Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Prof. Dr. Peter Marti
Institut für Baustatik und Konstruktion
D-BAUG, Studiengang Bauingenieurwissenschaften
Herbstsemester

BAUSTATIK I – KOLLOQUIUM 8, Lösung

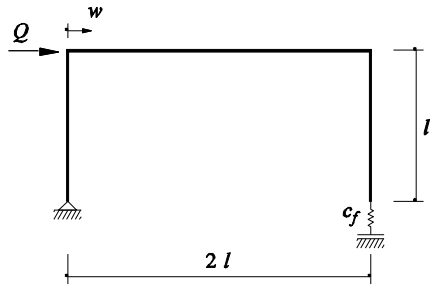
(101-0113)

Thema: Arbeitsgleichung

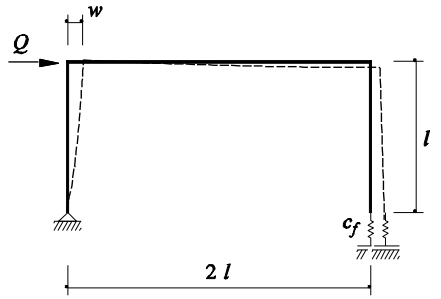
Aufgabe 1, Lösung

Gegeben: System und Einwirkung Q

Gesucht: Verformung w

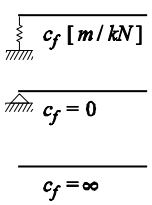


Qualitative Verformungslinie

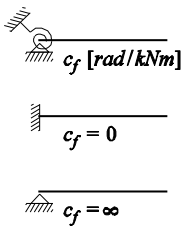


Merke:

c_f [m/kN]:
Einsenkung pro Einheitskraft

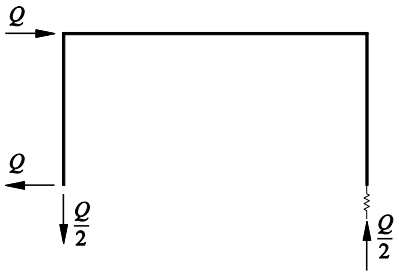


c_f [rad/kNm]:
Verdrehung pro Einheitsmoment

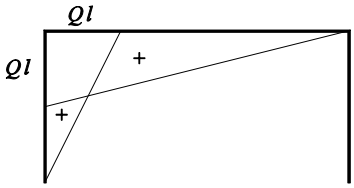


VZ:

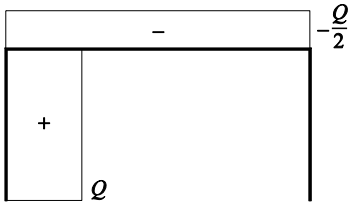
SKD:



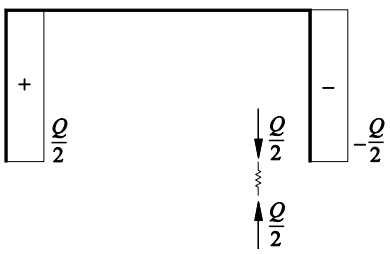
M:



V:

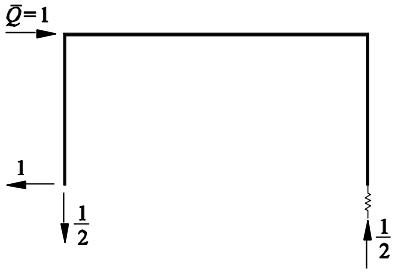


N:

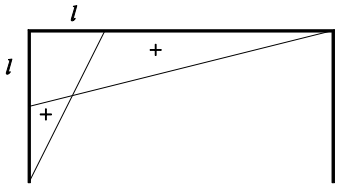


BZ:

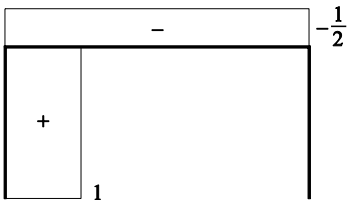
SKD:



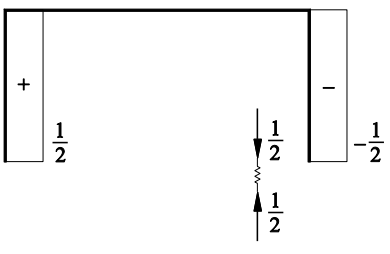
$\overline{M}$:



$\overline{V}$:



$\overline{N}$:



$$w = \int \overline{M} \cdot \frac{M}{EI} \cdot \mathrm{d}x + \int \overline{V} \cdot \frac{V}{GA^*} \cdot \mathrm{d}x + \int \overline{N} \cdot \frac{N}{EA} \cdot \mathrm{d}x + \overline{F} \cdot F \cdot c_f$$

$$w = \frac{1}{3} \cdot l \cdot \frac{Ql}{EI} \cdot l + \frac{1}{3} \cdot l \cdot \frac{Ql}{EI} \cdot 2l + 1 \cdot \frac{Q}{GA^*} \cdot l + \left(-\frac{1}{2}\right)\left(-\frac{Q}{2GA^*}\right) \cdot 2l + \frac{1}{2} \cdot \frac{Q}{2EA} \cdot l + \left(-\frac{1}{2}\right)\left(-\frac{Q}{2EA}\right) \cdot l + \frac{1}{2} \cdot \frac{Q}{2} \cdot c_f$$

$$\boxed{w = \frac{Ql^3}{EI} + \frac{3Ql}{2GA^*} + \frac{Ql}{2EA} + \frac{Q}{4} \cdot c_f}$$

Aufgabe 2, Lösung

Gegeben: System und Einwirkung:

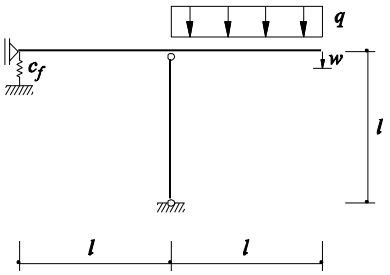
$L = 6$ m; $q = 2$ kN/m; $\Delta T_1 = \Delta T = 5°\mathrm{C}$; $\Delta T_2 = T_u - T_o = -10°\mathrm{C}$; $\alpha_T = 10^{-5}$ /°C

(ΔT wirkt am gesamten System; T_o und T_u wirken nur auf den horizontalen Träger)

HEA 260: $I = 105.4 \cdot 10^6$ mm^4; $A = 8680$ mm^2; $A^* = A_w = 1735$ mm^2; $h = 250$ mm

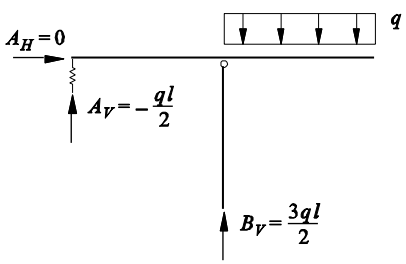
$E = 210$ kN/mm^2; $G = 81$ kN/mm^2; $c_f = 1$ mm/kN

Gesucht: Verschiebung *w*



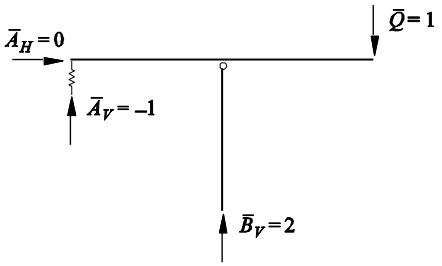
VZ:

SKD:

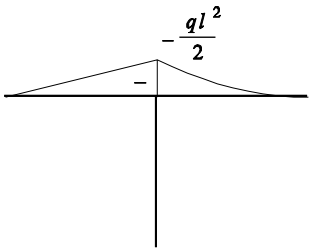


BZ:

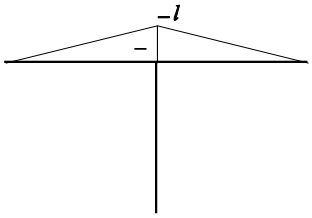
SKD:



M:



$\overline{M}$:



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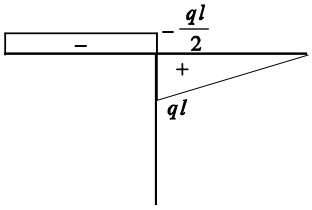
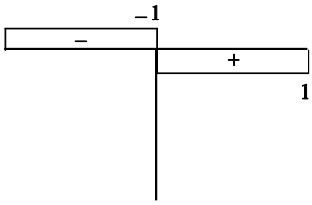
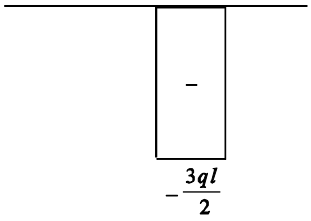


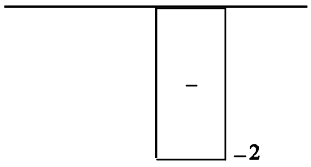
$\overline{V}$:



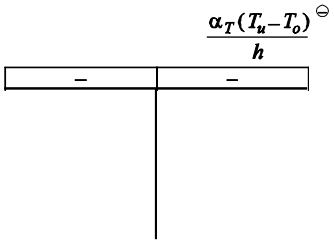
N:



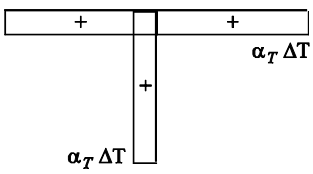
$\overline{N}$:



$\frac{\alpha_T \cdot (T_u - T_o)}{h}$:



$\alpha_T \cdot \Delta T$:



$$w = \int \overline{M} \cdot \frac{M}{EI} \cdot \mathrm{d}x + \int \overline{V} \cdot \frac{V}{GA^*} \cdot \mathrm{d}x + \int \overline{N} \cdot \frac{N}{EA} \cdot \mathrm{d}x + \int \overline{M} \cdot \frac{\alpha_T \cdot (T_u - T_o)}{h} \cdot \mathrm{d}x + \int \overline{N} \cdot \alpha_T \cdot \Delta T \cdot \mathrm{d}x + \overline{A_v} \cdot A_v \cdot c_f$$

$$w = \frac{1}{3} \cdot (-l) \cdot \left(-\frac{ql^2}{2EI} \right) \cdot l + \frac{1}{4} \cdot (-l) \cdot \left(-\frac{ql^2}{2EI} \right) \cdot l + (-1) \cdot \left(-\frac{ql}{2GA^*} \right) \cdot l + \frac{1}{2} \cdot 1 \cdot \frac{ql}{GA^*} \cdot l$$

$$+ (-2) \cdot \left(-\frac{3ql}{2EA} \right) \cdot l + 2 \cdot \frac{1}{2} \cdot (-l) \cdot \frac{\alpha_T \cdot (T_u - T_o)^{(-)}}{h} \cdot l + (-2) \cdot \alpha_T \cdot \Delta T \cdot l + (-1) \cdot \left(-\frac{ql}{2} \right) \cdot c_f$$

$$\boxed{w = \frac{7ql^4}{24EI} + \frac{ql^2}{GA^*} + \frac{3ql^2}{EA} - \frac{\alpha_T \cdot (T_u - T_o)^{(-)} \cdot l^2}{h} - 2 \cdot \alpha_T \cdot \Delta T \cdot l + \frac{ql}{2} \cdot c_f}$$

Merke: $T_u - T_o$ ist für $T_o > T_u$ negativ: (–)

$$w = \frac{7 \cdot 2 \cdot 10^{-3} \cdot (6 \cdot 10^3)^4}{24 \cdot 210 \cdot 105.4 \cdot 10^6} + \frac{2 \cdot 10^{-3} \cdot (6 \cdot 10^3)^2}{81 \cdot 1735} + \frac{3 \cdot 2 \cdot 10^{-3} \cdot (6 \cdot 10^3)^2}{210 \cdot 8680} - \frac{10^{-5} \cdot (-10) \cdot (6 \cdot 10^3)^2}{250}$$

$$-2 \cdot 10^{-5} \cdot 5 \cdot 6 \cdot 10^3 + \frac{2 \cdot 10^{-3} \cdot 6 \cdot 10^3 \cdot 1}{2}$$

$$w = w(M) + w(V) + w(N) + w(T_u - T_o) + w(\Delta T) + w(c_f)$$

$$\boxed{w = 34.16 \quad +0.51 \quad +0.12 \qquad +14.40 \qquad -0.60 \quad +6.00 = 54.59 \text{ mm}}$$

→ Der Einfluss der Verformungsanteile infolge V und N ist im Allgemeinen sehr gering und deshalb vernachlässigbar