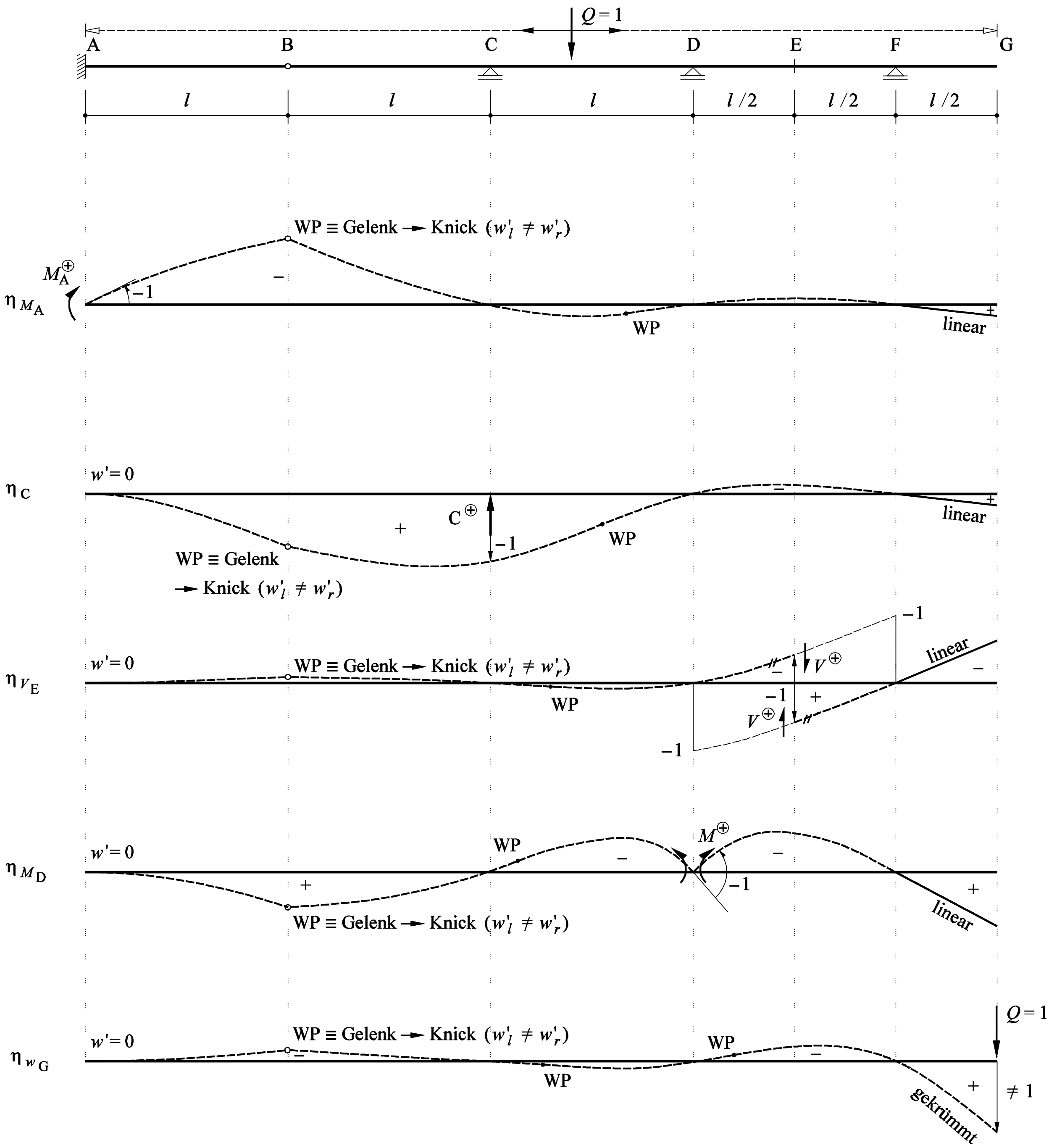
BAUSTATIK II – KOLLOQUIUM 3, Lösung

(101-0114)

Thema: Einflusslinien

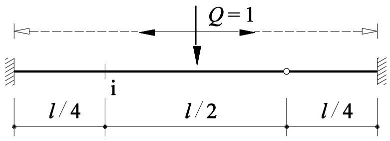
Aufgabe 1, Lösung



Aufgabe 2, Lösung

Gegeben: System $(l, EI = \text{konstant})$, wandernde Last $Q = 1$

Gesucht: Quantitativer Verlauf der Einflusslinien η_{M_i} und η_{V_i}
(Werte jeweils in den Viertelspunkten)



4 Lösungsvarianten gemäss Merkblatt

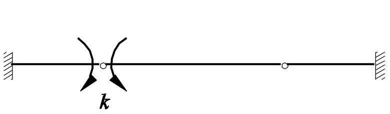
1. Anwendung der Methode Land mit Einführen einer Zustandsgrösse *k*

$\boxed{\eta_{M_\text{i}} = w(k)}$

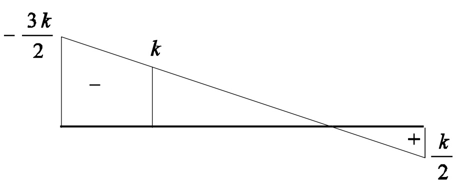
$\boxed{\eta_{V_\text{i}} = w(k)}$

für η_{M_i} :

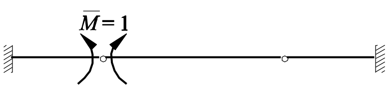
VZ:



M :



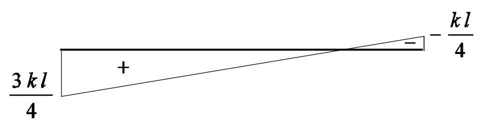
BZ:



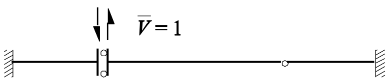
für η_{V_i} :

VZ:

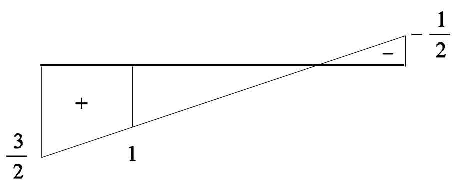
M :



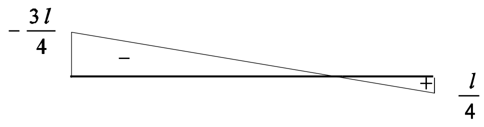
BZ:



$\overline{M}$:



$\overline{M}$:



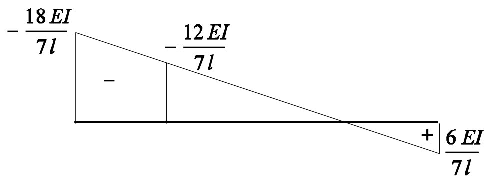
$$\varphi = \frac{1}{3EI}\cdot\frac{3}{2}\cdot\left(-\frac{3k}{2}\right)\cdot\frac{3l}{4} + \frac{1}{3EI}\cdot\left(-\frac{1}{2}\right)\cdot\frac{k}{2}\cdot\frac{l}{4}$$
$$= -\frac{7kl}{12EI}$$

$$\boxed{\varphi = -1} \quad \rightarrow \quad k = \frac{12EI}{7l}$$

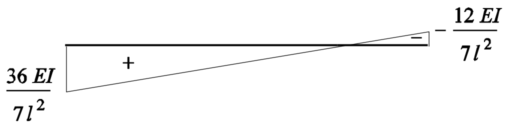
$$\delta = \frac{1}{3EI}\cdot\left(-\frac{3l}{4}\right)\cdot\frac{3kl}{4}\cdot\frac{3l}{4} + \frac{1}{3EI}\cdot\frac{l}{4}\cdot\left(-\frac{kl}{4}\right)\cdot\frac{l}{4}$$
$$= -\frac{7kl^3}{48EI}$$

$$\boxed{\delta = -1} \quad \rightarrow \quad k = \frac{48EI}{7l^3}$$

M :

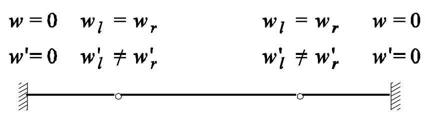


M :

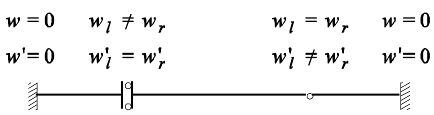


Mohrsche Analogie zu Ermittlung der Biegelinie w:

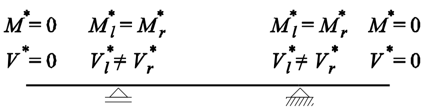
Grundträger:



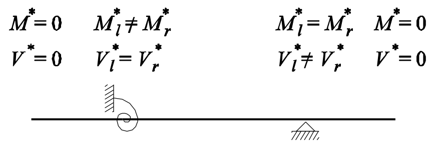
Grundträger:



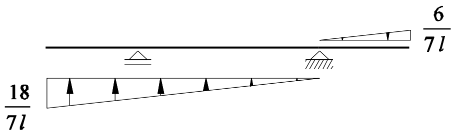
Analogieträger:



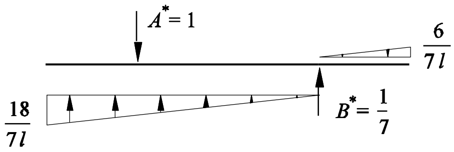
Analogieträger:



Analogieträger mit Belastung $q^* = \dfrac{M}{EI}$:



SKD:



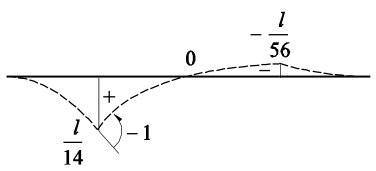
$$A^* = 1 \quad , \quad B^* = \frac{1}{7}$$

$$M^*(l/4) \;=\; \frac{l}{14} = w(l/4) \;= \eta_{M_\mathrm{i}}(l/4)$$

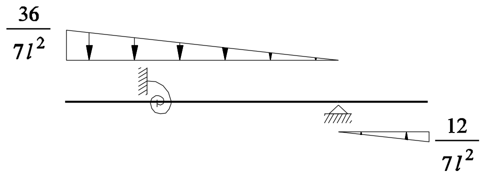
$$M^*(l/2) \;=\; 0 \;= w(l/2) \;= \eta_{M_\mathrm{i}}(l/2)$$

$$M^*(3l/4) = -\frac{l}{56} = w(3l/4) = \eta_{M_\mathrm{i}}(3l/4)$$

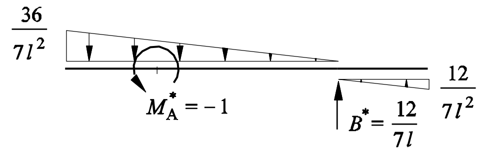
$$\boxed{\eta_{M_\mathrm{i}} = M^* = w}:$$



Analogieträger mit Belastung $q^* = \dfrac{M}{EI}$



SKD:



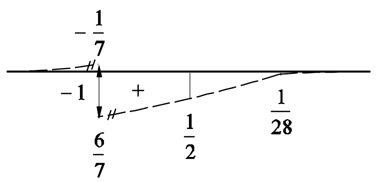
$$M_A^* = -1 \quad , \quad B^* = \frac{12}{7l}$$

$$M^*(l/4) \;= -\frac{1}{7} \;/\; \frac{6}{7} = w(l/4) \;= \eta_{V_\mathrm{i}}(l/4)$$

$$M^*(l/2) \;=\; \frac{1}{2} \qquad = w(l/2) \;= \eta_{V_\mathrm{i}}(l/2)$$

$$M^*(3l/4) = \frac{1}{28} \qquad = w(3l/4) = \eta_{V_\mathrm{i}}(3l/4)$$

$$\boxed{\eta_{V_\mathrm{i}} = M^* = w}:$$

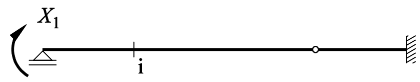


2. Anwendung der Methode Land mit Kraftmethode:

$$\boxed{\eta_{M_\mathrm{i}} = w = w_0 + X_1 \cdot w_1}$$

$$\boxed{\eta_{V_\mathrm{i}} = w = w_0 + X_1 \cdot w_1}$$

GS und ÜG (gilt für beide Einflusslinien!):



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Institut für Baustatik und Konstruktion
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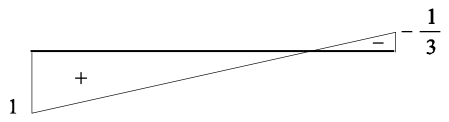
Frühjahrssemester

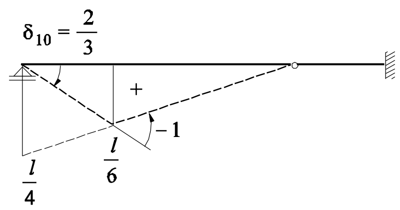
$M_1(X_1 = 1)$:



Merke: δ_{11} ist für dieses GS unveränderlich: $\delta_{11} = \frac{1}{3} \cdot 1 \cdot \frac{1}{EI} \cdot \frac{3l}{4} + \frac{1}{3} \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3EI}\right) \cdot \frac{l}{4} = \underline{\frac{7l}{27EI}}$

<u>für η_{M_i}:</u>

w_0:



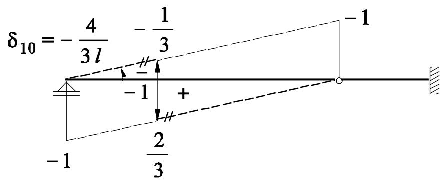
$\delta_{10} = \frac{2}{3}$ (aus Zeichnung)

$\delta_{11} = \frac{7l}{27EI}$ (unveränderlich)

$X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{2}{3} \cdot \frac{27EI}{7l} = \underline{-\frac{18EI}{7l}}$

<u>für η_{V_i}:</u>

w_0:



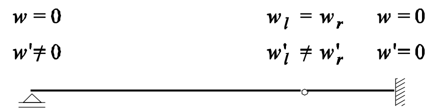
$\delta_{10} = -\frac{4}{3l}$ (aus Zeichnung)

$\delta_{11} = \frac{7l}{27EI}$ (unveränderlich)

$X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{4}{3l} \cdot \frac{27EI}{7l} = \underline{\frac{36EI}{7l^2}}$

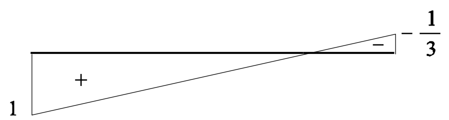
Mohrsche Analogie zur Ermittlung von w_1 am GS infolge $X_1 = 1$ (gilt für beide Einflusslinien!):

Grundträger:

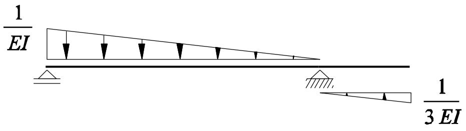


Analogieträger:

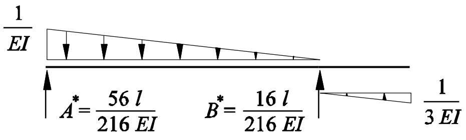
$M_1(X_1 = 1)$:



Analogieträger mit Belastung $q^* = \dfrac{M}{EI}$:



SKD:



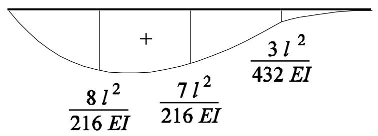
$$A^* = \frac{56l}{216EI} \quad , \quad B^* = \frac{16l}{216EI}$$

$$M^*(l/4) = \frac{8l^2}{216EI} = w_1(l/4)$$

$$M^*(l/2) = \frac{7l^2}{216EI} = w_1(l/2)$$

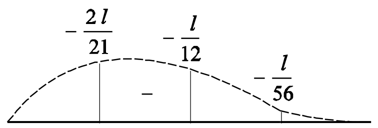
$$M^*(3l/4) = \frac{3l^2}{432EI} = w_1(3l/4)$$

$M^* = w_1$:

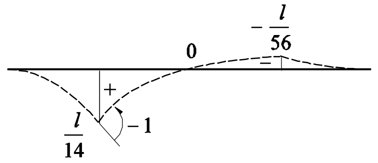


für η_{M_i}:

$X_1 \cdot w_1$:

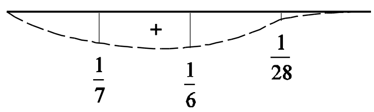


$\boxed{\eta_{M_i} = w = w_0 + X_1 \cdot w_1}$:

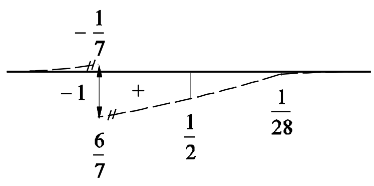


für η_{V_i}:

$X_1 \cdot w_1$:



$\boxed{\eta_{V_i} = w = w_0 + X_1 \cdot w_1}$:



3. Allgemeine Berechnung nach Kraftmethode

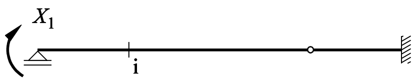
$$M_i = M_{i0} + X_1 \cdot M_{i1} \quad \rightarrow \quad \boxed{\eta_{M_i} = \eta_{M_{i0}} + \eta_{X_1} \cdot M_{i1}}$$

$$V_i = V_{i0} + X_1 \cdot V_{i1} \quad \rightarrow \quad \boxed{\eta_{V_i} = \eta_{V_{i0}} + \eta_{X_1} \cdot V_{i1}}$$

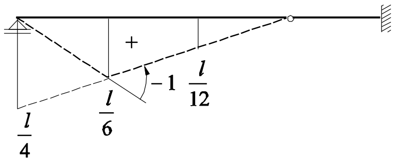
$$X_1 = -\frac{\delta_{10}}{\delta_{11}} \quad \rightarrow \quad \underline{\underline{\eta_{X_1}}} = -\frac{1}{\delta_{11}} \cdot \eta_{\delta_{10}} = -\frac{1}{\delta_{11}} \cdot \delta_{1x} = \underline{\underline{-\frac{1}{\delta_{11}} \cdot \delta_{x1}}}$$

$$\boxed{\begin{aligned}\eta_{M_i} &= \eta_{M_{i0}} + \eta_{X_1} \cdot M_{i1} \\ &= \eta_{M_{i0}} - \frac{1}{\delta_{11}} \cdot \delta_{x1} \cdot M_{i1}\end{aligned}} \qquad \boxed{\begin{aligned}\eta_{V_i} &= \eta_{V_{i0}} + \eta_{X_1} \cdot V_{i1} \\ &= \eta_{V_{i0}} - \frac{1}{\delta_{11}} \cdot \delta_{x1} \cdot V_{i1}\end{aligned}}$$

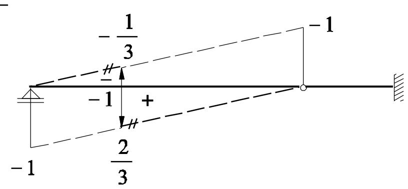
GS und ÜG (gilt für beide Einflusslinien!):



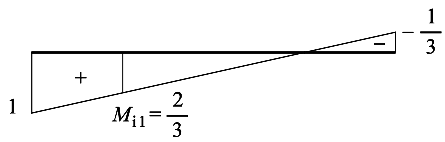
$\underline{\eta_{M_{i0}}}$:



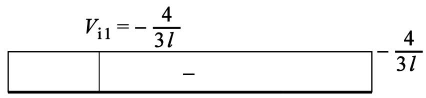
$\underline{\eta_{V_{i0}}}$:



$M_1(X_1 = 1)$:



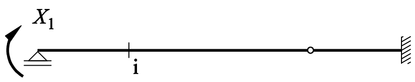
$V_1(X_1 = 1)$:



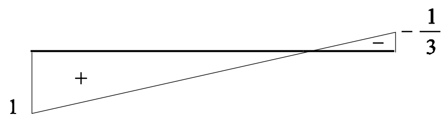
$$\delta_{11} = \frac{1}{3}\left(1 \cdot \frac{1}{EI} \cdot \frac{3l}{4} + \frac{1}{3} \cdot \frac{1}{3EI} \cdot \frac{l}{4}\right) = \underline{\underline{\frac{7l}{27EI}}} \qquad \text{(unveränderlich!)}$$

$\eta_{\delta_{10}} = \delta_{1x} = \delta_{x1}$: EL für δ_{10} = BL am GS infolge $X_1 = 1 \rightarrow$ mit Mohrscher Analogie

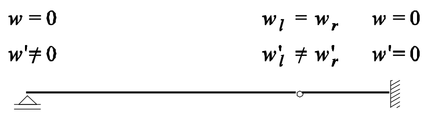
GS und ÜG:



$M_1(X_1 = 1)$:

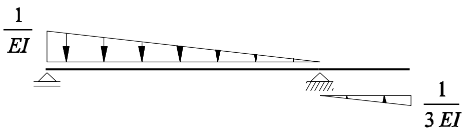


Grundträger:

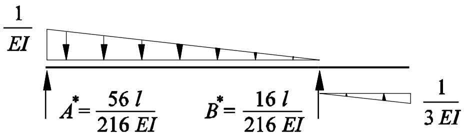


Analogieträger:

Analogieträger mit Belastung $q^* = \frac{M_1}{EI}$:



SKD:



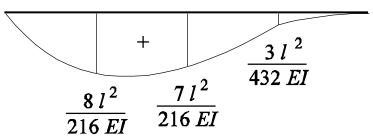
$$A^* = \frac{56l}{216EI} \quad , \quad B^* = \frac{16l}{216EI}$$

$$M^*(l/4) = \frac{8l^2}{216EI} = \delta_{x1}(l/4)$$

$$M^*(l/2) = \frac{7l^2}{216EI} = \delta_{x1}(l/2)$$

$$M^*(3l/4) = \frac{3l^2}{432EI} = \delta_{x1}(3l/4)$$

$M^* = \delta_{x1}$:

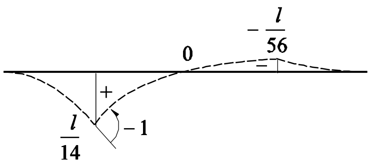


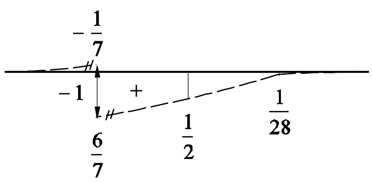
$$\eta_{M_i} = \eta_{M_{i0}} - \frac{1}{\delta_{11}} \cdot \delta_{x1} \cdot M_{i1}$$:

x	0	$l/4$	$l/2$	$3l/4$	l
$\eta_{M_{i0}}$	0	$\frac{l}{6}$	$\frac{l}{12}$	0	0
δ_{11}	←	←	$\frac{7l}{27EI}$	→	→
δ_{x1}	0	$\frac{8l^2}{216EI}$	$\frac{7l^2}{216EI}$	$\frac{3l^2}{432EI}$	
M_{i1}	←	←	$\frac{2}{3}$	→	→
η_{M_i}	0	$\frac{l}{14}$	0	$-\frac{l}{56}$	0

$$\eta_{V_i} = \eta_{V_{i0}} - \frac{1}{\delta_{11}} \cdot \delta_{x1} \cdot V_{i1}$$:

x	0	$l/4$	$l/2$	$3l/4$	l
$\eta_{V_{i0}}$	0	$-\frac{1}{3}, \frac{2}{3}$	$\frac{1}{3}$	0	0
δ_{11}	←	←	$\frac{7l}{27EI}$	→	→
δ_{x1}	0	$\frac{8l^2}{216EI}$	$\frac{7l^2}{216EI}$	$\frac{3l^2}{432EI}$	
V_{i1}	←	←	$-\frac{4}{3l}$	→	→
η_{V_i}	0	$-\frac{1}{7}, \frac{6}{7}$	$\frac{1}{2}$	$\frac{1}{28}$	0





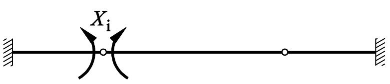
4. Kraftmethode: Einführen der gesuchten Zustandsgrösse als Überzählige Grösse X_i

$M_i = X_i$:

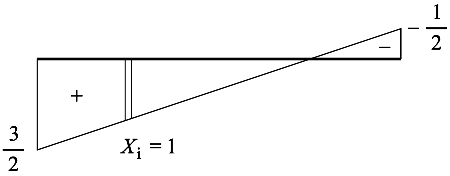
$$\eta_{M_i} = \eta_{X_i} = -\frac{1}{\delta_{ii}} \cdot \delta_{ix} = -\frac{1}{\delta_{ii}} \cdot \delta_{xi}$$

für η_{M_i} :

GS und ÜG:



$M_i(X_i = 1)$:

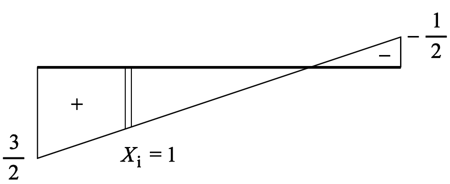


$$\delta_{ii} = \frac{1}{EI} \cdot \left(\frac{1}{3} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3l}{4} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{l}{4} \right)$$

$$= \underline{\frac{28l}{48EI}}$$

δ_{xi} :mit Mohrscher Analogie

$M_i(X_i = 1)$:



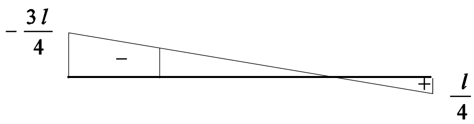
$V_i = X_i$:

$$\eta_{V_i} = \eta_{X_i} = -\frac{1}{\delta_{ii}} \cdot \delta_{ix} = -\frac{1}{\delta_{ii}} \cdot \delta_{xi}$$

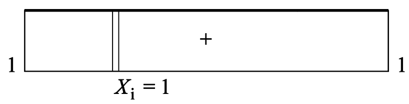
für η_{V_i} :

GS und ÜG:

$M_i(X_i = 1)$:



$V_i(X_i = 1)$:

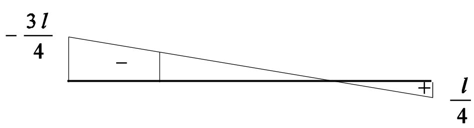


$$\delta_{ii} = \frac{1}{EI} \cdot \left(\frac{1}{3} \cdot \frac{3l}{4} \cdot \frac{3l}{4} \cdot \frac{3l}{4} + \frac{1}{3} \cdot \frac{l}{4} \cdot \frac{l}{4} \cdot \frac{l}{4} \right)$$

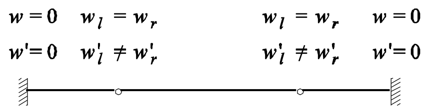
$$= \underline{\frac{7l^3}{48EI}}$$

δ_{xi} :mit Mohrscher Analogie

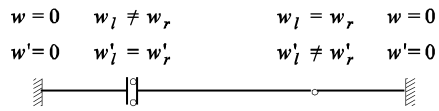
$M_i(X_i = 1)$:



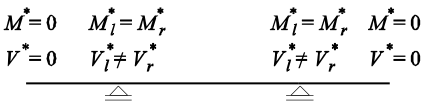
Grundträger:

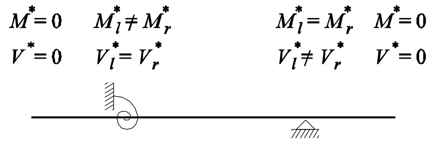


Grundträger:

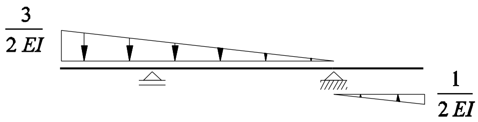


Analogieträger: Analogieträger:

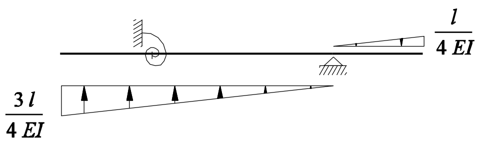




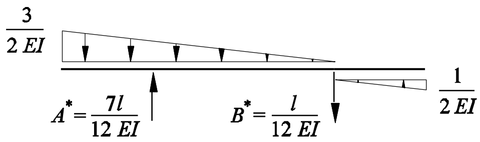
Analogieträger mit Belastung $q^* = \frac{M}{EI}$:



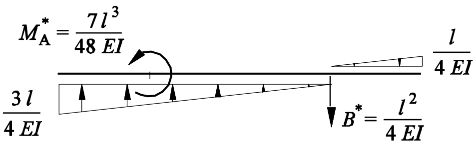
Analogieträger mit Belastung $q^* = \frac{M}{EI}$



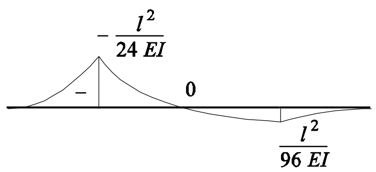
SKD:



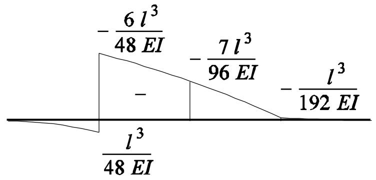
SKD:



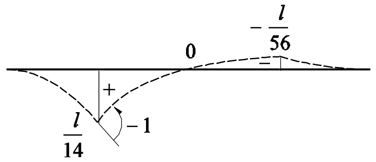
$M^* = \delta_{xi}$:



$M^* = \delta_{xi}$:



$$\boxed{\eta_{M_i} = -\frac{1}{\delta_{ii}} \cdot \delta_{xi}} :$$



$$\boxed{\eta_{V_i} = -\frac{1}{\delta_{ii}} \cdot \delta_{xi}} :$$