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Herbstsemester

BAUSTATIK I – KOLLOQUIUM 7, Lösung

(101-0113)

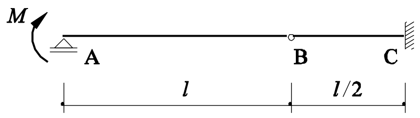
Thema: Mohrsche Analogie

Aufgabe und Lösung

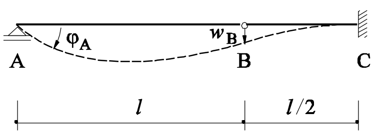
Gegeben: System und Einwirkung

Gesucht: a) Rotation φ_A im Punkt A

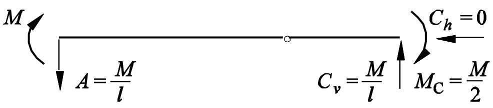
b) Durchbiegung w_B im Punkt B



Qualitative Verformungslinie des gegebenen Trägers infolge der Einwirkung *M*:



SKD:



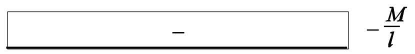
$$\sum F_v = 0: \qquad -A + C_v = 0$$

$$\sum M(\mathrm{C}) = 0: \qquad M_\mathrm{C} - A \cdot \frac{3l}{2} + M = 0$$

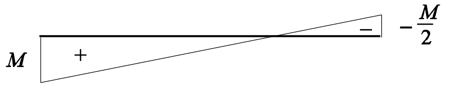
$$M(\mathrm{B})_{(\mathrm{links})} = 0: \qquad M - A \cdot l = 0$$

$$\rightarrow \quad \boxed{A = \frac{M}{l}, \quad C_v = \frac{M}{l}, \quad M_\mathrm{C} = \frac{M}{2}}$$

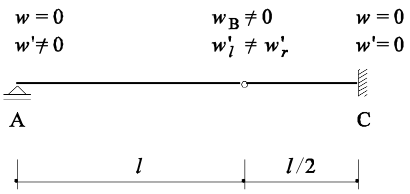
$V = M'$:



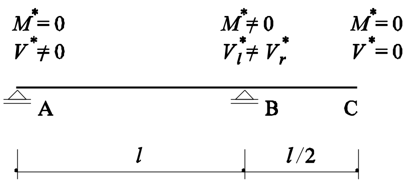
M :



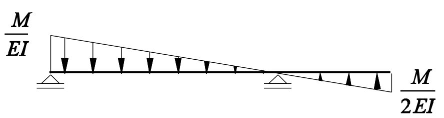
Gegebener Träger:



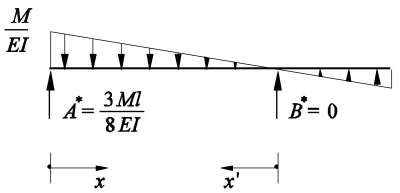
Analogieträger:



Analogieträger mit Belastung $q^* = \dfrac{M}{EI}$:



SKD:

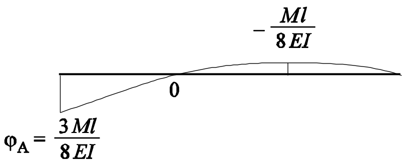


$$\sum F_v = 0: \qquad A^* + B^* = \frac{1}{2}\cdot\frac{M}{EI}\cdot l - \frac{1}{2}\cdot\frac{M}{2EI}\cdot\frac{l}{2} = \frac{3Ml}{8EI}$$

$$\sum M(\mathrm{A}) = 0: \quad B^* \cdot L - \frac{1}{2}\cdot\frac{M}{EI}\cdot l\cdot\frac{l}{3} + \frac{1}{2}\cdot\frac{M}{2EI}\cdot\frac{l}{2}\cdot\left(l + \frac{2}{3}\cdot\frac{l}{2}\right) = 0$$

$$\rightarrow \quad \boxed{\begin{aligned} B^* &= 0 \\ A^* &= \frac{3Ml}{8EI}\end{aligned}}$$

$V^* = M^{*\prime} = w' = \varphi$ (= Neigung der Biegelinie):

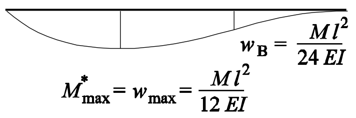


$$V^*(x=0) = \boxed{\frac{3Ml}{8EI} = \frac{dw}{dx}(x=0) = \varphi_\mathrm{A}}$$

$$V^*(x=\frac{l}{2}) = \frac{3Ml}{8EI} - \frac{1}{2}\cdot\left(\frac{M}{EI} + \frac{M}{2EI}\right)\cdot\frac{l}{2} = 0$$

$$V^*(x=l) = \frac{3Ml}{8EI} - \frac{1}{2}\cdot\frac{M}{EI}\cdot l = -\frac{Ml}{8EI}$$

$M^* = w$ (= Durchbiegung):



$$M^*(x=0) \; = 0$$

$$M^*(x=\frac{l}{2}) = M^*(x'=\frac{l}{2}) \qquad (B^* = 0!)$$

$$= \frac{1}{2}\cdot\frac{M}{2EI}\cdot\frac{l}{2}\cdot\left(\frac{l}{2} + \frac{2}{3}\cdot\frac{l}{2}\right) - \frac{1}{2}\cdot\frac{M}{2EI}\cdot\frac{l}{2}\cdot\frac{1}{3}\cdot\frac{l}{2}$$

$$= \frac{Ml^2}{12EI} = w_{max}$$

$$M^*(x=l) = M^*(x'=0)$$

$$= \frac{1}{2}\cdot\frac{M}{2EI}\cdot\frac{l}{2}\cdot\frac{2}{3}\cdot\frac{l}{2} = \boxed{\frac{Ml^2}{24EI} = w_\mathrm{B}}$$

N.B. Kein Knick in M-Linie, da $B^* = 0$

Kontrolle: $M^*_{max} = w_{max} \quad \leftrightarrow \quad V^* = 0$:

$$\frac{3Ml}{8EI} - \frac{1}{2}\cdot\left(\frac{M}{EI} + \frac{M}{EI}\cdot\frac{x'}{l}\right)\cdot\left(l - x'\right) = 0$$

$$\frac{3Ml}{8EI} - \frac{M}{2EI\cdot l}\cdot\left(l + x'\right)\cdot\left(l - x'\right) = 0$$

$$\frac{3l}{8} - \frac{l^2 - x'^2}{2l} = 0 = -\frac{l}{8} + \frac{x'^2}{2l} \rightarrow \boxed{x' = \frac{l}{2} \rightarrow w_{max} = \frac{Ml^2}{12EI}}$$