BAUSTATIK II – KOLLOQUIUM 2, Lösung

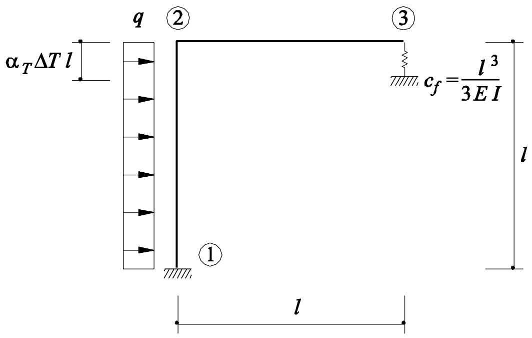
(101-0114)

Thema: Verformungsmethode

Aufgabe 1, Lösung

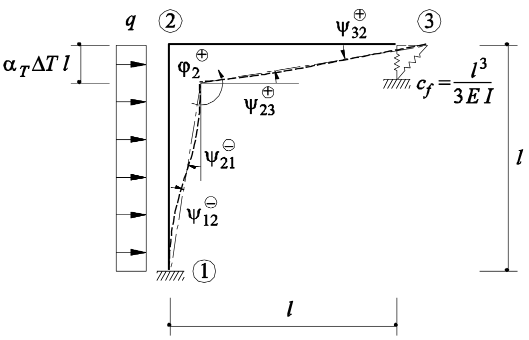
Gegeben: System (l, EI = konstant, $c_f = \dfrac{l^3}{3EI}$) , Einwirkungen q und $\alpha_T \cdot \Delta T = \dfrac{ql^3}{6EI}$

Gesucht: Schnittkraftlinien

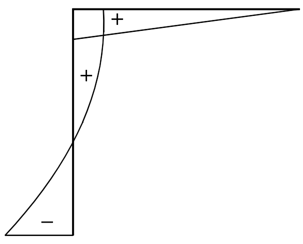


$\boxed{n=1}$

Verformungen qualitativ:



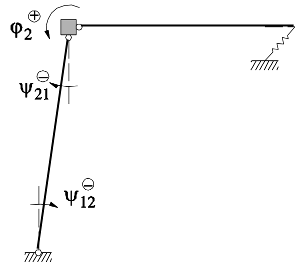
Momente qualitativ:



Das System ist einfach statisch unbestimmt; bei der Berechnung mit der Kraftmethode müsste nur eine ÜG eingeführt werden.

Da das System horizontal verschieblich gelagert ist (es braucht **eine** Festhaltekraft, um eine horizontale Verschiebung zu verhindern), ergeben sich bei der Berechnung mit der Deformationsmethode zwei Unbekannte: ein Knotendrehwinkel φ_2 und ein Stabdrehwinkel $\psi_{12} = \psi_{21}$.

Berechnung mit der Deformationsmethode:



System verschieblich

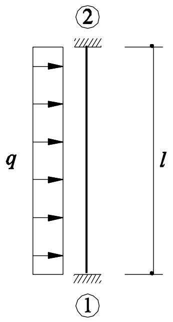
$\rightarrow$ Unbekannte: $\varphi_2, \psi_{12} = \psi_{21}$

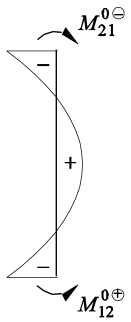
Der Stabdrehwinkel $\psi_{23} = \psi_{32}$ ist aus der Längenänderung des Stabes 1 – 2 **bekannt**:

$$\psi_{23} = \psi_{32} = \frac{1}{l} \cdot \alpha_T \cdot \Delta T \cdot l = \alpha_T \cdot \Delta T = \frac{ql^3}{6EI}$$

1. Festeinspannmomente

Stab 1-2:





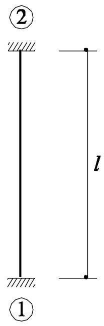
$$M_{21}^0 = -\frac{ql^2}{12}$$

$$M_{12}^0 = \frac{ql^2}{12}$$

Stab 2-3: keine Einwirkung $\rightarrow$ $M_{23}^0 = M_{32}^0 = 0$

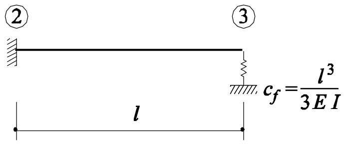
2. Stab- und Kreuzsteifigkeiten

Stab 1-2:



$$s_{21} = s_{12} = \frac{4EI}{l} \qquad t_{21} = t_{12} = \frac{2EI}{l}$$

Stab 2-3:



$$s_{23} = \frac{1}{\frac{l}{3EI} + \frac{c_f}{l^2}} = \frac{3EI}{2l} \qquad t_{23} = 0$$

(siehe Kolloquium 1, Aufgabe 2)

3. Stabendmomente:

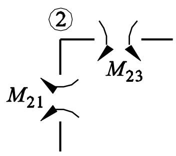
$$M_{ik} = M_{ik}^0 + s_{ik} \cdot \varphi_i + t_{ik} \cdot \varphi_k - \left(s_{ik} + t_{ik}\right) \cdot \psi_{ik}$$
$$M_{ki} = M_{ki}^0 + s_{ki} \cdot \varphi_k + t_{ki} \cdot \varphi_i - \left(s_{ki} + t_{ki}\right) \cdot \psi_{ki}$$

$$M_{12} = \frac{ql^2}{12} + \frac{2EI}{l} \cdot \varphi_2 - \left(\frac{4EI}{l} + \frac{2EI}{l}\right) \cdot \psi_{12}$$
$$M_{21} = -\frac{ql^2}{12} + \frac{4EI}{l} \cdot \varphi_2 - \left(\frac{4EI}{l} + \frac{2EI}{l}\right) \cdot \psi_{12} \qquad (\psi_{21} = \psi_{12})$$
$$M_{23} = \frac{3EI}{2l} \cdot \varphi_2 - \frac{3EI}{2l} \cdot \psi_{23} = \frac{3EI}{2l} \cdot \varphi_2 - \frac{3EI}{2l} \cdot \alpha_T \cdot \Delta \mathrm{T} = \frac{3EI}{2l} \cdot \varphi_2 - \frac{3EI}{2l} \cdot \frac{ql^3}{6EI}$$
$$= -\frac{ql^2}{4} + \frac{3EI}{2l} \cdot \varphi_2 \qquad \left(\psi_{23} = \frac{1}{l} \cdot \alpha_T \cdot \Delta \mathrm{T} \cdot l = \alpha_T \cdot \Delta \mathrm{T} = \frac{ql^3}{6EI}\right.$$: gegeben siehe Seite 2/11)

$$M_{32} = 0$$

4. Gleichgewichtsbedingungen:

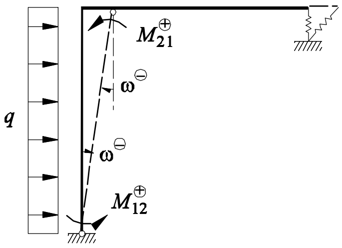
- Knotengleichgewicht:



$$\Sigma M_{2k} = 0 \quad \rightarrow \quad M_{21} + M_{23} = 0$$

$$\rightarrow -\frac{ql^2}{12} + \frac{4EI}{l} \cdot \varphi_2 - \frac{6EI}{l} \cdot \psi_{12} - \frac{ql^2}{4} + \frac{3EI}{2l} \cdot \varphi_2 = 0$$
$$\rightarrow \boxed{-\frac{ql^2}{3} + \frac{11EI}{2l} \cdot \varphi_2 - \frac{6EI}{l} \cdot \psi_{12} = 0} \qquad \text{(I)}$$

- Verschiebegleichgewicht:



$$W_e + W_i = 0$$

$$W_e = ql \cdot \frac{\omega l}{2}$$

$$W_i = -\omega \cdot \left(M_{12} + M_{21}\right)$$

(negativ, da ω^- und M_{12}^+, M_{21}^+ entgegengesetzt drehen)

$$W_e + W_i = ql \cdot \frac{\omega l}{2} - \omega \cdot \left(M_{12} + M_{21}\right) = 0$$

$$\to ql \cdot \frac{\omega l}{2} - \omega \cdot \left(\frac{ql^2}{12} + \frac{2EI}{l} \cdot \varphi_2 - \frac{6EI}{l} \cdot \psi_{12} - \frac{ql^2}{12} + \frac{4EI}{l} \cdot \varphi_2 - \frac{6EI}{l} \cdot \psi_{12}\right) = 0$$

$$\to \boxed{\frac{ql^2}{2} - \frac{6EI}{l} \cdot \varphi_2 + \frac{12EI}{l} \cdot \psi_{12} = 0} \qquad \text{(II)}$$

5. Auflösen des Gleichungssystems

$$\left.\begin{aligned} -\frac{ql^2}{3} + \frac{11EI}{2l} \cdot \varphi_2 - \frac{6EI}{l} \cdot \psi_{12} &= 0 \\ \frac{ql^2}{2} - \frac{6EI}{l} \cdot \varphi_2 + \frac{12EI}{l} \cdot \psi_{12} &= 0 \end{aligned}\right|$$

$$\to \boxed{\begin{aligned} \varphi_2 &= \frac{ql^3}{30EI} \\ \psi_{12} &= -\frac{ql^3}{40EI} \end{aligned}}$$

6. Endgültige Stabendmomente

$$M_{12} = \frac{ql^2}{12} + \frac{2ql^2}{30} + \frac{6ql^2}{40} = \underline{\underline{\frac{3ql^2}{10}}}$$

$$M_{21} = -\frac{ql^2}{12} + \frac{4ql^2}{30} + \frac{6ql^2}{40} = \underline{\underline{\frac{2ql^2}{10}}}$$

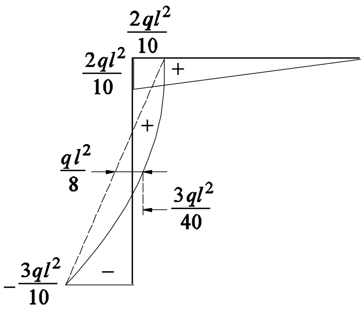
$$M_{23} = -\frac{ql^2}{4} + \frac{3ql^2}{2 \cdot 30} = \underline{\underline{-\frac{2ql^2}{10}}}$$

$$M_{32} = 0$$

Kontrolle: $\Sigma M_{2k} = 0$ i.O.

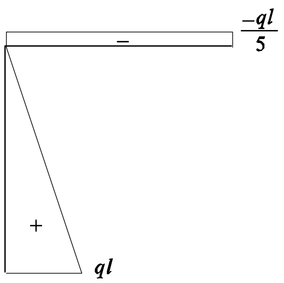
7. Schnittkraftlinien

M:



V:

Zeichnung nach
allg. Konvention: $V^+ \uparrow \overline{\quad} \downarrow$

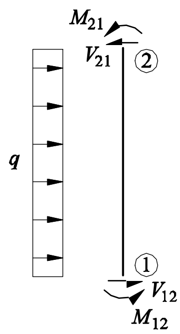


Berechnung nach
Konvention Verformungsmethode: $V^+ \downarrow \overline{\quad} \uparrow$

$$V_{21} \cdot l - \frac{ql^2}{2} + M_{21} + M_{12} = 0$$

$$V_{21} = \frac{ql}{2} - \left(\frac{M_{21} + M_{12}}{l} \right) = \underline{\underline{0}}$$

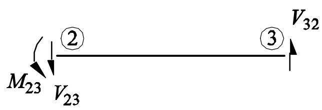
$$V_{12} = V_{21} - ql = \underline{\underline{-ql}}$$



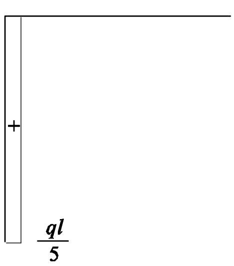
$$V_{23} \cdot l + M_{23} = 0$$

$$V_{23} = -\frac{M_{23}}{l} = \underline{\underline{\frac{ql}{5}}}$$

$$V_{32} = V_{23} = \underline{\underline{\frac{ql}{5}}}$$



N:



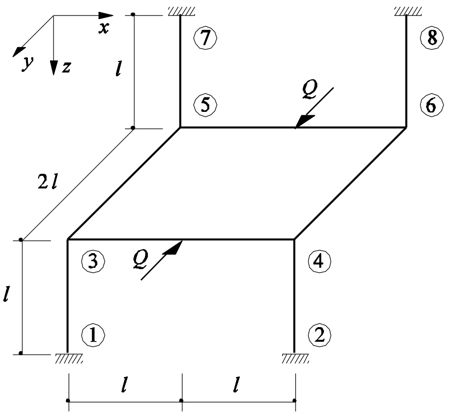
$$N_{23} = \underline{\underline{0}}$$

$$N_{21} = V_{23} = \underline{\underline{\frac{ql}{5}}}$$

Aufgabe 2, Lösung

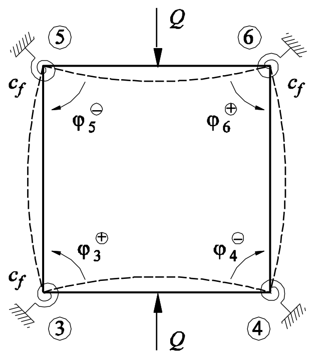
Gegeben: System (l, $EI =$ konstant, $GK = \dfrac{EI}{2}, GA^* \to \infty, EA \to \infty$) , Einwirkung Q

Gesucht: Schnittkraftlinien



System symmetrisch beansprucht
$\rightarrow$ keine Verschieblichkeit

Ersatzsystem:

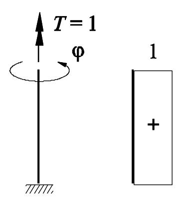


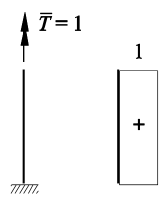
Ermittlung der Federnachgiebigkeit c_f [rad/kNm]:

$$\boxed{c_f = \frac{\varphi}{T} \qquad \rightarrow \qquad c_f = \varphi(T=1)}$$

Berechnung mit Arbeitsgleichung:

VZ: T : BZ: $\bar{T}$:

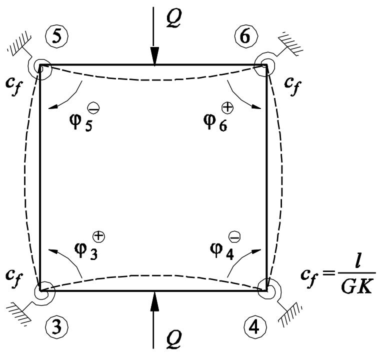




$$\varphi = \int \bar{T}\,\frac{T}{GK}\,dx = \frac{l}{GK}$$

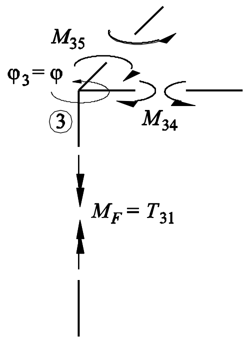
$$\rightarrow c_f = \frac{l}{GK} = \underline{\underline{\frac{2l}{EI}}}$$

Verformungen qualitativ am Ersatzsystem infolge Q:



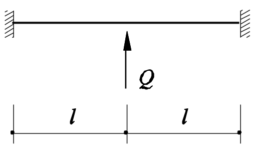
System unverschieblich
Symmetrie: $\varphi_3 = -\varphi_4 = -\varphi_5 = \varphi_6 = \varphi$
$\rightarrow$ Unbekannte: φ

$\rightarrow$ Es muss nur ein Knoten (z.B. Knoten 3) betrachtet werden:



1. Festeinspannmomente

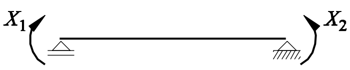
Stab 3-4:



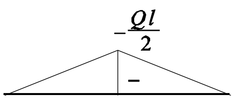
$\boxed{n = 2}$

$\rightarrow$ Berechnung mit der Kraftmethode

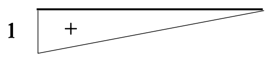
GS und ÜG:



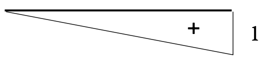
M_0 :



$M_1(X_1 = 1):$



$M_2(X_2 = 1):$



$$\delta_{10} = \delta_{20} = \frac{1}{4} \cdot 1 \cdot \frac{Ql}{2EI} \cdot 2l = -\frac{Ql^2}{4EI}$$

$$\delta_{11} = \frac{1}{3} \cdot 1 \cdot \frac{1}{EI} \cdot 2l = \frac{2l}{3EI} = \delta_{22}$$

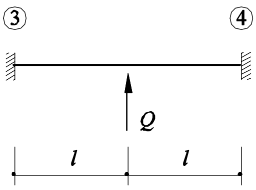
$$\delta_{12} = \delta_{21} = \frac{1}{6} \cdot 1 \cdot \frac{1}{EI} \cdot 2l = \frac{l}{3EI}$$

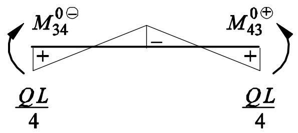
$$\delta_{10} + X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} = 0$$
$$\delta_{20} + X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} = 0$$

$$\left.\begin{array}{l} -\dfrac{Ql^2}{4EI} + X_1 \cdot \dfrac{2l}{3EI} + X_2 \cdot \dfrac{l}{3EI} = 0 \\ -\dfrac{Ql^2}{4EI} + X_1 \cdot \dfrac{l}{3EI} + X_2 \cdot \dfrac{2l}{3EI} = 0 \end{array}\right|$$

$$\rightarrow \boxed{X_1 = X_2 = \frac{Ql}{4}}$$

$M:$



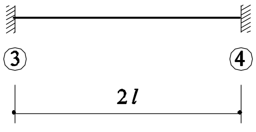


$$M_{34}^0 = -\frac{Ql}{4}$$

$$M_{43}^0 = \frac{Ql}{4}$$

2. Stab- und Kreuzsteifigkeiten

<u>Stab 3-4 , Stab 3-5</u>*:*



$$s_{34} = s_{43} = \frac{4EI}{2l} = \frac{2EI}{l} = s_{35} = s_{53}$$

$$t_{34} = t_{43} = \frac{2EI}{2l} = \frac{EI}{l} = t_{35} = t_{53}$$

3. Stabendmomente

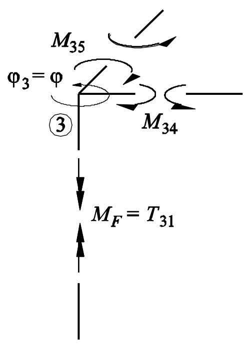
$$M_{ik} = M_{ik}^0 + s_{ik} \cdot \varphi_i + t_{ik} \cdot \varphi_k - (s_{ik} + t_{ik}) \cdot \psi_{ik}$$
$$M_{ki} = M_{ki}^0 + s_{ki} \cdot \varphi_k + t_{ki} \cdot \varphi_i - (s_{ki} + t_{ki}) \cdot \psi_{ki}$$

$$M_{34} = -\frac{QL}{4} + \frac{2EI}{l} \cdot \varphi - \frac{EI}{l} \cdot \varphi = -\frac{QL}{4} + \frac{EI}{l} \cdot \varphi$$
$$M_{35} = 0 + \frac{2EI}{l} \cdot \varphi - \frac{EI}{l} \cdot \varphi = \frac{EI}{l} \cdot \varphi$$

M_F in Feder wird wie ein Stabendmoment betrachtet:

$$M_F = \frac{\varphi}{c_f} = \frac{GK}{l} \cdot \varphi = \frac{EI}{2l} \cdot \varphi = T_{31}$$

4. Knotengleichgewicht



$$\Sigma M_{3k} = 0 \quad \rightarrow \quad M_{34} + M_{35} + M_F = 0$$

mit $M_F = T_{31}$: $\Sigma M_{3k} = M_{34} + M_{35} + T_{31} = 0$

$$\rightarrow -\frac{Ql}{4} + \frac{EI}{l} \cdot \varphi + \frac{EI}{l} \cdot \varphi + \frac{EI}{2l} \cdot \varphi = 0$$
$$\rightarrow -\frac{Ql}{4} + \frac{5EI}{2l} \cdot \varphi = 0$$

$$\rightarrow \boxed{\varphi = \frac{Ql^2}{10EI}}$$

5. Endgültige Stabendmomente

$$M_{34} = -\frac{Ql}{4} + \frac{EI}{l} \cdot \frac{Ql^2}{10EI} = -\underline{\underline{\frac{3Ql}{20}}}$$

$$M_{35} = \frac{EI}{l} \cdot \frac{Ql^2}{10EI} = \underline{\underline{\frac{Ql}{10}}}$$

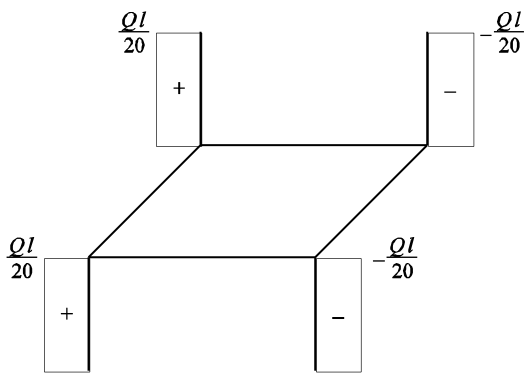
$$T_{31} = M_F = \frac{EI}{2l} \cdot \frac{Ql^2}{10EI} = \underline{\underline{\frac{Ql}{20}}}$$

Kontrolle: $\Sigma M_{3k} = 0$ i.O.

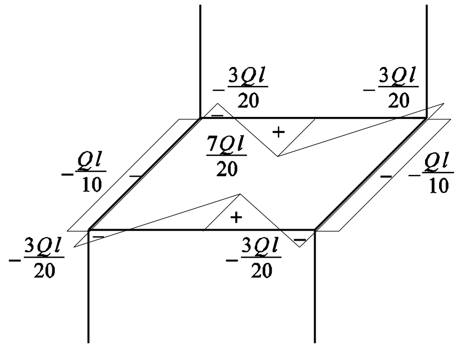
6. Schnittkraftlinien

Merke: Die Indices entsprechen jeweils dem lokalen Koordinatensystem der Stäbe

***T_x*:**

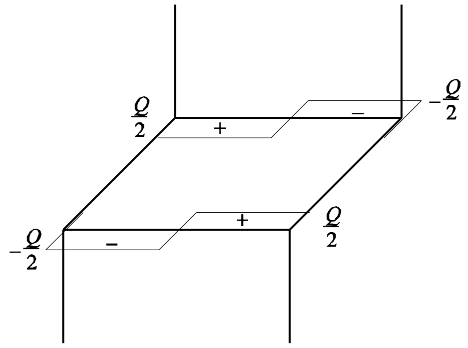


***M_z*:**



Bemerkung zur Wahl der Vorzeichen:
- Rahmen: - innen positiv
  - aussen negativ
- Stiele: - links positiv
  - rechts negativ

***V_y*:**



***N_x*:**