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System Effects in Portfolio Loss Estimation

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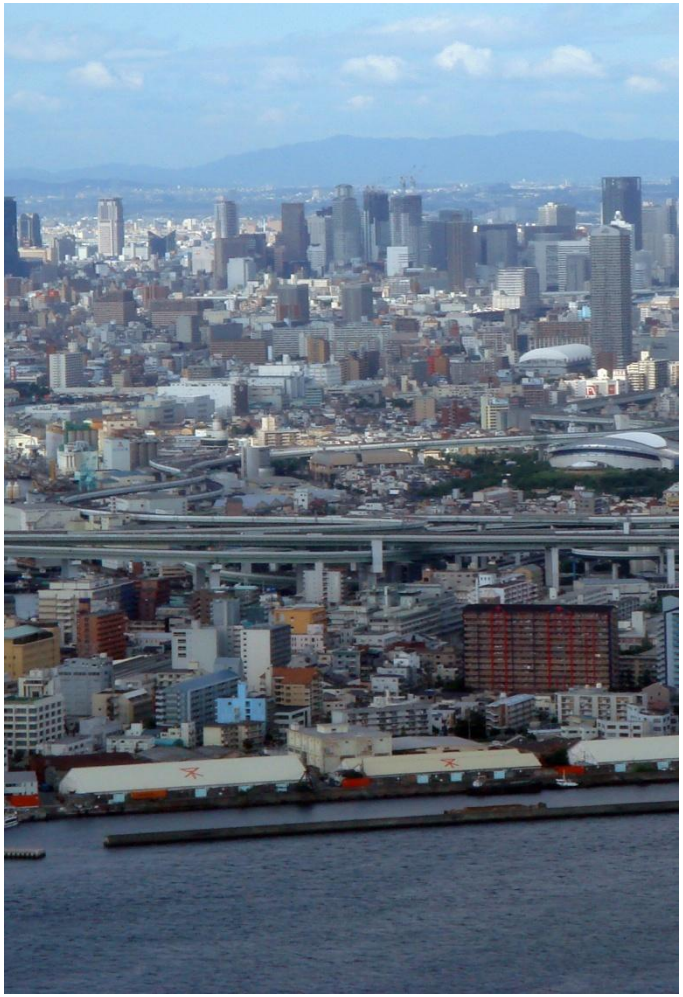


Overview

- *Introduction*
- *Dependencies in portfolios*
- *Hierarchical model formulation*
- *System effects in portfolios*
- *Conclusions*

Introduction

- Decision makers and stake holders managing portfolios of assets (road authorities, rail companies, etc.) need an overview of their risks for strategic planning.

- The aggregation of risks and the loss estimation are crucial requirements for the management.

- Loss estimation is influenced by two factors: expected value and the variance of losses.

- In principle decision-makers are faced with three questions:

Introduction

- How large is the probability of a total loss of the the portfolio?
- How large is the probability that a certain budget is exceeded?
- Are there dependencies and nonlinearities in the portfolio which lead to an increase of the risk?

Dependencies in Portfolios

- Geographic locations lead to common aleatoric effects:

  - Floods, earthquakes, strong winds, avalanches,…

  - common economic conditions.

Dependencies in Portfolios

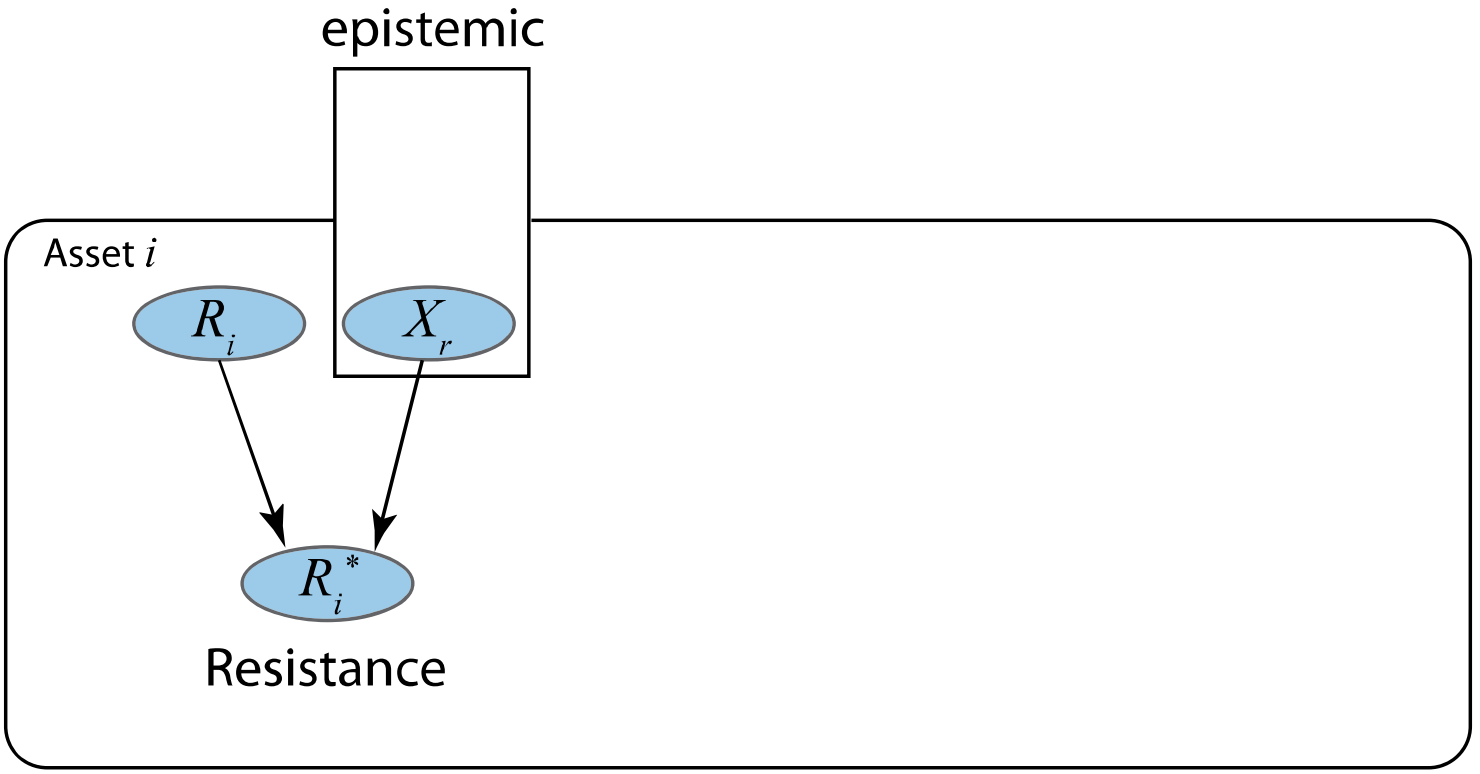
- Best practices lead to common epistemic effects.

  The set of available models is restricted; same models are used for the design for same failure modes. Design codes, standards, common procedures, …

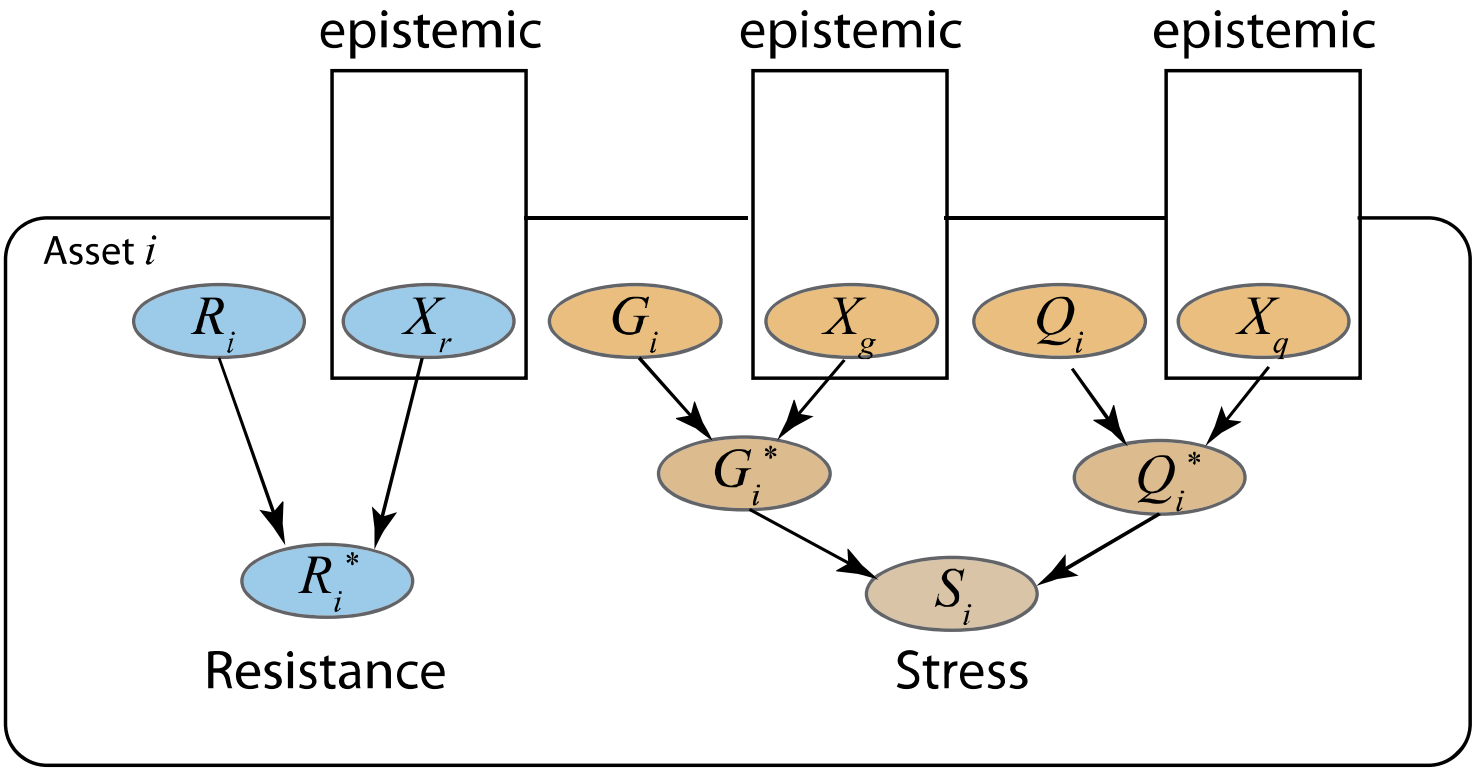
  The models are imperfect – the same effect realizes in the same models.

→ All common effects introduce dependencies in the model and have to be considered explicitly in a portfolio model.

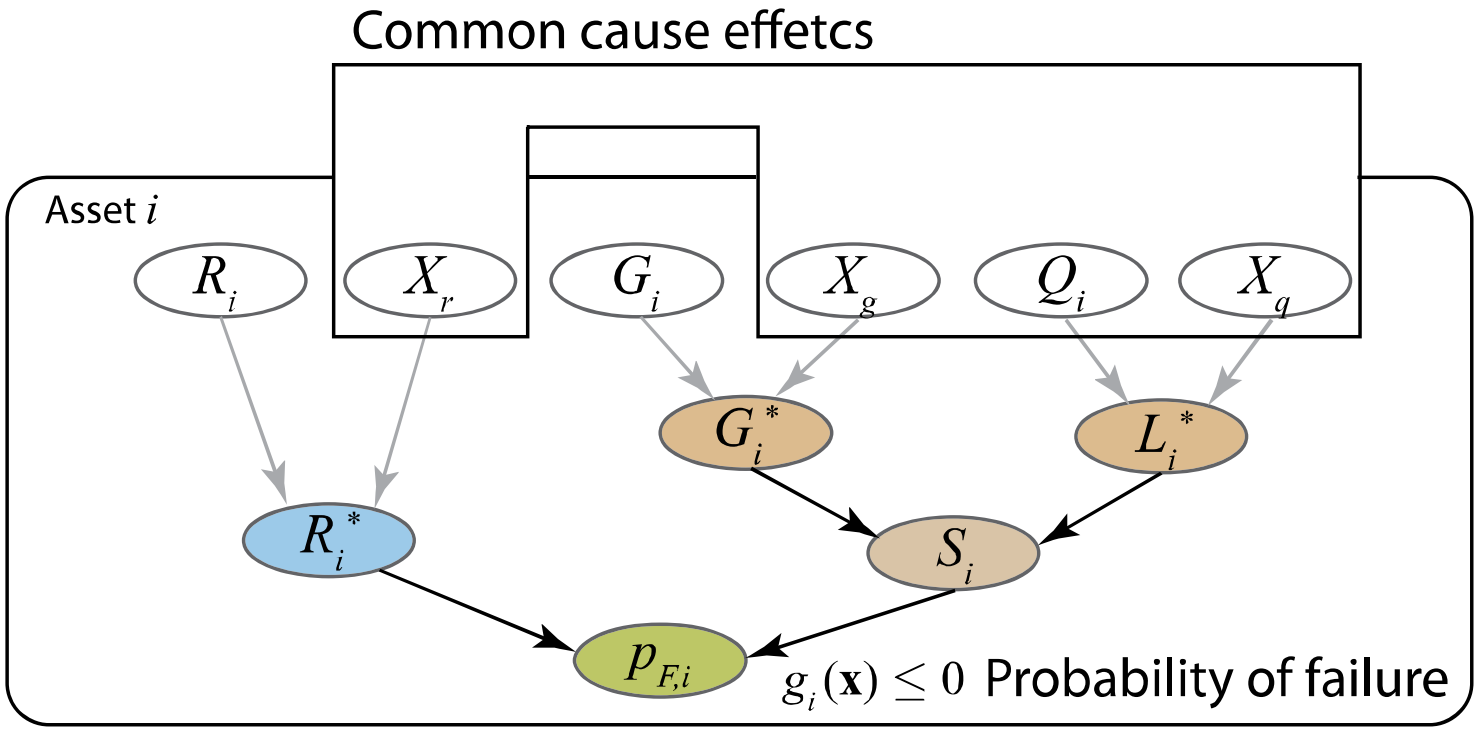
Dependencies in Portfolios



Dependencies in Portfolios



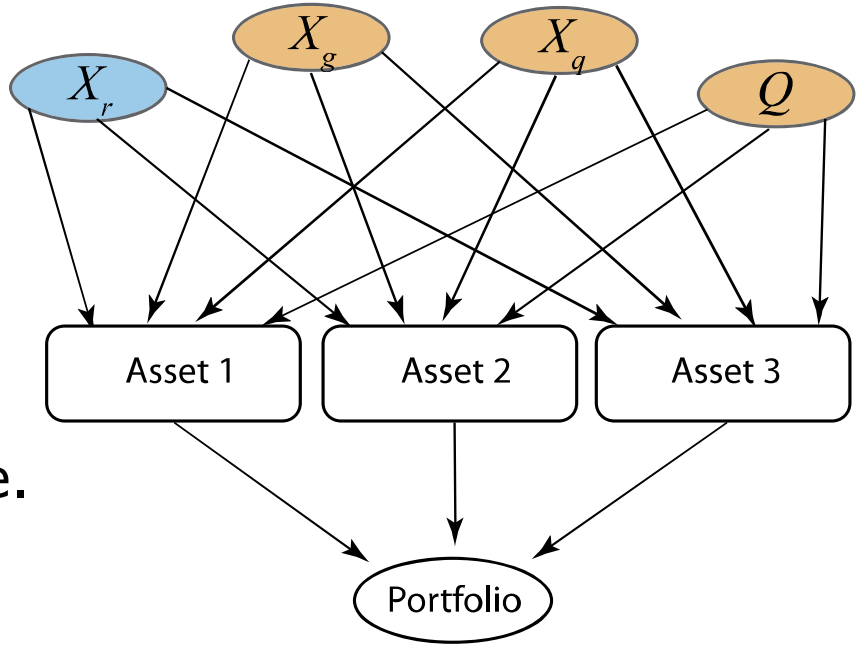
Dependencies in Portfolios



Hierarchical model formulation

Homogeneous portfolios:

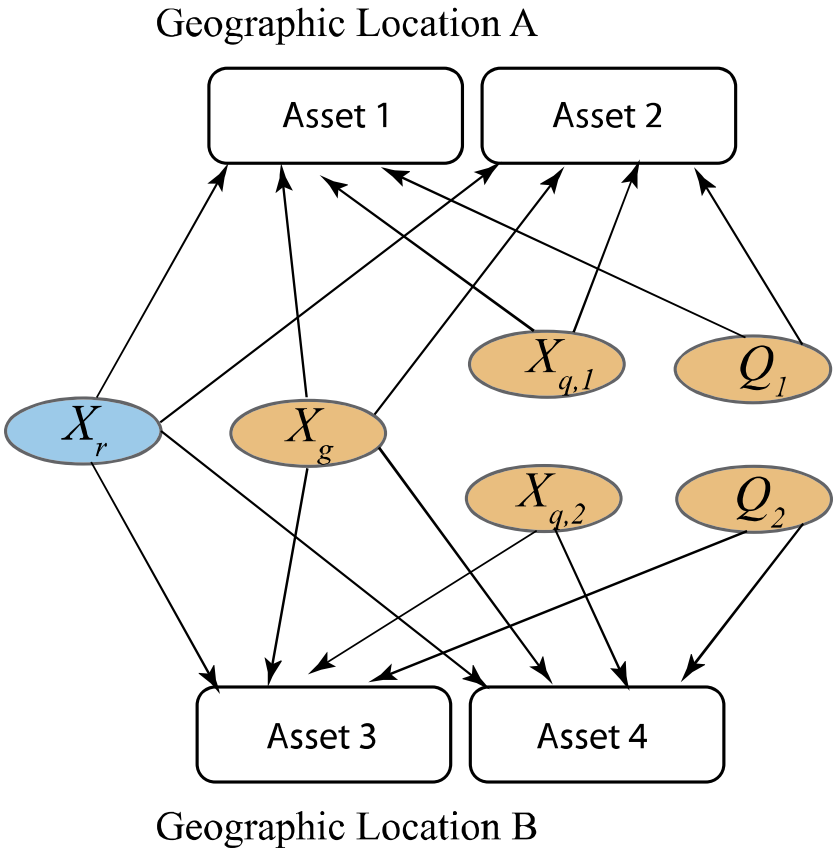
- Number of assets.
- Identical failure probabilities.
- Uniform dependency structure.



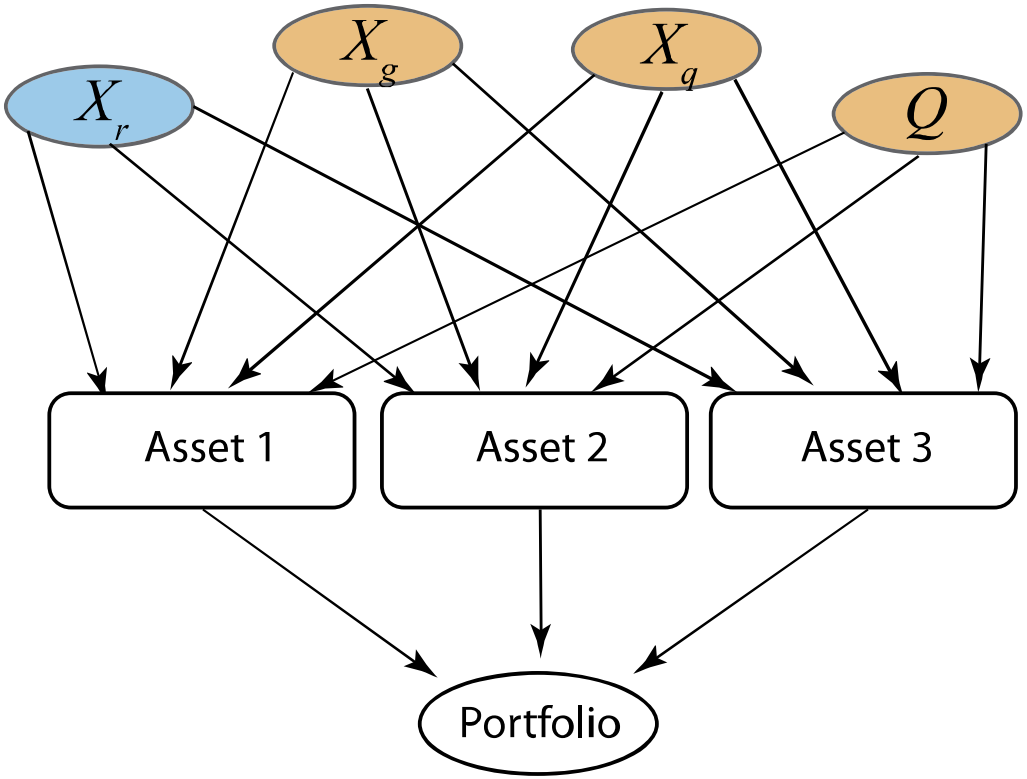
Hierarchical model formulation

Inhomogeneous portfolios:

- Number of assets.
- Different external conditions.
- Nonuniform dependency structure.



Hierarchical model formulation



$$p_N(n) = \binom{k}{n} \underbrace{\int_{g_1(\mathbf{x}) \le 0} \dots \int_{g_n(\mathbf{x}) \le 0}}_{n} \underbrace{\int_{g_{n+1}(\mathbf{x}) > 0} \dots \int_{g_k(\mathbf{x}) > 0}}_{n-k} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}$$

Hierarchical model formulation

Advantages:

- Existing sub-models can be used; dependencies are modeled on a higher hierarchical level.

- Hierarchical approach utilize causal relations among components.

- Failure probabilities of assets can be assessed conditionally independent.

Hierarchical model formulation

- Loss distribution function $p_N(n)$ can be assessed *almost* independent from the size a homogeneous portfolio:

$$p_N(n) = \int_{\boldsymbol{\theta}} \binom{k}{n} \left(p_F(F|\boldsymbol{\theta})\right)^n \left(1 - p_F(F|\boldsymbol{\theta})\right)^{k-n} f_{\boldsymbol{\theta}}(\boldsymbol{\theta})\, d\boldsymbol{\theta}$$

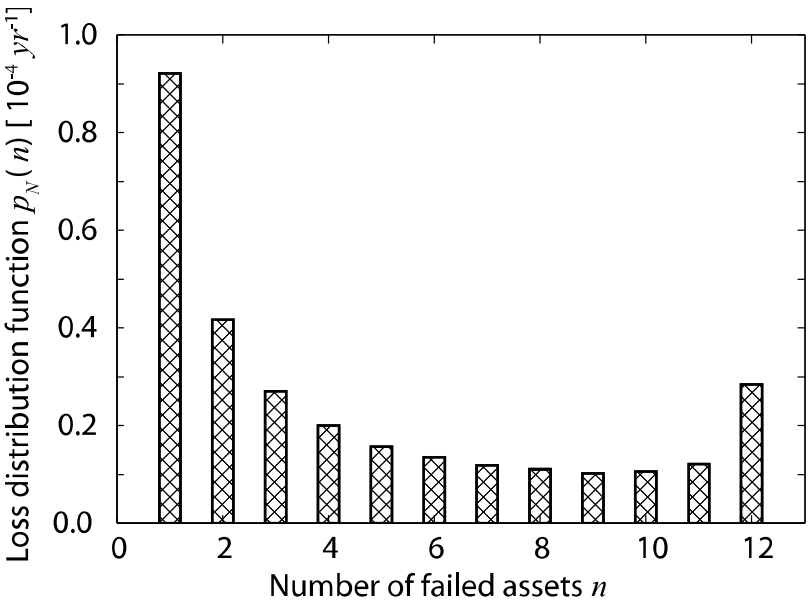
- Inhomogeneous portfolios can be divided into a set of homogeneous portfolios; each treated as a conditionally independent random variable:

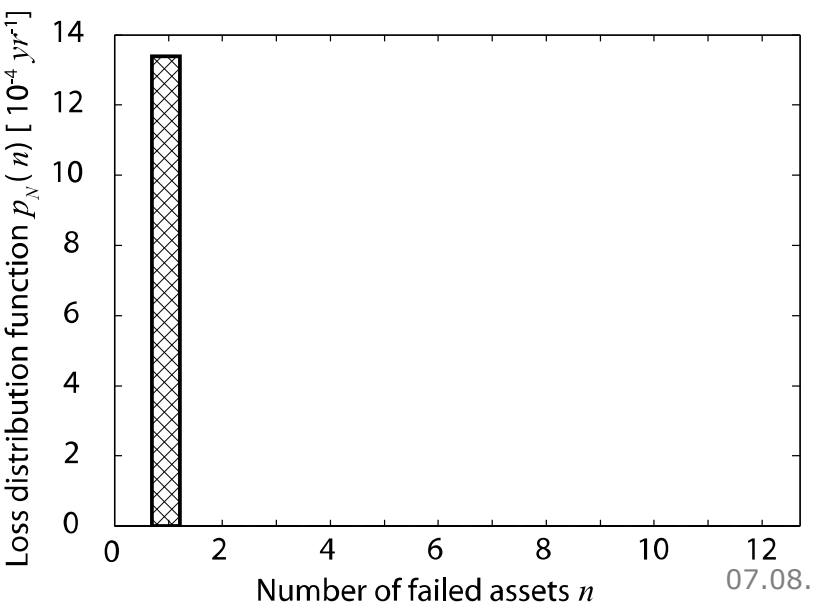
$$p_N(n) = \Pr(N_1+N_2=n) = \int_{\boldsymbol{\theta}} \sum_{i=0}^{n} \Pr(N_1=i \mid \boldsymbol{\theta}) \Pr(N_2=n-i \mid \boldsymbol{\theta}) f_{\boldsymbol{\theta}}(\boldsymbol{\theta})\, d\boldsymbol{\theta}$$

System Effects

- Homogeneous portfolio with 12 identical assets and subjected to an identical variant load.

- Distinct tail of the loss distribution.
  Expected loss is identical – nonlinearity of consequences will increase the expected losses.

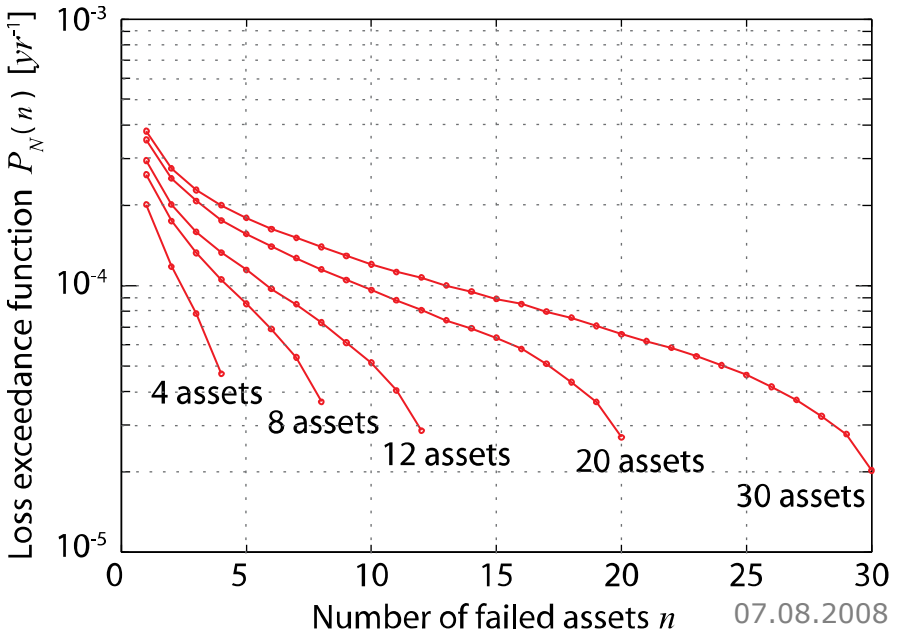




Homogeneous portfolio

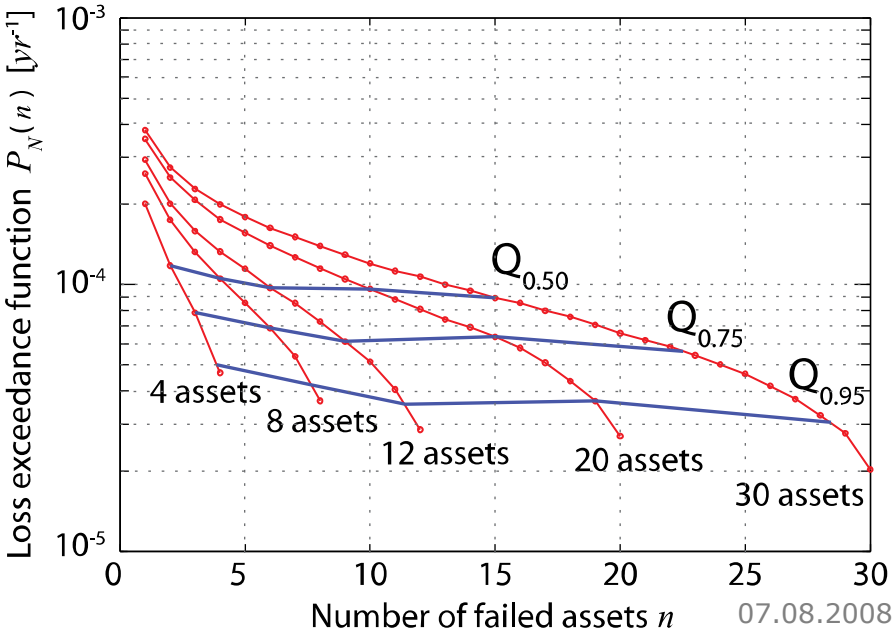
- Two effects are observable: systematic effects and unsystematic effects.

- Unsystematic effects vanish with the number of assets in the portfolio; systematic effects remain.



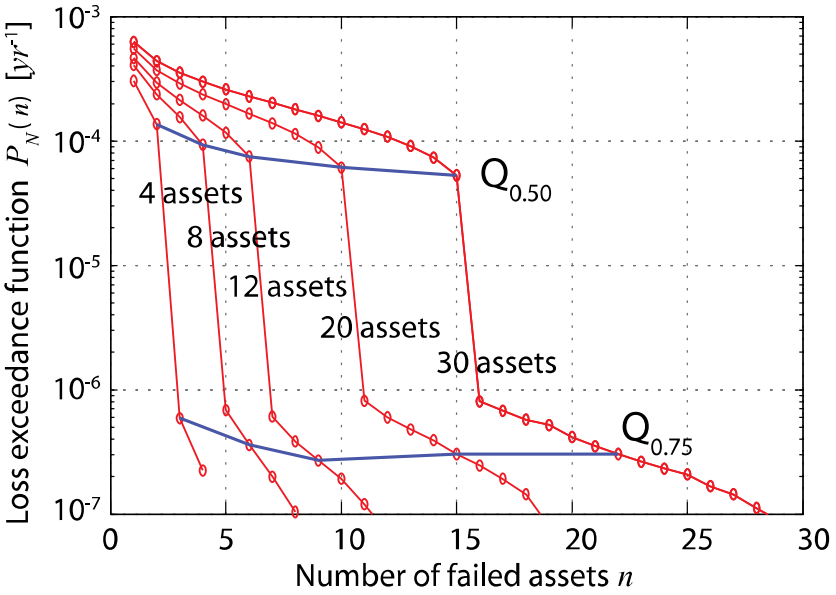
Homogeneous portfolio

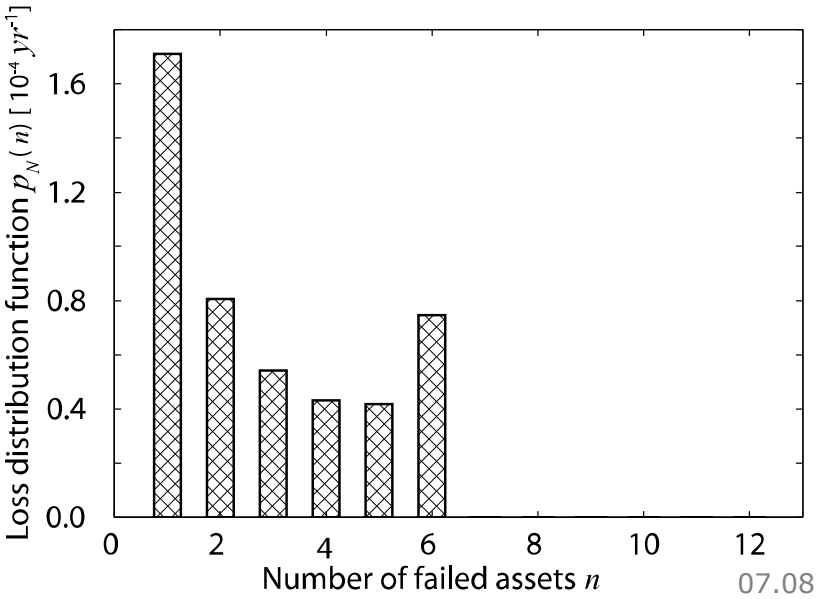
- Two effects are observable: systematic effects and unsystematic effects.
- Unsystematic effects vanish with the number of assets in the portfolio; systematic effects remain.
- Increasing the number of assets does not decrease the probability for large losses



Inhomogeneous portfolio

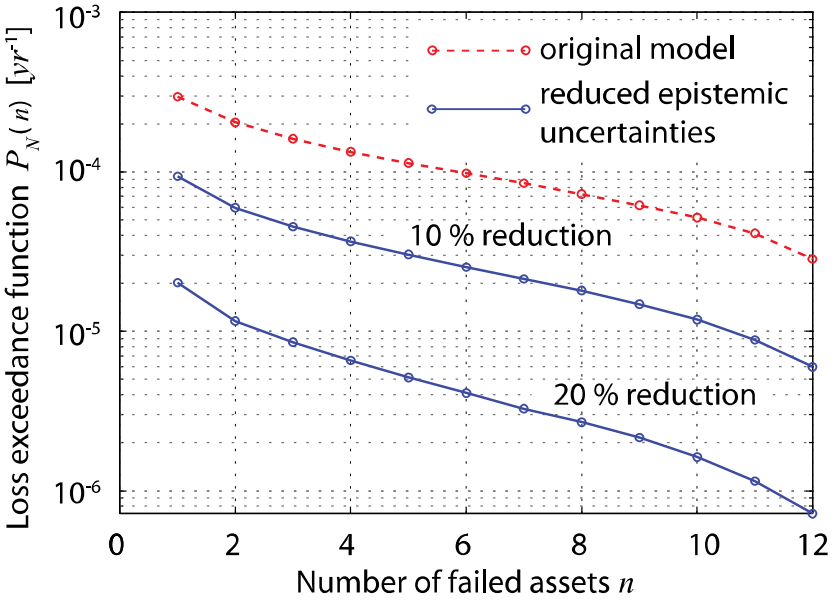
- Inhomogeneous portfolio; different geographical location, different variant loads.
- The probability that half of the portfolio is lost is decreased. Adding assets from the same population does not change significantly the probability of large losses.

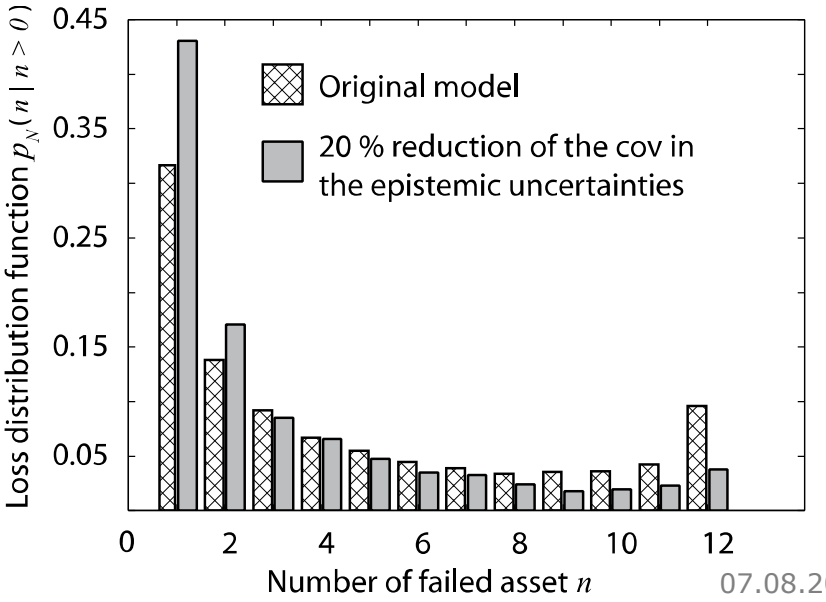




Risk reduction measure

- Risk reduction: Reduction of epistemic uncertainties in a portfolio.
- Two positive effects: Expected number of failures is decreased; Dependency is decreased.
- Might become rational to improve the best practice.





Conclusions

- Hierarchical approach for the modeling of portfolio losses is presented.
- Allows using conditional independence among assets in the portfolio.
- Different sources of common causes (geographical, best practices) lead to large variance in the loss distribution function.
- Especially for the aggregation of risks of importance; neglecting such common causes lead to sub optimal decisions if consequences behave nonlinear.

System Effects in Portfolio Loss Estimation

Thank you for your attention