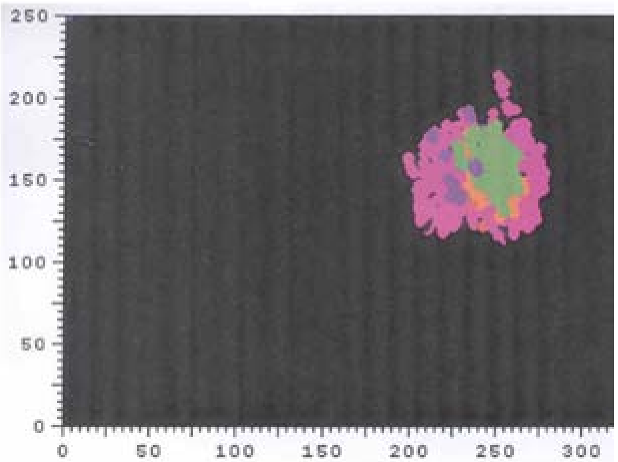

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Reliability Updating for Structures Subject to Localized Corrosion Defects

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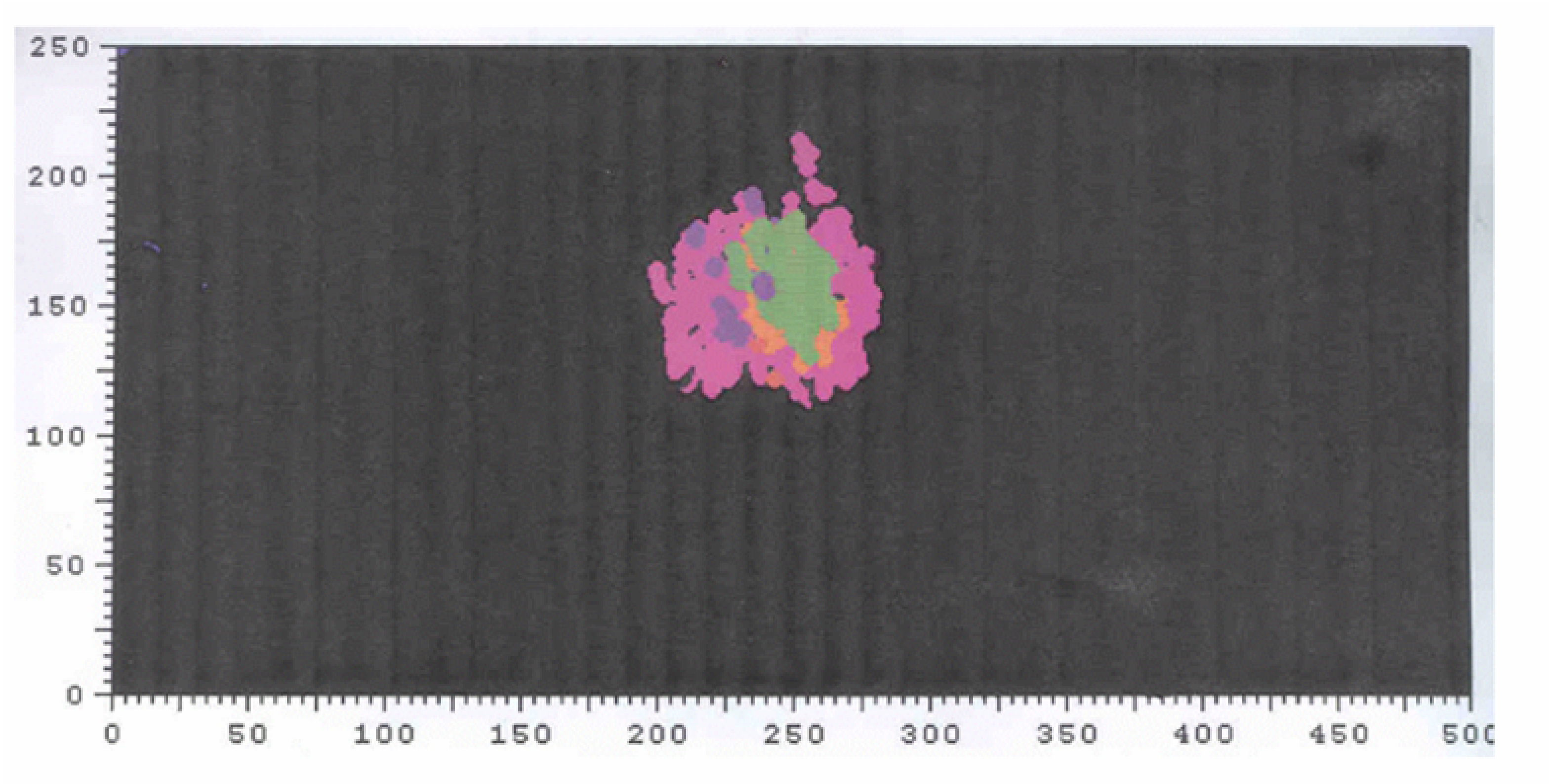
Motivation

- Inspection quality models have been derived for NDE methods aiming at the detection of flaws and cracks (POD, PFA ...)
- These models assume that the outcome of the inspection is binary (detection – no-detection)
- For inspections aiming at the detection of corrosion defects, this is not the case: the result is a measurement on a continuous scale
- During a JIP related to the assessment of the quality of inspections for corrosion, the question were:
    - What are the relevant indicators for the quality of the NDE?
    - How can these indicators be applied for reliability updating?
    - How can they be considered in inspection planning?

A typical inspection outcome

- A scanning method (ultrasonic based)
- Different colours indicate different wall thickness



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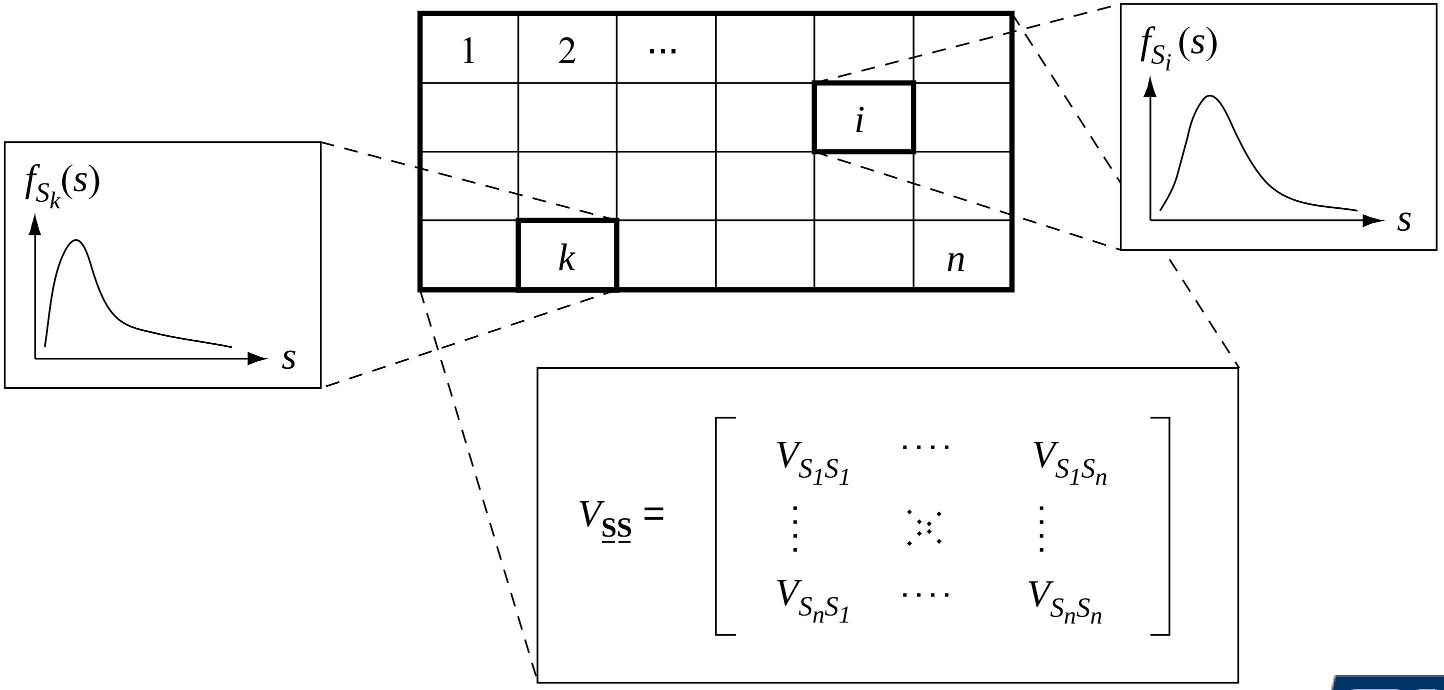
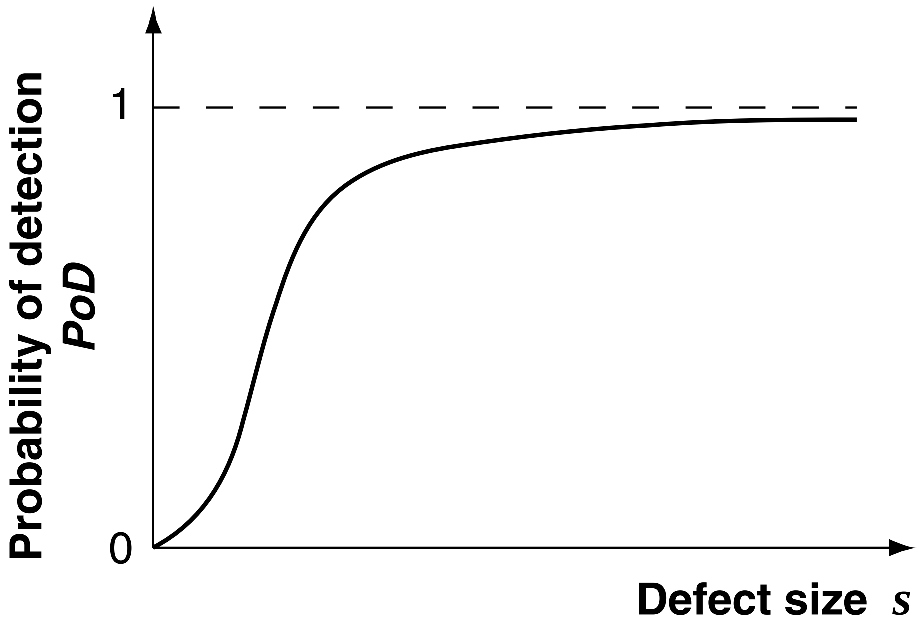
Characterisation of corrosion defects

- The structure is divided in individual elements
- Each element is characterised by its largest/deepest defect
- This is a function of time, described by corrosion models



Classical inspection performance models

- Probability of Detection (PoD), a function of defect size:



- Probability of False Alarm (PFA)
- Probability of Indication (PoI)

Classical inspection performance models

- Measurement uncertainty:
- Often considered through a measurement error ε_m , additive or multiplicative
- Here:

$$s = s_m - \varepsilon_m$$

Limit state functions for reliability updating

- Classically, two limit state functions are applied for reliability updating individually:
  - For the event of detection: $$g_D = z - \Phi^{-1}\left(PoD\left(s\right)\right)$$
  - For the measurement event: $$g_M = s - \left(s_m - \varepsilon_m\right)$$

    (this is an equality event)

- For localised corrosion defects, a measurement is always made, although it is unclear whether the deepest defect has actually been detected.

➢ Both LSF must be combined

- But how?

Reliability updating for localized corrosion

- The observable event is the detection of a defect with size s_O
- It is not clear, if s_O is the largest defect in the considered element
- Updating with s_O :

$$f''_S\left(s \middle| s_O\right) = \frac{L\left(s \middle| s_O\right) f'_S\left(s\right)}{f_{S_O}\left(s_O\right)}$$

- The corrosion model only describes the largest defect in the element

➢ The the likelihood function and the constant $f_{S_O}\left(s_O\right)$ cannot be determined for all *s*. It is only know that

$$L\left(s \middle| s_O\right) = \mathrm{h}_O\left(s, s_O\right), \qquad s_O \leq s$$

$$L\left(s \middle| s_O\right) = 0, \qquad s_O > s$$

(Measurement uncertainty is neglected at first)

Reliability updating for localized corrosion

- Assumption: $f_S''(s|s_O) = f_S'(s), \qquad s > s_O$

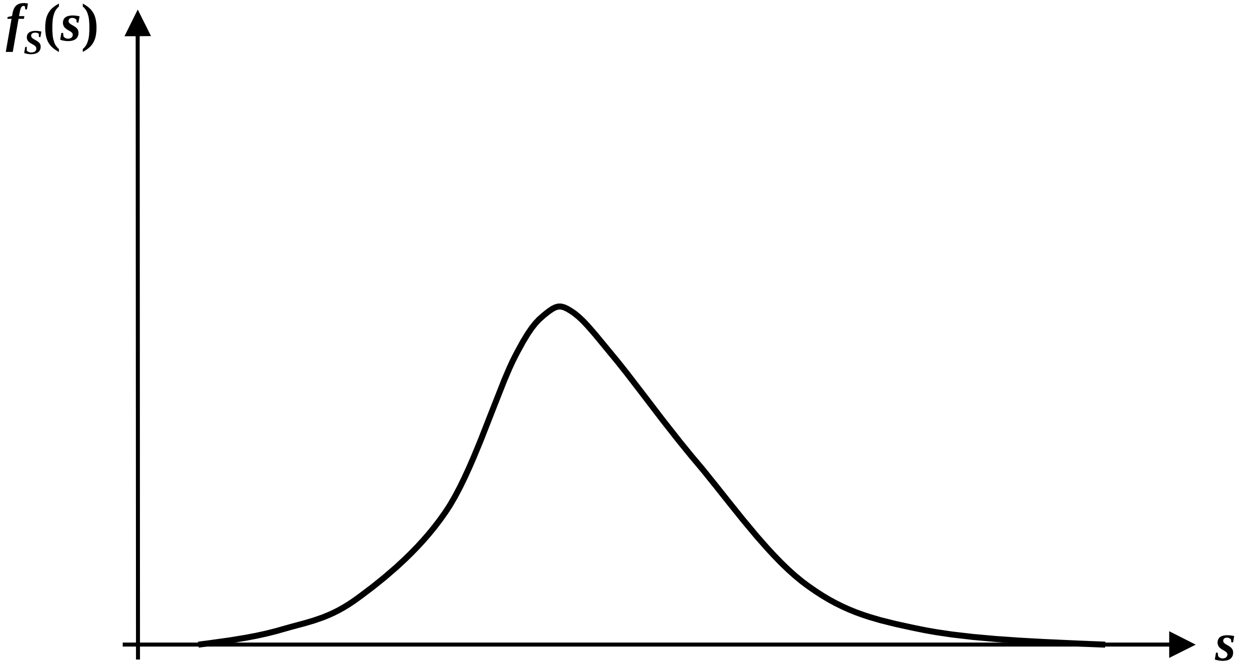
  (Reasonable when individual defects within an element are considered independent)

- the full posterior pdf is then

$$f_S''(s|s_O) = \begin{cases} 0\,, & s < s_O \\ F_S'(s_O)\,\delta(s - s_O)\,, & s = s_O \\ f_S'(s)\,, & s \geq s_O \end{cases}$$

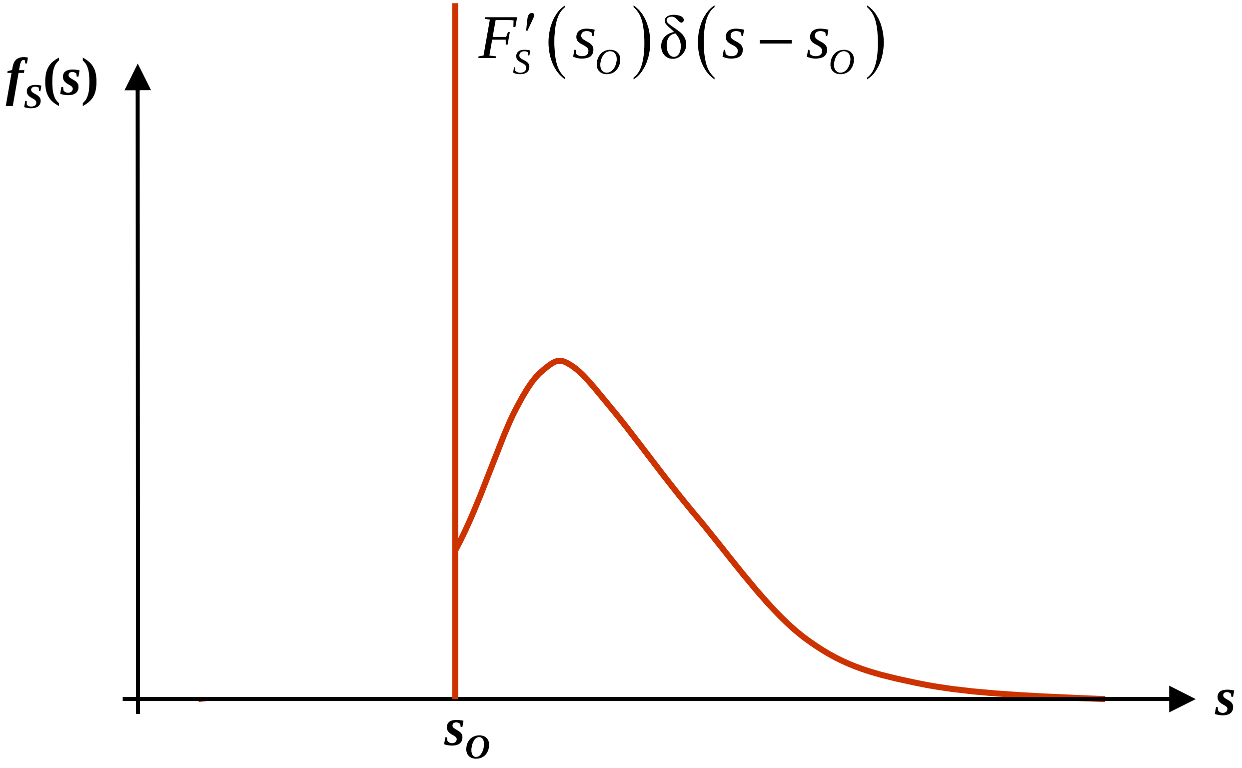
Reliability updating for localized corrosion

- Prior model



Reliability updating for localized corrosion

- After the event s_O - Without measurement uncertainty



Reliability updating for localized corrosion

- The event that no detect is larger than s_O is denoted by L
- Additionally to s_O , the event of no-detection of a defect larger than s_O is observed, denoted by $\bar{I}$
- The final posterior pdf of s is obtained by use of the likelihood function

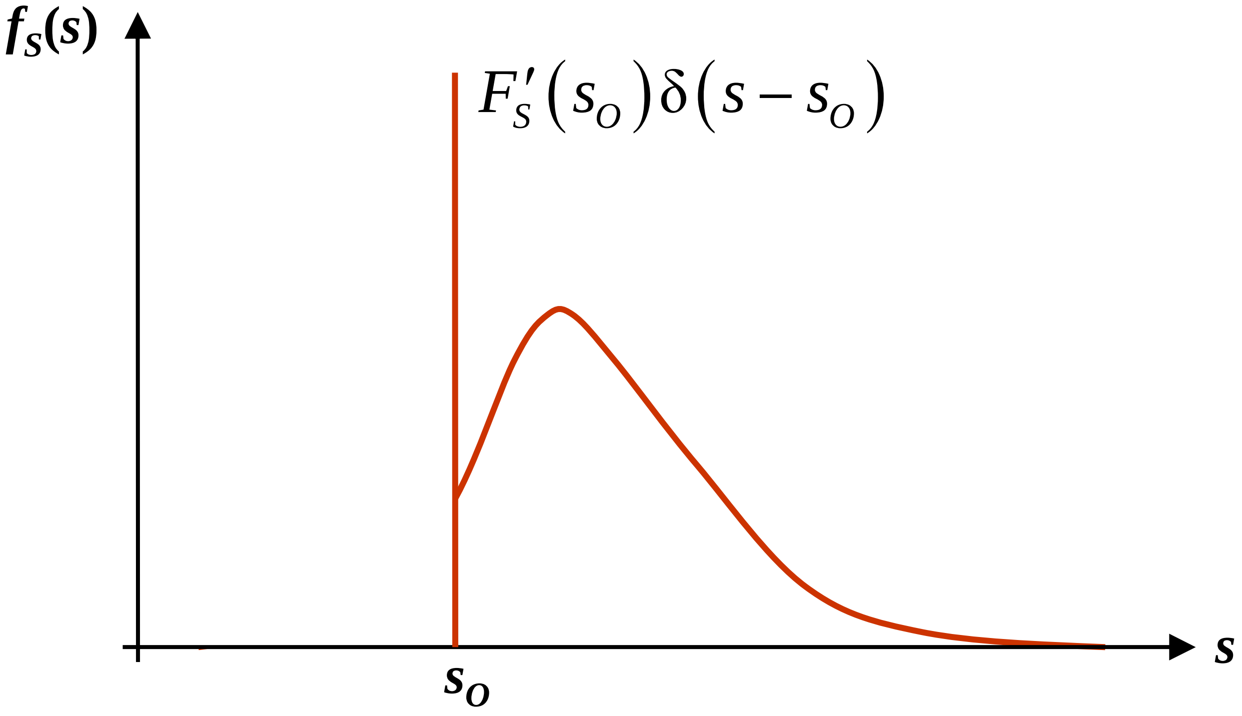
$$P\left(\bar{I} \middle| s \cap s_O\right) = \begin{cases} 1\,; & s \leq s_O \\ \left(1 - PoD(s)\right); & s > s_O \end{cases}$$

- and results in

$$f_S''\left(s \middle| \bar{I} \cap s_O\right) = \begin{cases} 0\,, & s < s_O \\ F_S'(s_O)\,\delta(s - s_O)\,P\left(\bar{I} \middle| s_O\right)^{-1}, & s = s_O \\ \left(1 - PoI(s)\right) f_S'(s)\,P\left(\bar{I} \middle| s_O\right)^{-1}, & s > s_O \end{cases}$$

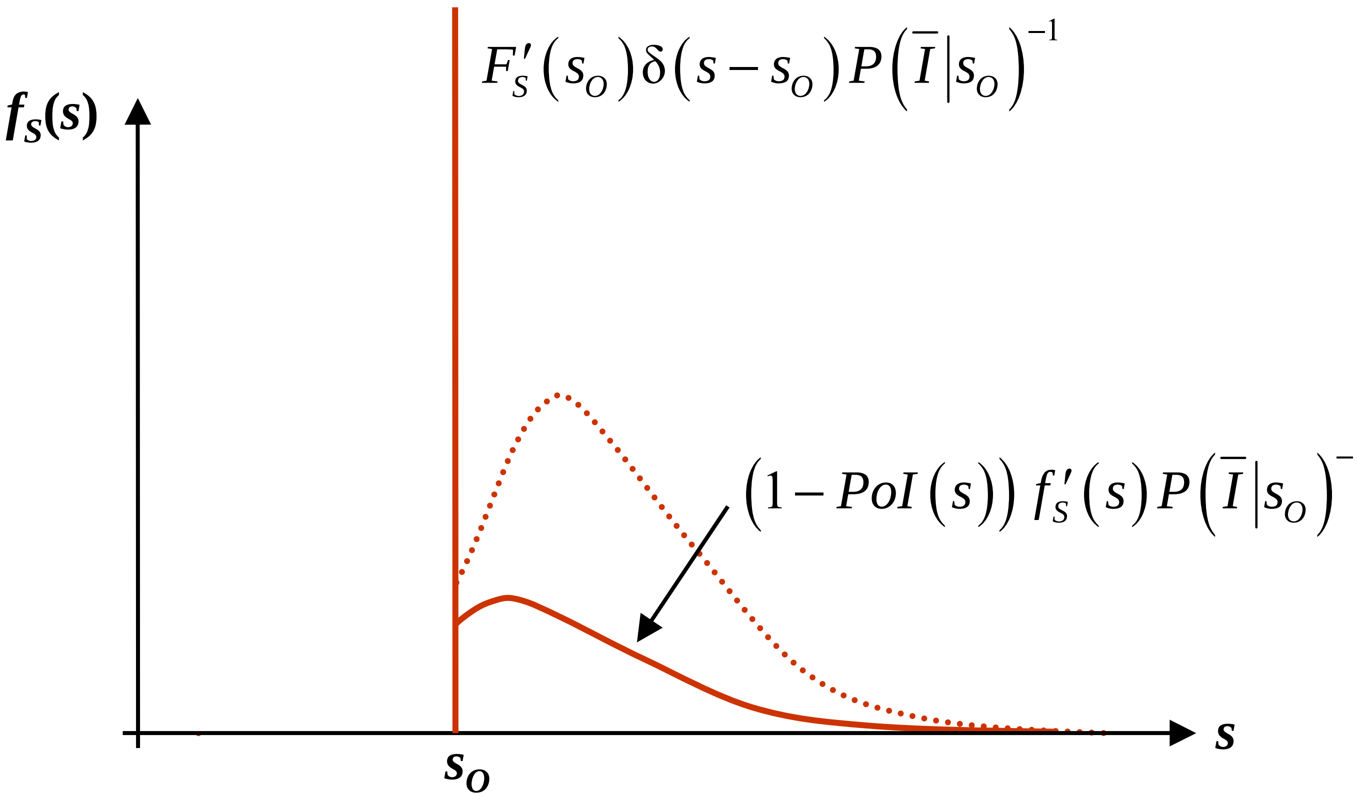
Reliability updating for localized corrosion

- After the event s_O - Without measurement uncertainty



Reliability updating for localized corrosion

- After the events s_O and $\bar{I}$ - Without measurement uncertainty



Reliability updating for localized corrosion

- Application of structural reliability: Update the event of failure
- The event of no-detection of the largest defect is introduced as

$$\overline{I_S} = \left\{ \overline{D} \cap L \right\}$$

- Then with some mathematical manipulation it is shown that

$$P\left(F \middle| \overline{I} \cap s_O \cap \overline{I_S} \right) = P\left(F \middle| \overline{I_S} \right)$$

$$P\left(F \middle| \overline{I} \cap s_O \cap I_S \right) = P\left(F \middle| s_O \cap \overline{L} \right)$$

- It follows that

$$P\left(F \middle| s_O \cap \overline{I} \right) = P\left(F \middle| \overline{I_S} \right) P\left(\overline{I_S} \right) + P\left(F \middle| M \right) P\left(I_S \right)$$

Reliability updating for localized corrosion

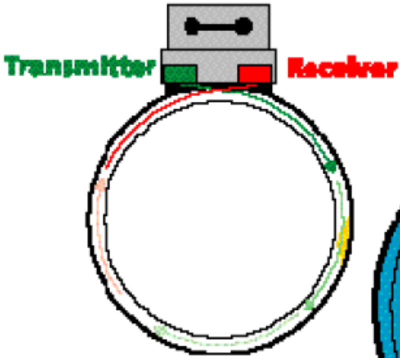
$$P\left(F \middle| s_O \cap \overline{I}\right) = P\left(F \middle| \overline{I_S}\right) P\left(\overline{I_S}\right) + P\left(F \middle| M\right) P\left(I_S\right)$$

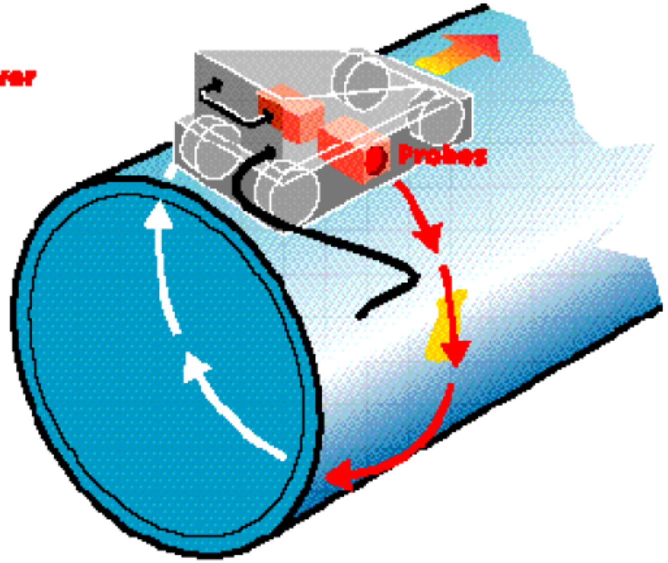
no-detection of the largest defect

measurement of the detected defect

- Both events can be described by the classical limit state functions describing inspection outcomes.
- Application of structural reliability analysis is facilitated

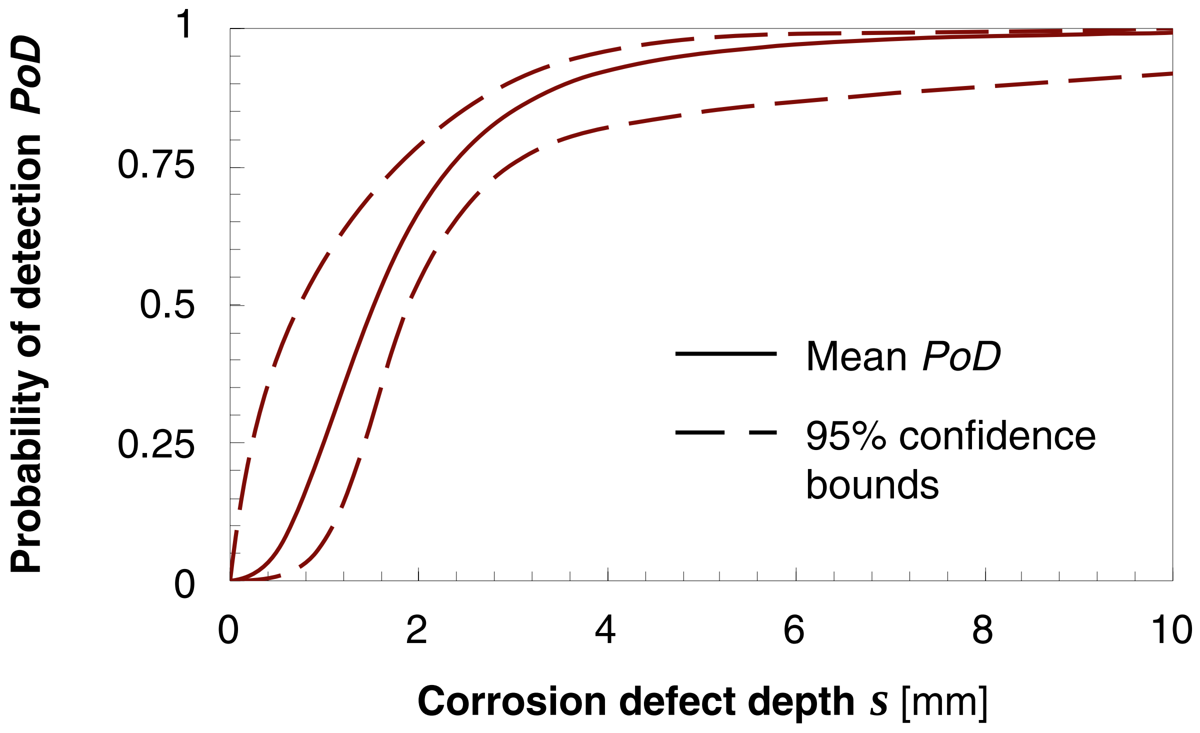
Example





Example – Inspection quality model

- Data obtained from a JIP
- Measurement uncertainty: $\varepsilon_m \sim \mathrm{N}(0, 0.8\mathrm{mm})$
- PoD: Probability of detection of the largest defect



Example - Deterioration model

- CO_2 corrosion in a pipeline $$g = d_{cr} - d_C(t)$$
- DeWaards-Miliams model: $$d_C(t) = X_M r_{CO_2} t$$

$$r_{CO_2} = 10^{\left(5.8 - 1710/T_o + 0.67 \cdot \log_{10} f_{CO_2}\right)}$$

$$f_{CO_2} = P_{CO_2} \cdot 10^{P_o\left(0.0031 - 1.4/T_o\right)}$$

$$P_{CO_2} = n_{\mathrm{CO_2}} P_o$$

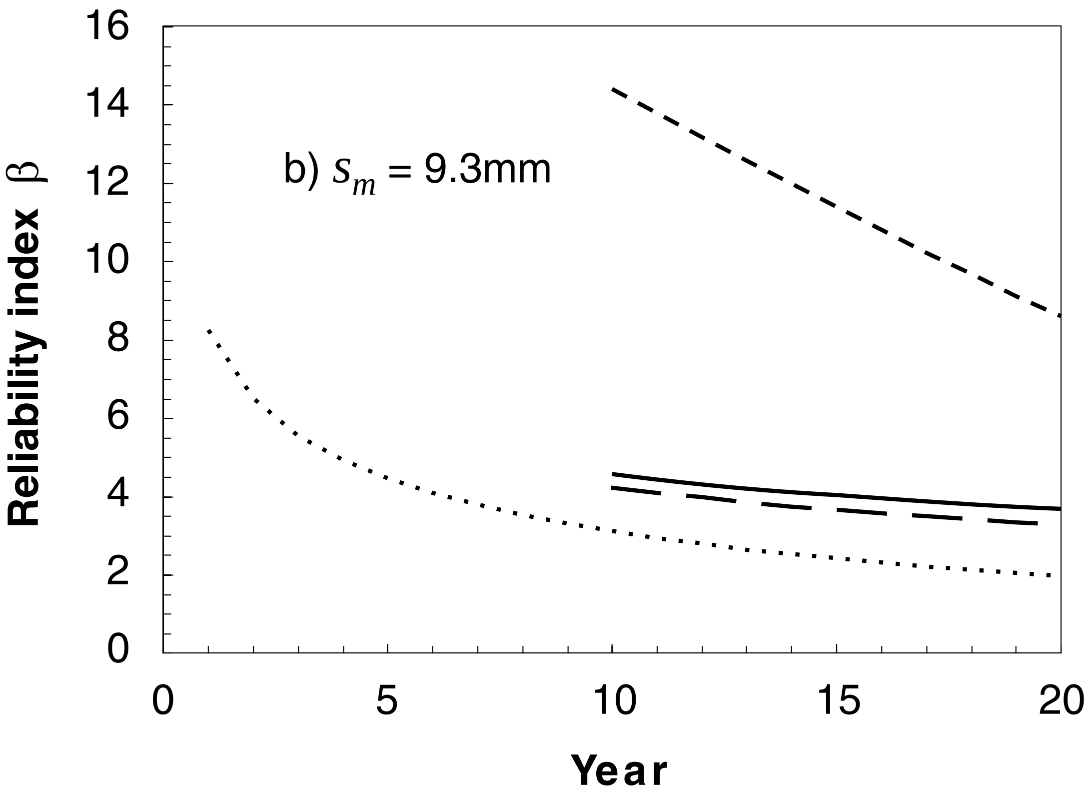
Table 1. Parameters of the corrosion model.

Parameters	Dimension	Mean	St. dev.	Dist. type
d	mm	30	1.5	Normal
T_o	K	303	3	Normal
P_o	bar	100	10	Normal
n_{CO_2}	-	0.01	-	Determ.
X_M	-	0.4	0.32	Weibull

Example – Results

- Measurement

  9.3mm



- - - - - - Without inspection

— – Updating based on the no-indication event

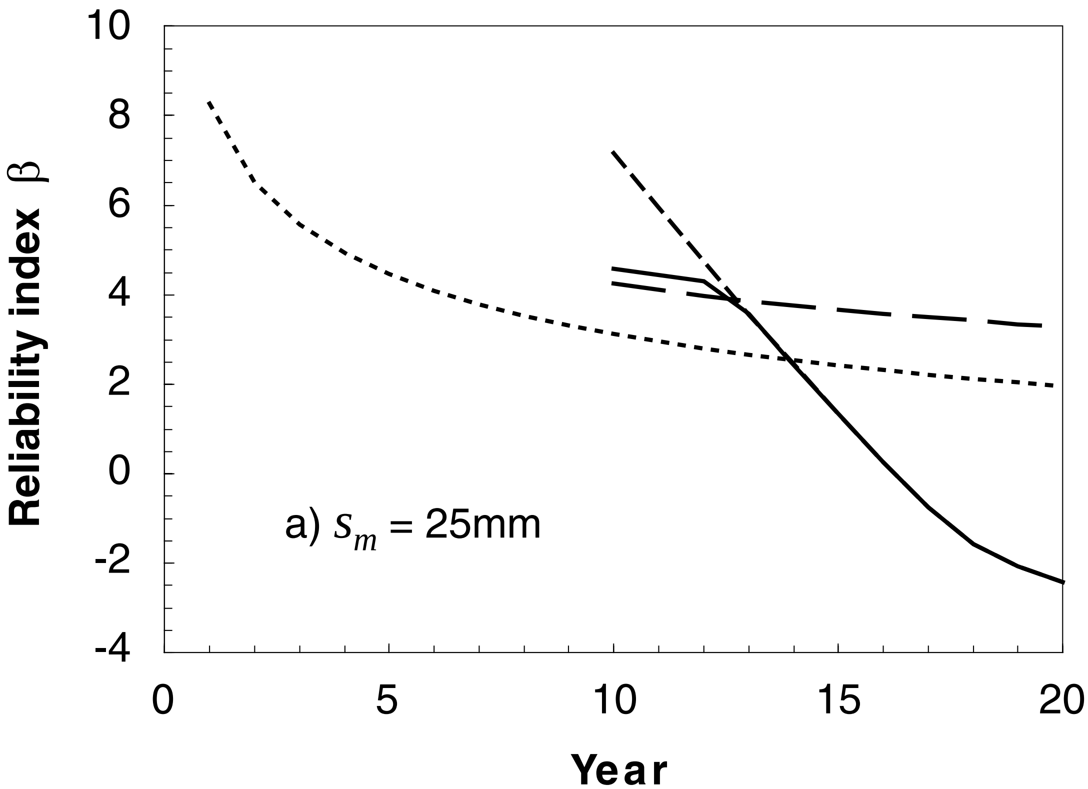
- - - - Updating based on the measurement of s_m

—— Investigated model: Updating based on the combined events

Example – Results

- Measurement

  25mm



Without inspection

Updating based on the no-indication event

Updating based on the measurement of s_m

Investigated model: Updating based on the combined events

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Conclusions

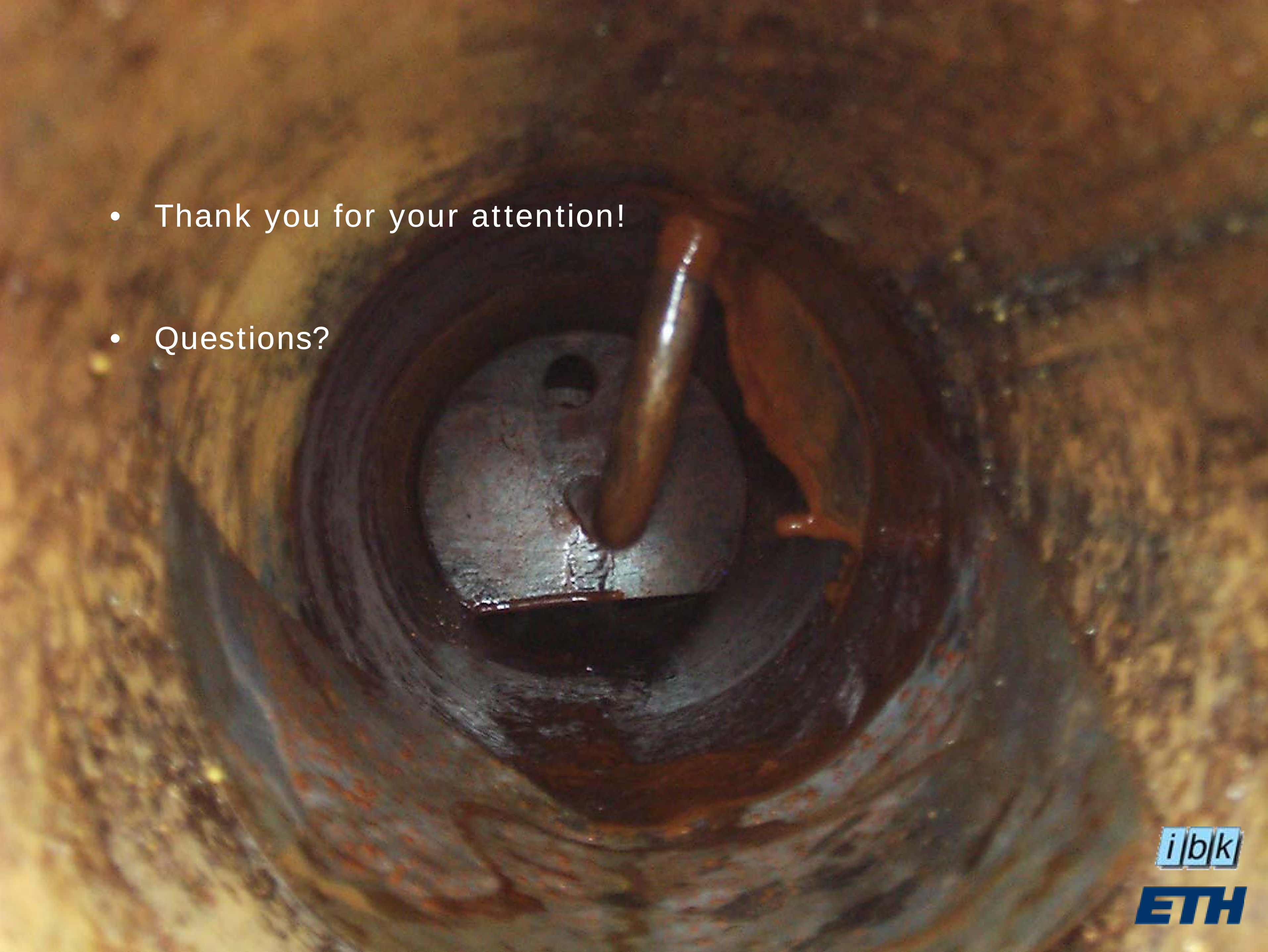
- The reliability for localized corrosion can be updated by calculating the following probabilities by SRA:

$$P\left(\overline{I_S}\right) = P\left(\overline{D} \cap L\right)$$

$$P\left(F \middle| \overline{I_S}\right) = P\left(F \middle| \overline{D} \cap L\right)$$

$$P\left(F \middle| M\right)$$

- This requires the standard models for corrosion reliability and for inspection perfomance
- The updated probability can be approximated by considering the measurement and the indication event individually
- This facilitates an application of the models in inspection planning



- Thank you for your attention!
- Questions?