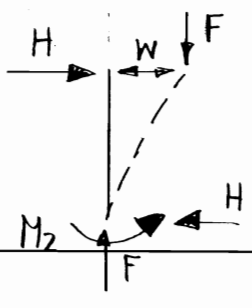
Knicklänge: $l_K = 2 \cdot h = 2 \cdot 6m = \underline{12 m}$

Eulersche Knicklast: $Q_E = \frac{\pi^2 EI}{l_K^2} = \frac{\pi^2 \cdot 55350 kNm^2}{12^2 m^2} = \underline{3794 kN}$

Vergrösserungsfaktor, Vianello: $\frac{1}{1 - \frac{Q}{Q_E}} = \frac{1}{1 - \frac{1200}{3794}} = \underline{1.46}$

Stützkopfauslenkung $W = 1.46 \cdot 65 mm = \underline{\underline{95 mm}}$

$$M_2 \text{ (Einspannung)} = M_1 + F \cdot W = -300 kNm - 1200 kN \cdot 0.095 m$$
$$= -300 kNm - 114 kNm$$
$$= \underline{-414 kNm}$$



3 Spannungen Stützenfuss

$M_2 = -414\ kNm$
$N = -1200\ kN$
$V = 50\ kN$

$$\left.\begin{array}{l}\\\\\end{array}\right\} \sigma = \frac{N}{A} + \frac{My}{Iy} \cdot z$$

$$\sigma(z=-150\,mm) = \frac{-1200\ kN}{12000\ mm^2} + \frac{-414\ kNm}{270 \cdot 10^6\ mm^4} \cdot -150\ mm$$

$$= -100\ N/mm^2 + 230\ N/mm^2 = \underline{\underline{130\ N/mm^2}}$$

$$\sigma(z=+150\,mm) = -100\ N/mm^2 - 230\ N/mm^2 = \underline{\underline{-330\ N/mm^2}}$$

N.B **Querschnitt** [mm]