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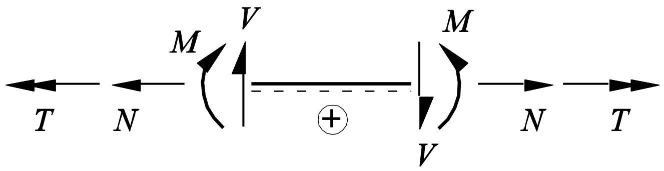
Herbstsemester

BAUSTATIK I – KOLLOQUIUM 2, Lösung

(101-0113)

Thema: Reaktionen und Schnittgrössen

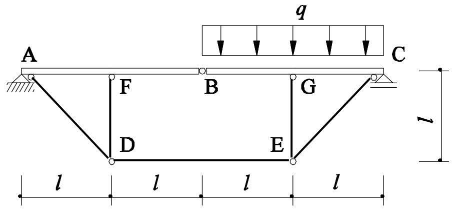
Vorzeichenkonvention:



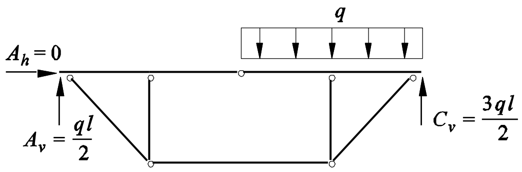
Aufgabe 1, Lösung

Gegeben: System und Einwirkung

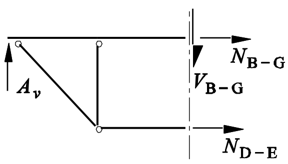
Gesucht: Reaktionen (SKD) und Schnittgrössen (N, V, M)



SKD:



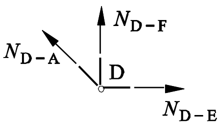
$$\sum F_v = 0 \quad \rightarrow \quad A_v + C_v - 2ql = 0$$
$$\sum F_h = 0 \quad \rightarrow \quad A_h = 0$$
$$\sum M(\mathrm{A}) = 0 \quad \rightarrow \quad C_v \cdot 4l - 2ql \cdot 3l = 0$$
$$M_\mathrm{B} = 0 \quad \rightarrow \quad A_v \cdot 2l - N_\text{D-E} \cdot l = 0$$



$$\rightarrow \quad \boxed{\begin{aligned} C_v &= \frac{3ql}{2} \\ A_v &= \frac{ql}{2} \\ A_h &= 0 \\ N_\text{D-E} &= ql \end{aligned}}$$

Ermittlung der Schnittgrössen:

Knoten D:



$$N_{\text{D-E}} = N_{\text{E-D}} = ql \qquad \text{(siehe oben)}$$

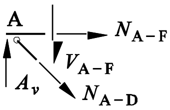
$$\sum F_h = 0 \;\rightarrow\; \frac{N_{\text{D-A}}}{\sqrt{2}} - N_{\text{D-E}} = 0$$

$$\rightarrow\; N_{\text{D-A}} = N_{\text{D-E}} \cdot \sqrt{2} = \underline{\underline{ql \cdot \sqrt{2}}}$$

$$\sum F_v = 0 \;\rightarrow\; \frac{N_{\text{D-A}}}{\sqrt{2}} + N_{\text{D-F}} = 0$$

$$\rightarrow\; N_{\text{D-F}} = -\frac{N_{\text{D-A}}}{\sqrt{2}} = \underline{\underline{-ql}}$$

Knoten A:



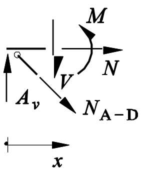
$$\sum F_h = 0 \;\rightarrow\; N_{\text{A-F}} + \frac{N_{\text{A-D}}}{\sqrt{2}} = 0$$

$$\rightarrow\; N_{\text{A-F}} = -\frac{N_{\text{A-D}}}{\sqrt{2}} = -\frac{N_{\text{D-A}}}{\sqrt{2}} = \underline{\underline{-ql}}$$

$$\sum F_v = 0 \;\rightarrow\; A_v - \frac{N_{\text{A-D}}}{\sqrt{2}} - V_{\text{A-F}} = 0$$

$$\rightarrow\; V_{\text{A-F}} = A_v - \frac{N_{\text{A-D}}}{\sqrt{2}} = \frac{ql}{2} - ql = \underline{\underline{-\frac{ql}{2}}}$$

Stab A – F:



$$\sum F_h = 0 \;\rightarrow\; N + \frac{N_{\text{A-D}}}{\sqrt{2}} = 0$$

$$\rightarrow\; N = -\frac{N_{\text{A-D}}}{\sqrt{2}} = \underline{\underline{-ql}}$$

$$\sum F_v = 0 \;\rightarrow\; A_v - \frac{N_{\text{A-D}}}{\sqrt{2}} - V = 0$$

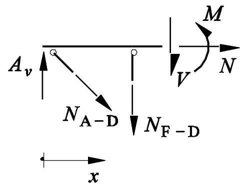
$$\rightarrow\; V = A_v - \frac{N_{\text{A-D}}}{\sqrt{2}} = \frac{ql}{2} - ql = \underline{\underline{-\frac{ql}{2}}}$$

$$\sum M = 0 \;\rightarrow\; M = A_v \cdot x - \frac{N_{\text{A-D}}}{\sqrt{2}} \cdot x$$

$$\rightarrow\; M(x=0) = M_{\text{A-F}} = \underline{\underline{0}}$$

$$\rightarrow\; M(x=l) = M_{\text{F-A}} = \frac{ql}{2} \cdot l - ql \cdot l = \underline{\underline{-\frac{ql^2}{2}}}$$

Stab F – B:



$$\sum F_h = 0 \;\rightarrow\; N + \frac{N_{\text{A-D}}}{\sqrt{2}} = 0$$

$$\rightarrow\; N = -\frac{N_{\text{A-D}}}{\sqrt{2}} = \underline{\underline{-ql}}$$

$$\sum F_v = 0 \rightarrow\; A_v - \frac{N_{\text{A-D}}}{\sqrt{2}} - N_{\text{F-D}} - V = 0$$

$$\rightarrow\; V = A_v - \frac{N_{\text{A-D}}}{\sqrt{2}} - N_{\text{F-D}}$$

$$= \frac{ql}{2} - ql + ql = \underline{\underline{\frac{ql}{2}}}$$

$$\sum M = 0 \;\rightarrow\; M = A_v \cdot x - \frac{N_{\text{A-D}}}{\sqrt{2}} \cdot x - N_{\text{F-D}} \cdot (x - l)$$

$$= \frac{ql}{2} \cdot x - \frac{ql\sqrt{2}}{\sqrt{2}} \cdot x + ql \cdot (x - l)$$

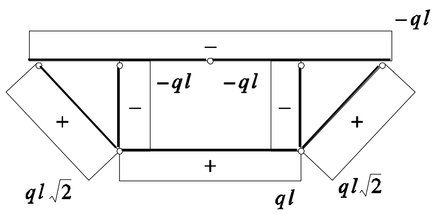
$$\rightarrow\; M(x = l) = M_{\text{F-B}} = \underline{\underline{-\frac{ql^2}{2}}}$$

$$\rightarrow\; M(x = 2l) = M_{\text{B-F}} = \underline{\underline{0}}$$

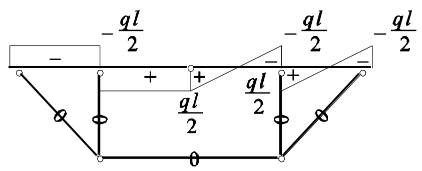
(Kontrolle: im Gelenk muss $M = 0$ sein!)

→ Analoges Vorgehen für die übrigen Stäbe.

N:



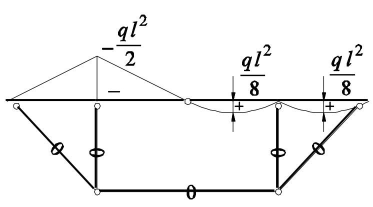
V:



Merke:

- Bei F und G: Einzelkraft aus vertikalen Stäben:
  → Knick in *M*-Linie
  → Sprung in *V*-Linie
- Bei Gelenk B: $M = 0$
  (kein Knick in *M*-Linie resp. kein Sprung in *V*-Linie, da keine Einzelkraft angreift)

M:



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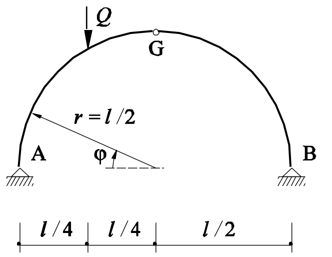
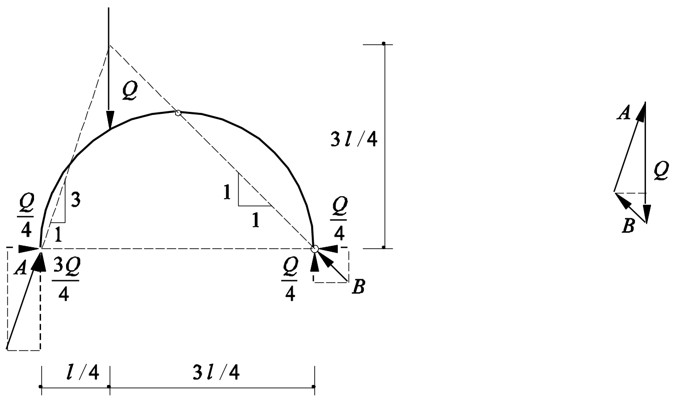
Aufgabe 2, Lösung

Gegeben: System und Einwirkung

Gesucht: Reaktionen (SKD) und Schnittgrössen (N, V, M)

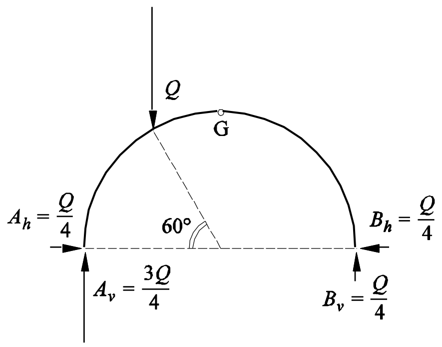


Grafische Lösung mit Hilfe eines Seilpolygons und eines Kräftepolygons:



Analytische Lösung:

SKD:



$$\sum F_v = 0 \quad \rightarrow \quad A_v + B_v - Q = 0$$

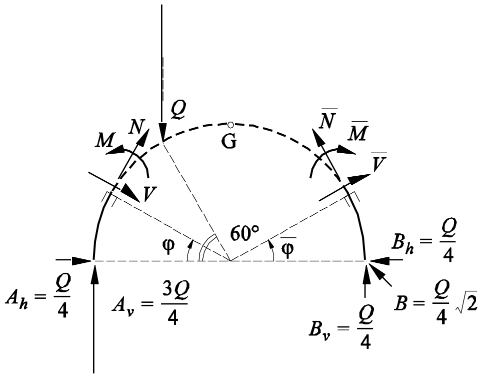
$$\sum F_h = 0 \quad \rightarrow \quad A_h = B_h$$

$$\sum M(\mathrm{A}) = 0 \quad \rightarrow \quad B_v \cdot l - Q \cdot \frac{l}{4} = 0$$

$$M_{\mathrm{G}} = 0 \quad \rightarrow \quad B_v \cdot \frac{l}{2} - B_h \cdot \frac{l}{2} = 0$$

$$\rightarrow \quad \boxed{B_v = \frac{Q}{4}, \quad B_h = \frac{Q}{4}, \quad A_h = \frac{Q}{4}, \quad A_v = \frac{3Q}{4}}$$

Ermittlung der Schnittgrössen:

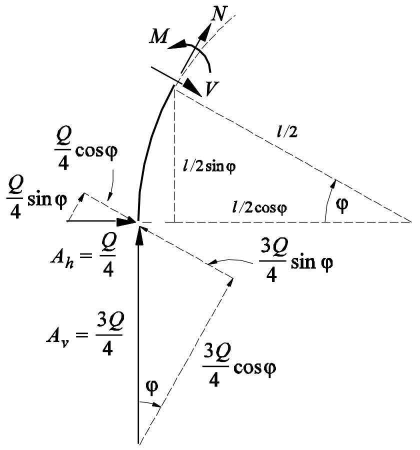


$$\boxed{\varphi = 0 \div 60^{o}}$$

$$N = -A_v \cdot \cos\varphi - A_h \cdot \sin\varphi$$
$$= -\frac{3Q}{4} \cdot \cos\varphi - \frac{Q}{4} \cdot \sin\varphi = -\frac{Q}{4}\left(\sin\varphi + 3\cos\varphi\right)$$

$$V = A_v \cdot \sin\varphi - A_h \cdot \cos\varphi$$
$$= \frac{3Q}{4} \cdot \sin\varphi - \frac{Q}{4} \cdot \cos\varphi = \frac{Q}{4}\left(3\sin\varphi - \cos\varphi\right)$$

$$M = A_v\left(\frac{l}{2} - \frac{l}{2} \cdot \cos\varphi\right) - A_h \cdot \frac{l}{2} \cdot \sin\varphi$$
$$= \frac{3Q}{4}\left(\frac{l}{2} - \frac{l}{2} \cdot \cos\varphi\right) - \frac{Q}{4} \cdot \frac{l}{2} \cdot \sin\varphi$$
$$= \frac{Ql}{8}\left(3 - 3\cos\varphi - \sin\varphi\right)$$



$$\boxed{\overline{\varphi} = 0 \div 120^{o}}$$

$$\overline{N} = -B_v \cdot \cos\overline{\varphi} - B_h \cdot \sin\overline{\varphi}$$
$$= -\frac{Q}{4} \cdot \cos\overline{\varphi} - \frac{Q}{4} \cdot \sin\overline{\varphi} = -\frac{Q}{4}\left(\sin\overline{\varphi} + \cos\overline{\varphi}\right)$$

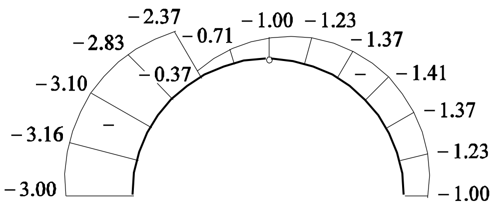
$$\overline{V} = -B_v \cdot \sin\varphi + B_h \cdot \cos\varphi$$
$$= -\frac{Q}{4} \cdot \sin\varphi + \frac{Q}{4} \cdot \cos\varphi = \frac{Q}{4}\left(-\sin\overline{\varphi} + \cos\overline{\varphi}\right)$$

$$\overline{M} = B_v\left(\frac{l}{2} - \frac{l}{2} \cdot \cos\overline{\varphi}\right) - B_h \cdot \frac{l}{2} \cdot \sin\overline{\varphi}$$
$$= \frac{Q}{4}\left(\frac{l}{2} - \frac{l}{2} \cdot \cos\overline{\varphi}\right) - \frac{Q}{4} \cdot \frac{l}{2} \cdot \sin\overline{\varphi}$$
$$= \frac{Ql}{8}\left(1 - \cos\overline{\varphi} - \sin\overline{\varphi}\right)$$

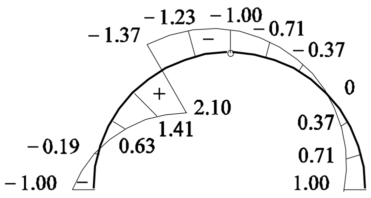
Zusammenstellung:

φ	$\bar{\varphi}$	$N[Q/4]$	$V[Q/4]$	$M[Q\,l/8]$
0°		-3.00	-1.00	0
15°		-3.16	-0.19	-0.16
30°		-3.10	0.63	-0.10
45°		-2.83	1.41	0.17
60°		-2.37	2.10	0.63
	120°	-0.37	-1.37	0.63
	105°	-0.71	-1.23	0.29
	90°	-1.00	-1.00	0
	75°	-1.23	-0.71	-0.23
	60°	-1.37	-0.37	-0.37
	45°	-1.41	0	-0.41
	30°	-1.37	0.37	-0.37
	15°	-1.23	0.71	-0.23
	0°	-1.00	1.00	0

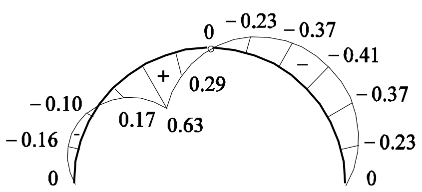
N: $[Q/4]$



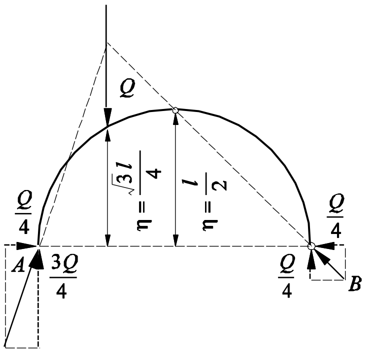
V: $[Q/4]$

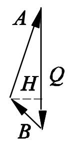


M: $[Ql/8]$



Lösungsweg gemäss Skript Ergänzungsblätter Bogen und Seile (Seite 1:)



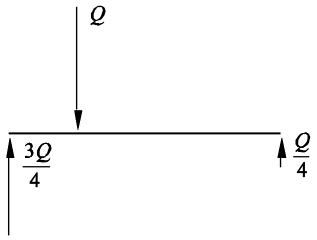


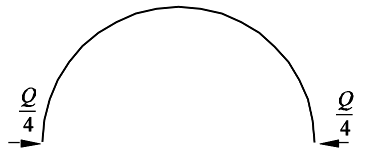
$$H = \frac{Q}{4}$$

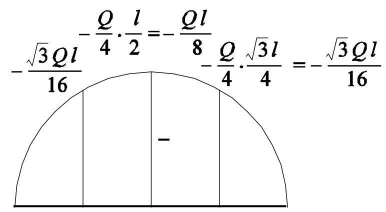
Moment am Bogen durch Überlagerung: $\boxed{M = M_0 + M(H) = M_0 - H \cdot \eta}$

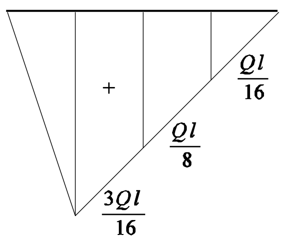
$\underline{M_0}$: (am einfachen Balken)

$\underline{M(H) = -H \cdot \eta}$: (am Bogen)

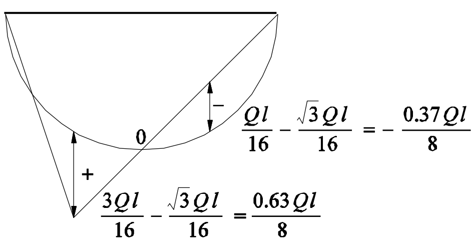








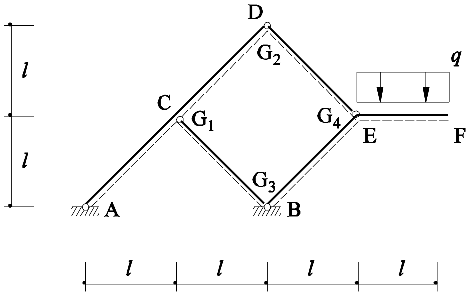
$\underline{M = M_0 + M(H) = M_0 - H \cdot \eta}$:



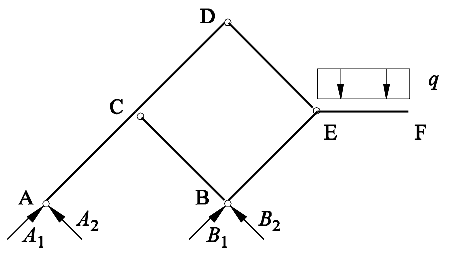
Aufgabe 3, Lösung

Gegeben: System und Einwirkung

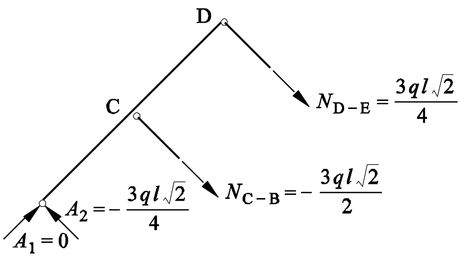
Gesucht: Reaktionen (SKD) und Schnittgrössen (N, V, M)



SKD:



SKD(1): Teilsystem (1)

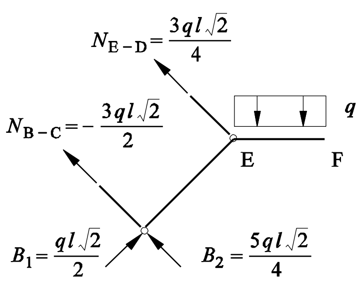


$$\sum F_{\nearrow} = 0 \quad \rightarrow A_1 = 0$$

$$\sum F_{\nwarrow} = 0 \quad \rightarrow A_2 - N_{\text{C-B}} - N_{\text{D-E}} = 0$$

$$\sum M(\text{A}) = 0 \quad \rightarrow N_{\text{C-B}} \cdot l\sqrt{2} + N_{\text{D-E}} \cdot 2l\sqrt{2} = 0$$

SKD(2): Teilsystem (2)



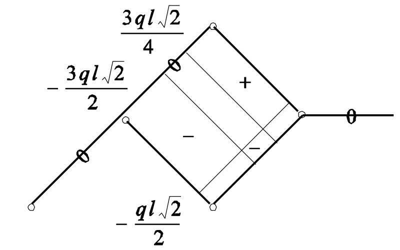
$$\sum F_{\nearrow} = 0 \quad \rightarrow B_1 - \frac{ql}{\sqrt{2}} = 0$$

$$\sum F_{\nwarrow} = 0 \quad \rightarrow B_2 + N_{\text{B-C}} + N_{\text{E-D}} - \frac{ql}{\sqrt{2}} = 0$$

$$\sum M(\text{B}) = 0 \quad \rightarrow ql \cdot \frac{3l}{2} - N_{\text{E-D}} \cdot l\sqrt{2} = 0$$

$$\rightarrow \quad \boxed{\begin{array}{ll} B_1 = \dfrac{ql\sqrt{2}}{2} & \\ N_{\text{E-D}} = N_{\text{D-E}} = \dfrac{3ql\sqrt{2}}{4} & \text{(Zug)} \\ A_1 = 0 & \\ N_{\text{C-B}} = N_{\text{B-C}} = -\dfrac{3ql\sqrt{2}}{2} & \text{(Druck)} \\ A_2 = -\dfrac{3ql\sqrt{2}}{4} & \\ B_2 = \dfrac{5ql\sqrt{2}}{4} & \end{array}}$$

N:



Bekannt sind somit:

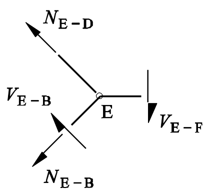
$$N_{\text{E-D}} = N_{\text{D-E}} = \frac{3ql\sqrt{2}}{4}$$

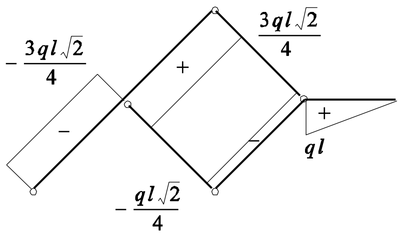
$$N_{\text{C-B}} = N_{\text{B-C}} = -\frac{3ql\sqrt{2}}{2}$$

$$V_{\text{E-F}} = ql$$

Gleichgewicht am Knoten E:

V:





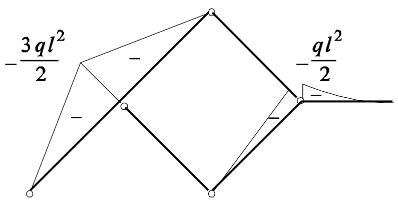
$$\sum F_{\nearrow} = 0: \quad N_{\text{E-B}} + \frac{V_{\text{E-F}}}{\sqrt{2}} = 0$$

$$\rightarrow \; N_{\text{E-B}} = -\frac{V_{\text{E-F}}}{\sqrt{2}} = \underline{\underline{-\frac{ql\sqrt{2}}{2}}}$$

M:

$$\sum F_{\nwarrow} = 0: \quad V_{\text{E-B}} + N_{\text{E-D}} - \frac{V_{\text{E-F}}}{\sqrt{2}} = 0$$

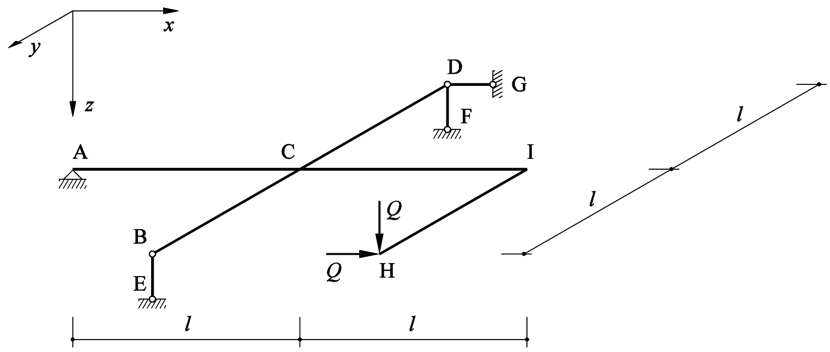
$$\rightarrow \; V_{\text{E-B}} = \frac{V_{\text{E-F}}}{\sqrt{2}} - N_{\text{E-D}} = \underline{\underline{-\frac{ql\sqrt{2}}{4}}}$$



Aufgabe 4, Lösung

Gegeben: Räumliches System und Einwirkung

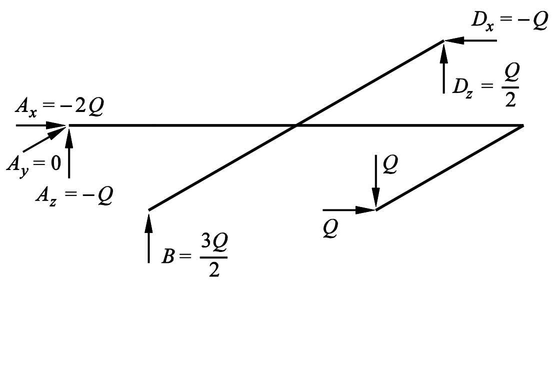
Gesucht: Reaktionen (SKD) und Schnittgrössen (*M*, *T*, *V*, *N*)



Merke:

In den Pendelstäben E-B , F-D und G-D wirken nur Normalkräfte. Die Auflagerkräfte können somit in die Punkte B und D reduziert werden.

SKD:



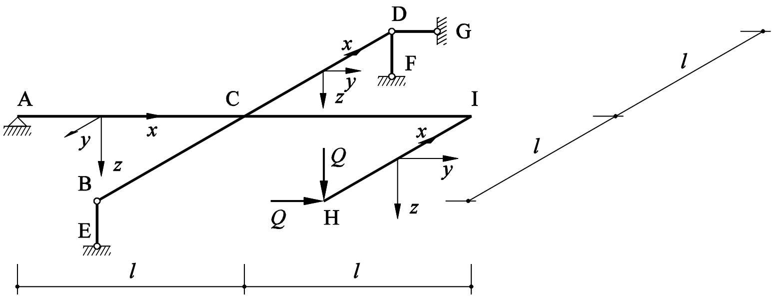
Im globalen Koordinatensystem gilt:

$$\sum F_x = 0 \quad \rightarrow \quad A_x - D_x + Q = 0$$
$$\sum F_y = 0 \quad \rightarrow \quad A_y = 0$$
$$\sum F_z = 0 \quad \rightarrow \quad A_z + B + D_z - Q = 0$$
$$\sum M_y(\mathrm{A}) = 0 \quad \rightarrow \quad B \cdot l + D_z \cdot l - Q \cdot 2l = 0$$
$$\sum M_x(\mathrm{A}) = 0 \quad \rightarrow \quad B \cdot l - D_z \cdot l - Q \cdot l = 0$$
$$\sum M_z(\mathrm{A}) = 0 \quad \rightarrow \quad D_x \cdot l + Q \cdot l = 0$$

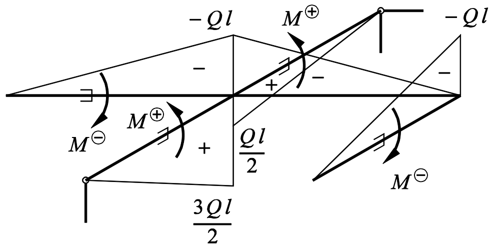
$$\rightarrow \quad \boxed{\begin{aligned} A_y &= 0 \\ B &= \frac{3Q}{2} \\ D_z &= \frac{Q}{2} \\ A_z &= -Q \\ D_x &= -Q \\ A_x &= -2Q \end{aligned}}$$

Merke:

Die Schnittgrössen $M_y, M_z, T_x, V_z, V_y, N_x$ werden **immer** bezüglich den lokalen Koordinatensystemen mit der *x*-Achse in Stabrichtung und der *z*-Achse nach unten aufgezeichnet.



***M*_y:**



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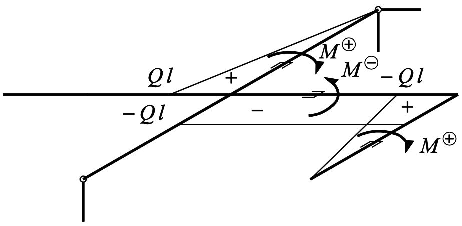
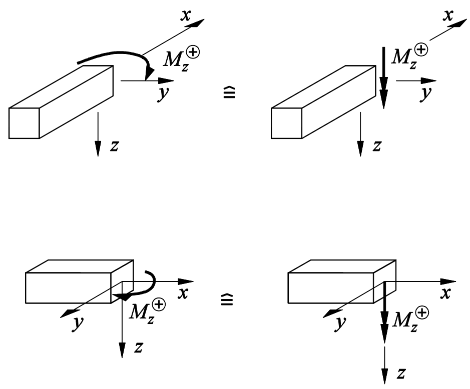


Erklärung für die Vorzeichen:



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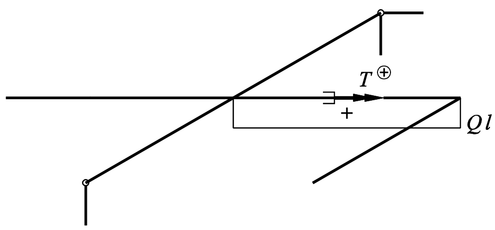
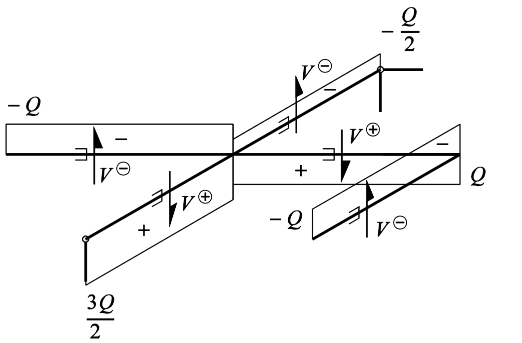
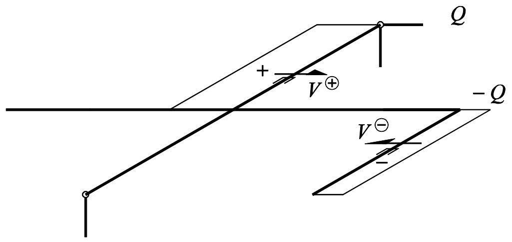


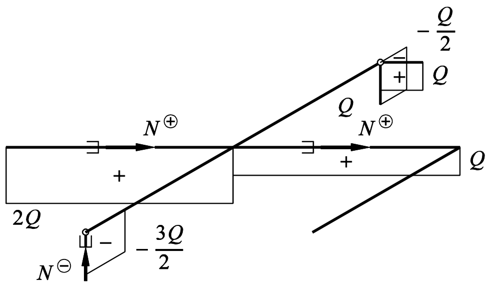
V_z**:**


$-\frac{Q}{2}$
$-Q$
Q
$-Q$
$\frac{3Q}{2}$
+
−
$V^{\ominus}$
$V^{\oplus}$

V_y**:**


Q
$-Q$
+
−
$V^{\oplus}$
$V^{\ominus}$

N_x**:**


$-\frac{Q}{2}$
Q
Q
Q
$2Q$
$-\frac{3Q}{2}$
+
−
$N^{\oplus}$
$N^{\ominus}$