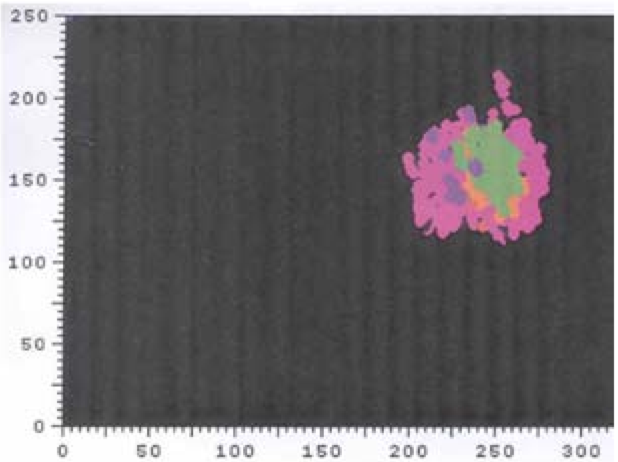


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# Temporal variability in corrosion modeling and reliability updating

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## Motivation

- Corrosion models are applied in many industries, for both
  - Design
  - Inspection & maintenance planning
- To optimize inspection and maintenance activities, a probabilistic approach is required
- Most of the applied models are simplistic and/or deterministic
- They do not consider neither **spatial variability** nor **temporal variability**

1) What are the implications of using simplistic models in this context?

2) How can the temporal variability be accounted for?


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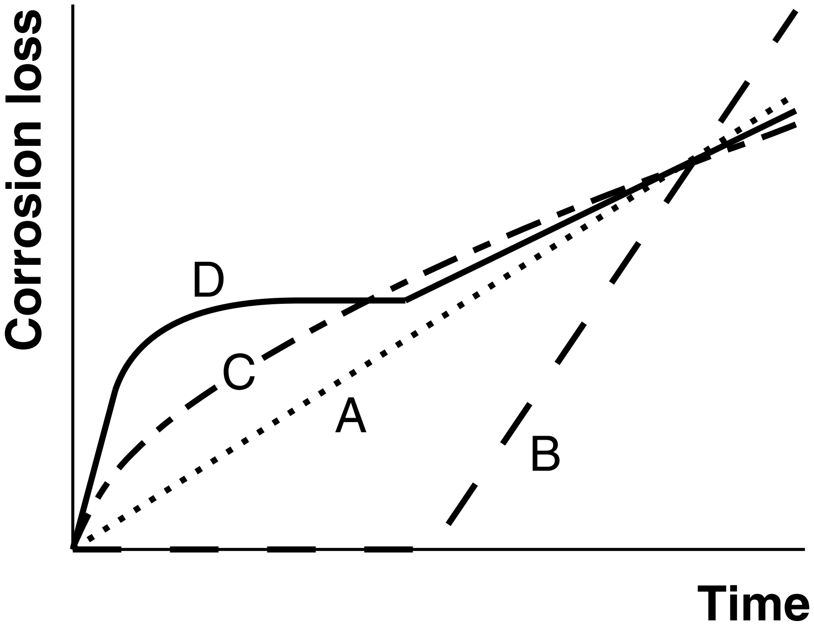
## Modelling of corrosion

- Typical models from the literature:




A: Constant corrosion rate

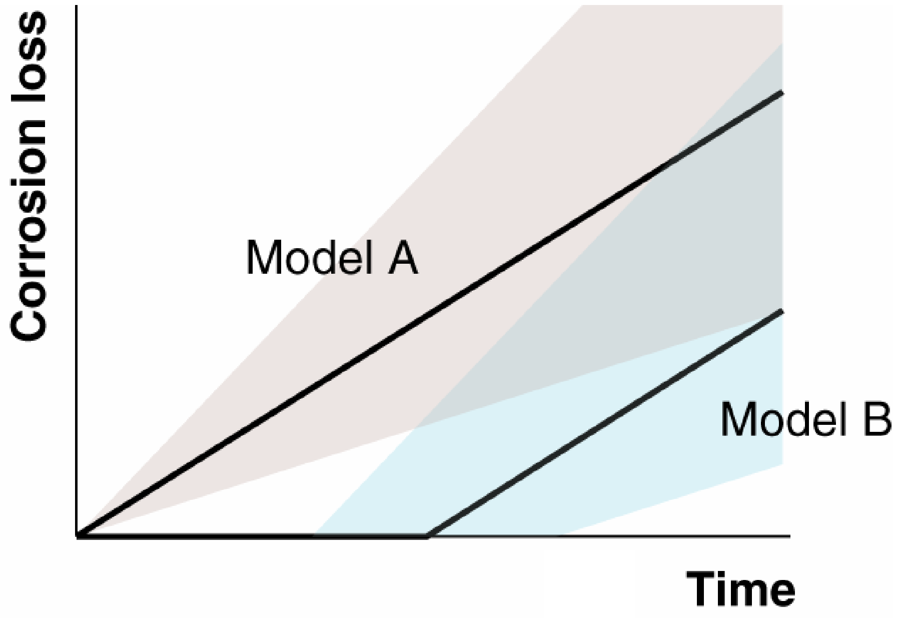
B: Constant corrosion rate with initiation period

C: Corrosion loss follows a power law

D: A model considering different driving mechanisms

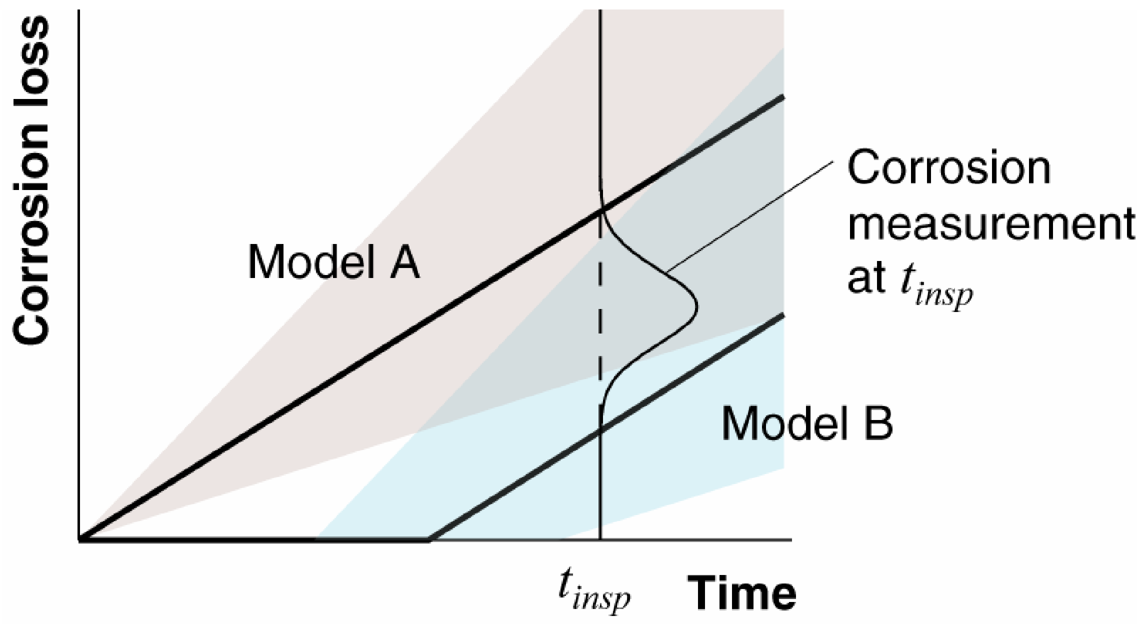
## Probabilistic modelling of corrosion

- Comparing two (simple) models:
  - Model A: constant corrosion rate
  - Model B: As A, with initiation period
- Model A can be considered as conservative compared to B



## Probabilistic modelling of corrosion

- Influence of an inspection result on the reliability as evaluated with the two models




## Example

$$g = d_{cr} - d_C(t)$$

$$g_m = d_m - d_C(t)$$

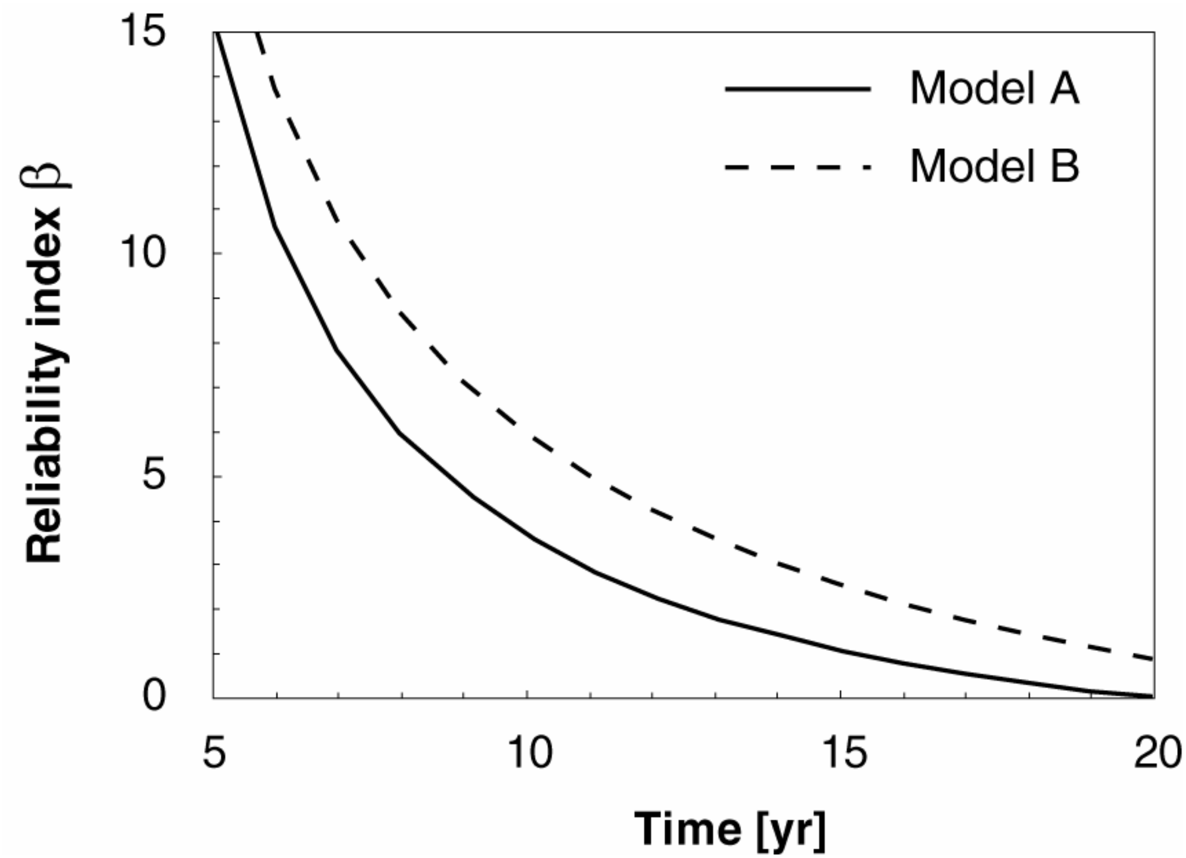
$$d_C(t) = \begin{cases} 0, & t < t_I \\ r(t - t_I) & t \geq t_I \end{cases}$$

| Parameter | | Dim. | $\mu$ | $\sigma$ | Dist. |
|---|---|---|---|---|---|
| Corrosion rate $r$ | | mm/yr | 1 | 0.3 | W |
| Initiation time $t_I$ | Model A | yr | 0 | 0 | - |
| | Model B | yr | 5 | 2 | LN |
| Critical depth $d_{cr}$ | | mm | 20 | - | D |
| Insp. time $t_{Insp}$ | | yr | 8 | - | D |
| Corrosion measurement $d_m$ | | mm | 6 | 1 | N |

$\mu$ : Mean value; $\sigma$ : standard deviation; W: Weibull distr.; LN: Lognormal distr.; D: deterministic; N: Normal distr.

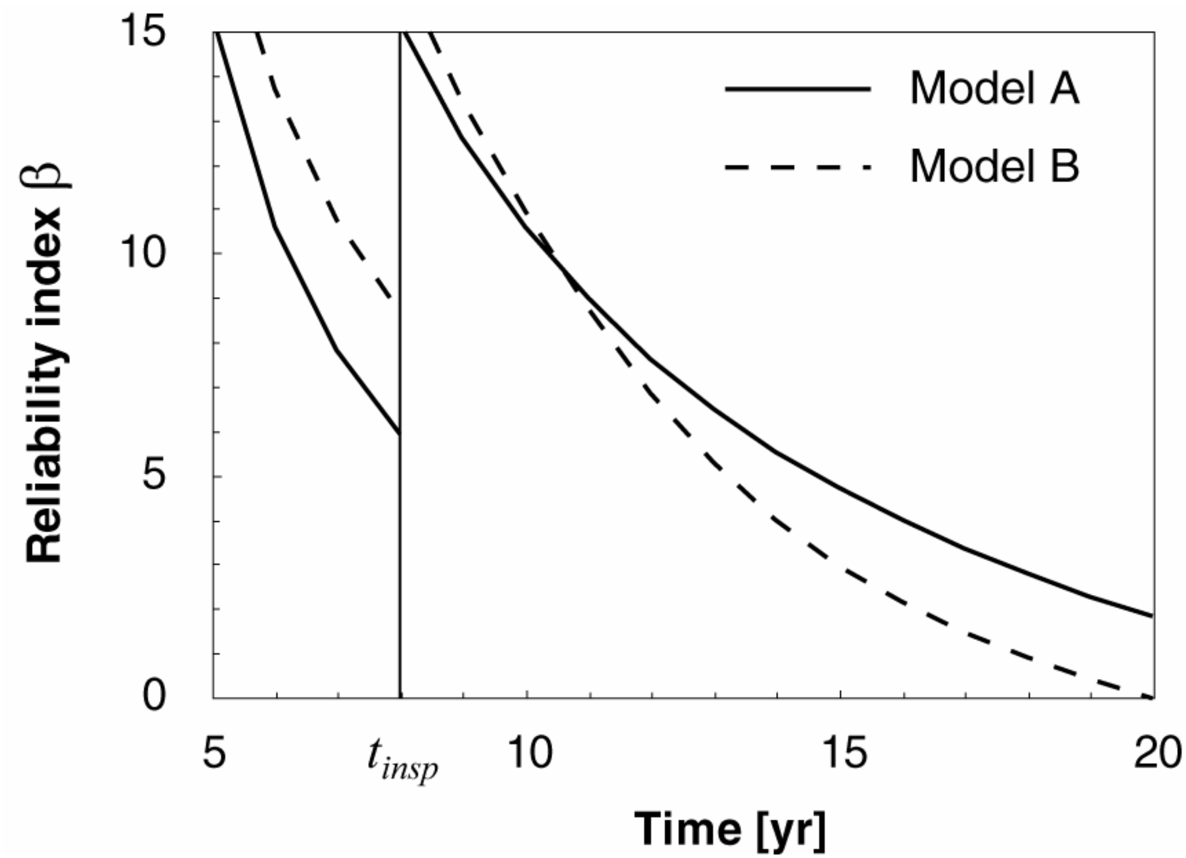
## Results example

- Without inspection: Model A is conservative



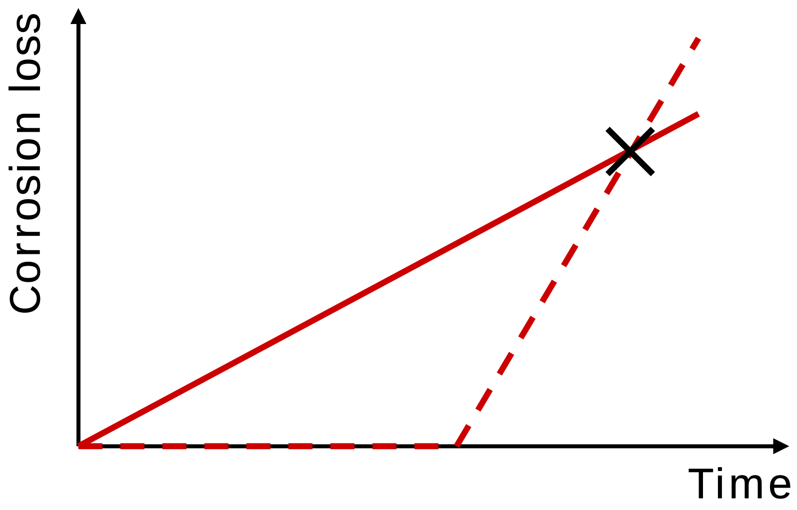
## Results example

- With inspection: Model A is no longer conservative



## Probabilistic modelling of corrosion

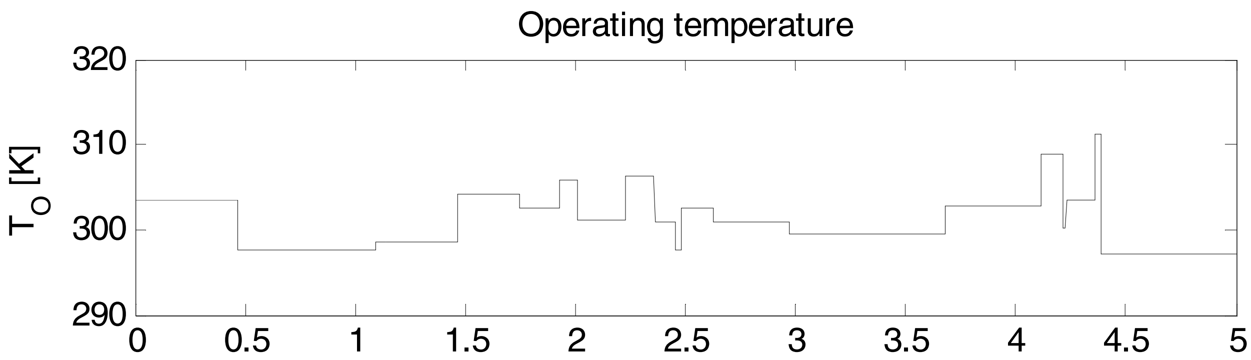
- The non-conservatism of model A can be illustrated by considering only the measurement event deterministically:




- This points out the importance of an appropriate phenomenological model of the corrosion deterioration

## Temporal variability in corrosion modeling

- So far no temporal variability in the model has been considered, i.e., all random variables were assumed constant with time.
- In the real world, influencing factors vary with time, e.g.,
  - Chemical composition of the environment
  - Temperature
  - Pressures




- What is the influence of these variation and how can it be considered?

## Temporal variability in corrosion modeling

- **$CO_2$ corrosion in a pipeline**
- DeWaards-Miliams model:

$$g = d_{cr} - d_C(t)$$

$$d_C(t) = X_M r_{CO_2} t$$

$$r_{CO_2} = 10^{\left(5.8 - 1710/T_o + 0.67 \cdot \log_{10} f_{CO_2}\right)}$$

$$f_{CO_2} = P_{CO_2} \cdot 10^{P_o\left(0.0031 - 1.4/T_o\right)}$$

$$P_{CO_2} = n_{\mathrm{CO}_2} P_o$$

- Parameters considered as stochastic processes:
  - Temperature
  - Pressure

## Temporal variability in corrosion modeling

- Temperature & pressure are modelled by conditional Poisson square wave processes.
- The underlying Poisson processes are fully correlated.
- Additionally a correlation factor of 0.8 between the amplitudes of temperature & pressure at any point in time is assumed.
- For an ordinary Poisson square wave processes it is

$$\mathrm{E}\left[X(t)\right] = \mathrm{E}\left[Y\right]$$

$$\mathrm{Cov}\left[X(t_1), X(t_2)\right] = \mathrm{Var}\left[Y\right]\exp\left|-\nu\left(t_2 - t_1\right)\right|$$

- The mean values of the amplitude of T and P are uncertain themselves. Therefore it is:

$$\mathrm{Cov}\left[X(t_1), X(t_2)\right] = \mathrm{Var}\left[Y\right]\mathrm{e}^{\left|-\nu\left(t_2 - t_1\right)\right|} + \mathrm{Var}\left[\mu_Y\right]\left(1 - \mathrm{e}^{\left|-\nu\left(t_2 - t_1\right)\right|}\right)$$

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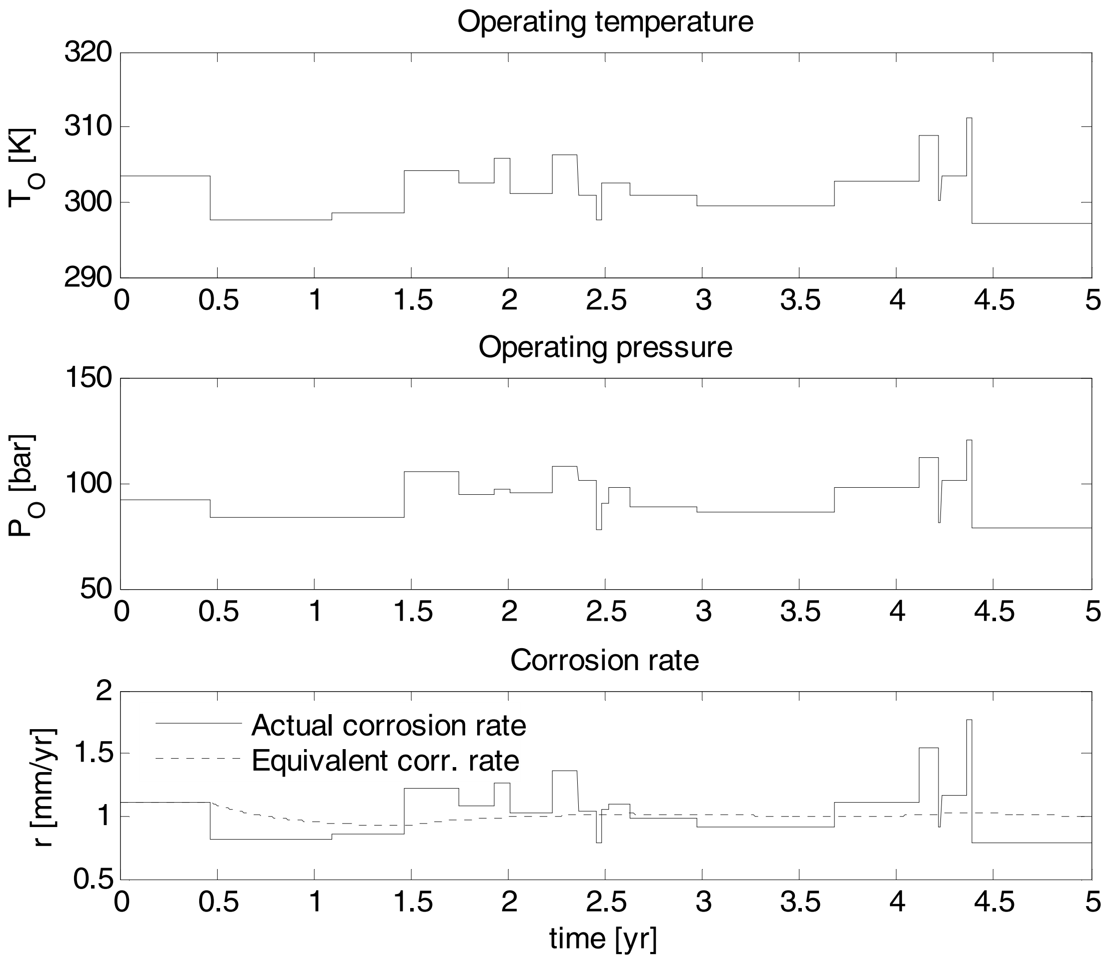
# Temporal variability in corrosion modeling

- One realisation of the random processes:



## Temporal variability in corrosion modeling

- The corrosion rate follows also a conditional Poisson square wave process
- For one realisation of the corrosion rate, an equivalent corrosion rate can be computed:

$$r_e(t) = \frac{d_C(t)}{t} = \frac{1}{t}\int_0^t r_{CO_2}(t)\,\mathrm{d}\,t$$

- The moments of such an integration are obtained as

$$\mathrm{E}\left[r_e(t)\right] = \frac{1}{t}\int_0^t \mathrm{E}\left[r_{CO_2}(t)\right]\mathrm{d}\,t = \mathrm{E}\left[Y_r\right]$$

$$\mathrm{Var}\left[r_e(t)\right] = \frac{1}{t^2}\int_0^t\int_0^t \mathrm{Cov}\left[r_{CO_2}(t_1), r_{CO_2}(t_2)\right]\mathrm{d}\,t_1\,\mathrm{d}\,t_2$$

# Temporal variability in corrosion modeling

- The variance for the considered process is obtained as

$$\mathrm{Var}\left[r_e(t)\right] = \mathrm{Var}\left[\mu_{Y_r}\right] + 2\left(\mathrm{Var}\left[Y_r\right] - \mathrm{Var}\left[\mu_{Y_r}\right]\right)\left(\frac{1}{\nu t} + \frac{\left(\mathrm{e}^{-\nu t} - 1\right)}{(\nu t)^2}\right)$$

- To calculate the moments of equivalent corrosion rate at any time, the following must be evaluated numerically:

| | | |
|---|---|---|
| $\mathrm{E}\left[Y_r\right]$ | Constant | |
| $\mathrm{Var}\left[Y_r\right]$ | The random-point-in-time variability | $\sigma_{r_e}{}^2(\nu t = 0)$ |
| $\mathrm{Var}\left[\mu_{Y_r}\right]$ | The variability related to the time-invariant variability | $\sigma_{r_e}{}^2(\nu t = \infty)$ |

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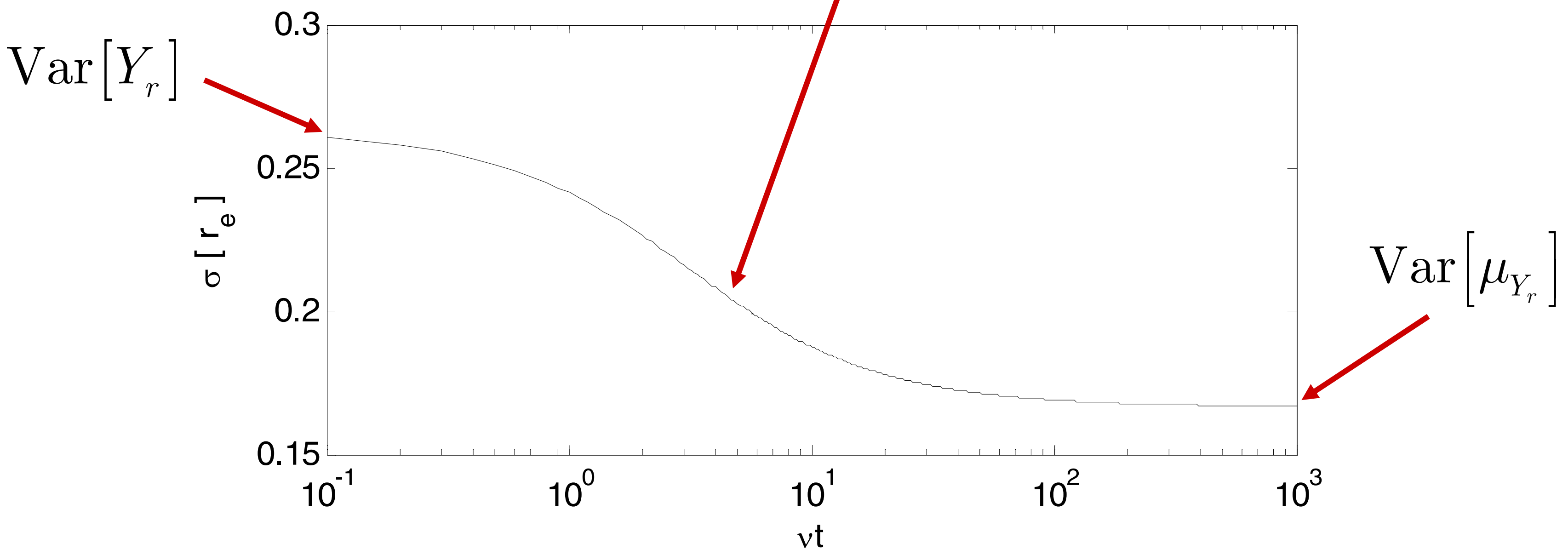
# Temporal variability in corrosion modeling

$$\mathrm{Var}\left[r_e(t)\right] = \mathrm{Var}\left[\mu_{Y_r}\right] + 2\left(\mathrm{Var}\left[Y_r\right] - \mathrm{Var}\left[\mu_{Y_r}\right]\right)\left(\frac{1}{\nu t} + \frac{\left(e^{-\nu t} - 1\right)}{(\nu t)^2}\right)$$



## Temporal variability in corrosion modeling

- The equivalent corrosion rate $r_e(t)$ can now be calculated at any point in time
- The time-variant analysis can be replaced by a time-invariant reliability analysis with $r_e(t)$

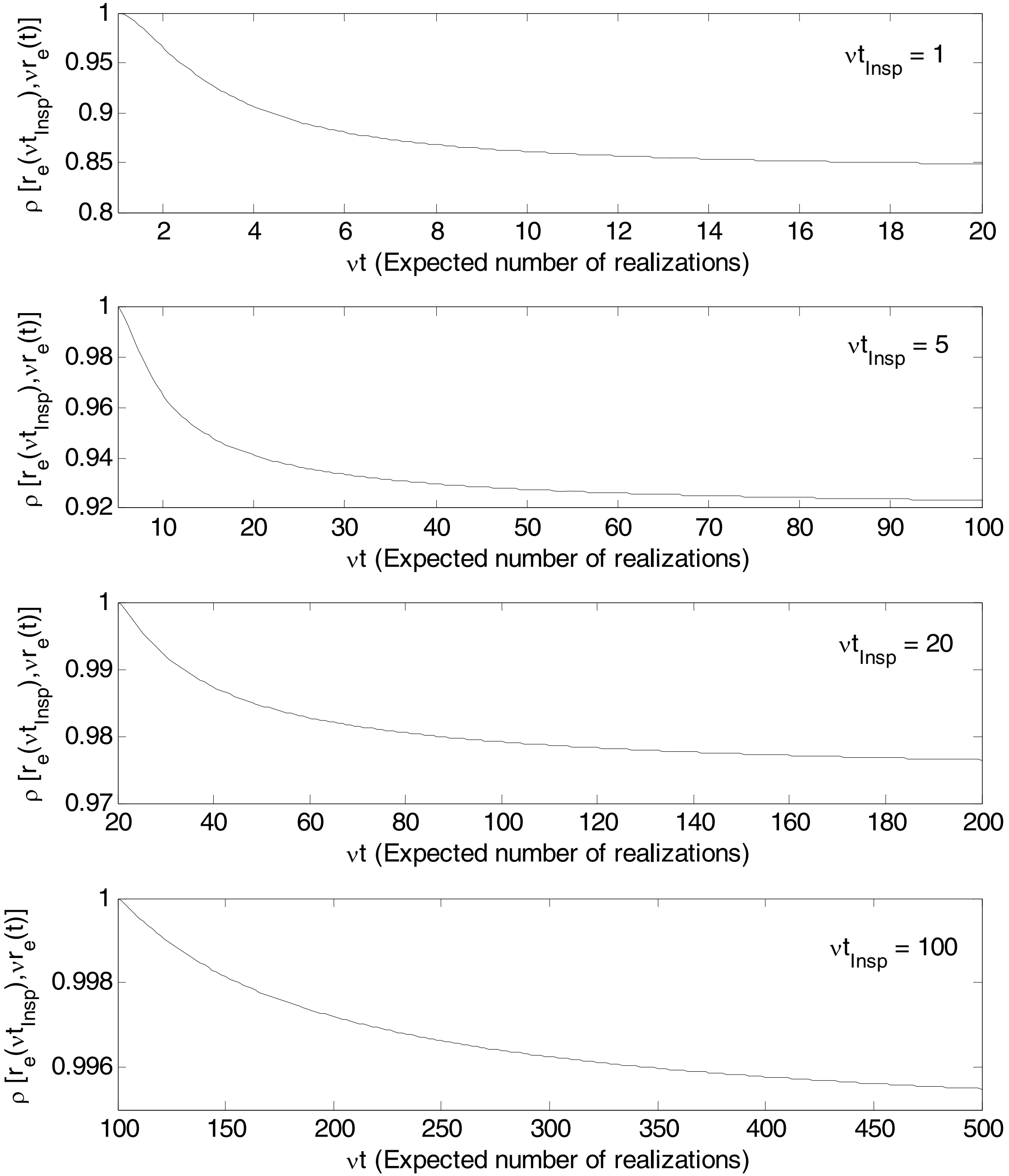
- How to consider the temporal variability in reliability updating?

# Temporal variability in corrosion reliability updating

- The application of equivalent values for the calculation of corrosion reliability may not be appropriate when considering reliability updating.
- This is investigated by calculating the correlation between the equivalent corrosion rate before and after the inspection:

$$\mathrm{Cov}\left[r_e\left(t_{insp}\right), r_e\left(t\right)\right] = \frac{1}{t_{insp}t}\int_0^{t_{insp}} \mathrm{d}\,t_1 \int_0^{t} \mathrm{d}\,t_2\, \mathrm{Cov}\left[r_{CO_2}\left(t_1\right), r_{CO_2}\left(t_2\right)\right]$$

$$= \mathrm{Var}\left[\mu_{Y_r}\right] + \left(\mathrm{Var}\left[Y_r\right] - \mathrm{Var}\left[\mu_{Y_r}\right]\right)\left(\frac{2}{\nu t} + \frac{1}{\nu t_{insp}\nu t}\left(-\mathrm{e}^{-\nu\left(t-t_{insp}\right)} + \mathrm{e}^{-\nu t} + \mathrm{e}^{-\nu t_{insp}} - 1\right)\right)$$



ρ [r_e(νt_Insp), νr_e(t)]
νt_Insp = 1
1
0.95
0.9
0.85
0.8
2 4 6 8 10 12 14 16 18 20
νt (Expected number of realizations)
ρ [r_e(νt_Insp), νr_e(t)]
νt_Insp = 5
1
0.98
0.96
0.94
0.92
10 20 30 40 50 60 70 80 90 100
νt (Expected number of realizations)
ρ [r_e(νt_Insp), νr_e(t)]
νt_Insp = 20
1
0.99
0.98
0.97
20 40 60 80 100 120 140 160 180 200
νt (Expected number of realizations)
ρ [r_e(νt_Insp), νr_e(t)]
νt_Insp = 100
1
0.998
0.996
100 150 200 250 300 350 400 450 500
νt (Expected number of realizations)

## Discussion & conclusions

- The use of an appropriate corrosion model is crucial for inspection & maintenance planning
- Temporal variability can be considered through the use of an equivalent corrosion rate as proposed
- The equivalent corrosion rate is derived for one example, but can be extended to other corrosion models
- The equivalent corrosion rate principle is also valid for reliability updating
- The same considerations apply also for other deterioration mechanisms