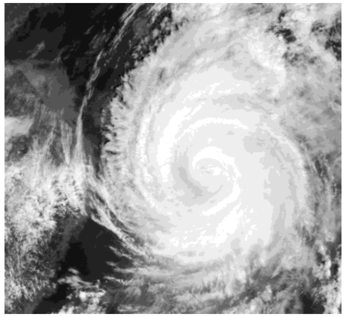
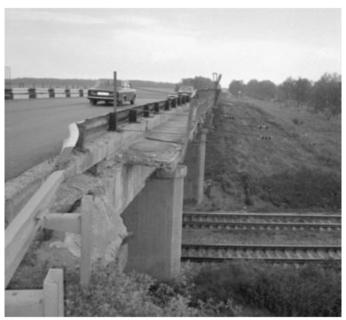











Risk & Safety in Engineering

Dr. Jochen Köhler

Contents of Today's Lecture

- **Introduction to Classical Reliability Theory**
- **Structural Reliability**
  - The fundamental case
  - Safety Margin

Introduction to Classical Reliability Theory

Initially the reliability theory was developed for systems with a large number of (semi-) identical components subject to the same exposure conditions

- electrical systems (bulbs, switches, ..)

and later on to

- nuclear power installations (valves, pipes, pumps, ..)

- chemical plants (pipes, pressure vessels, valves, pumps,..)

- manufacturing plants (pumps, compressors, conveyers,..)

Introduction to Classical Reliability Theory

For such systems the probability of a componential failure may be assessed in a frequentistic manner –

from observed failure rates

- number of failures per component operation hours

Due to the characteristics of the failure mechanism

a steady deterioration as function of time/use

for the considered components the main concern was centred around the statistical modelling of the time till failure

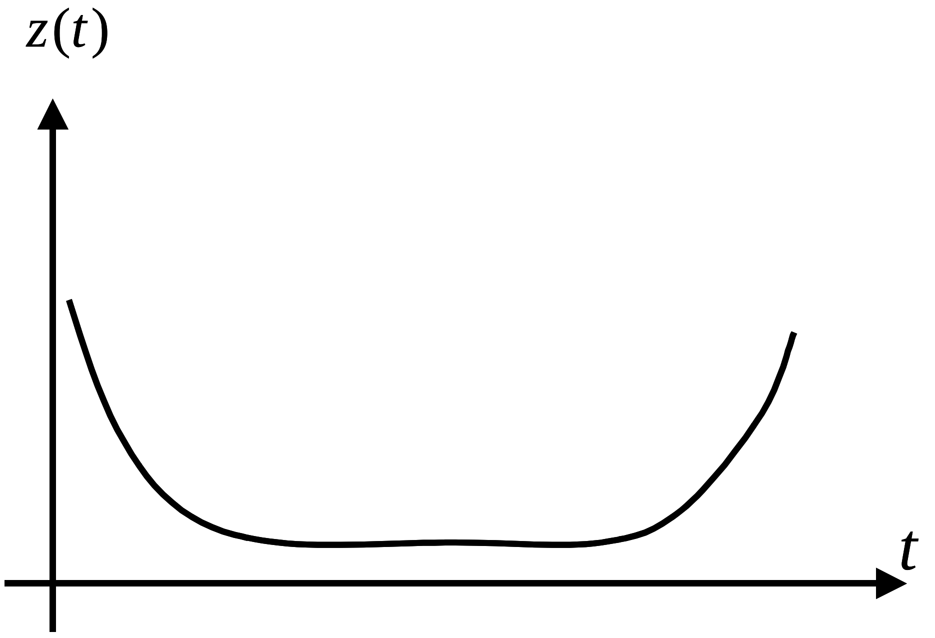
Introduction to Classical Reliability Theory

The failure rate function is typically use to represent the hazard of failure over time.

First there is a rate of failure due to birth defects.

Then there is a steady state.

Then deterioration starts and the failure rate increases.



Introduction to Classical Reliability Theory

For technical systems such as structures and structural components the classical reliability analysis is of limited use because

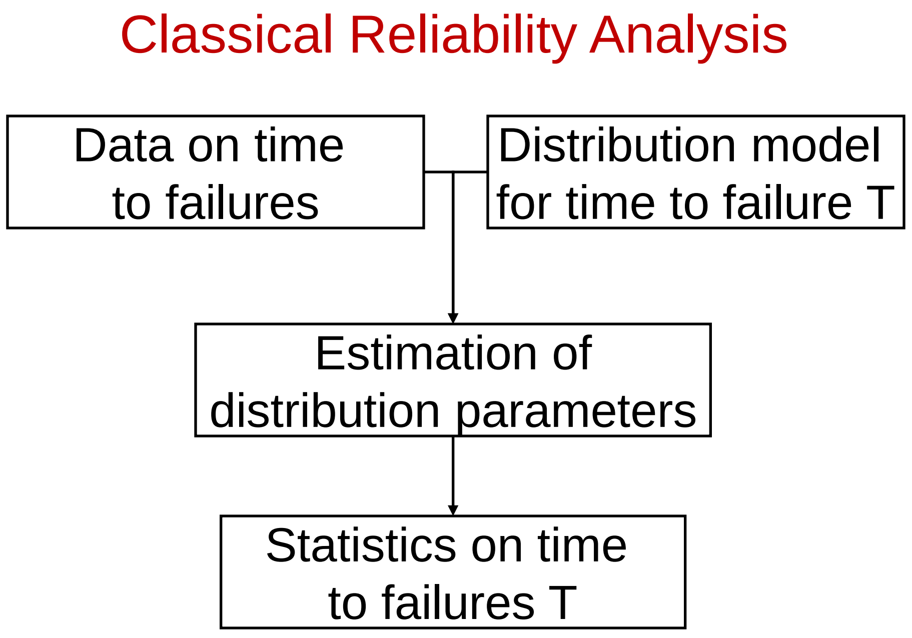
- all structural components are unique

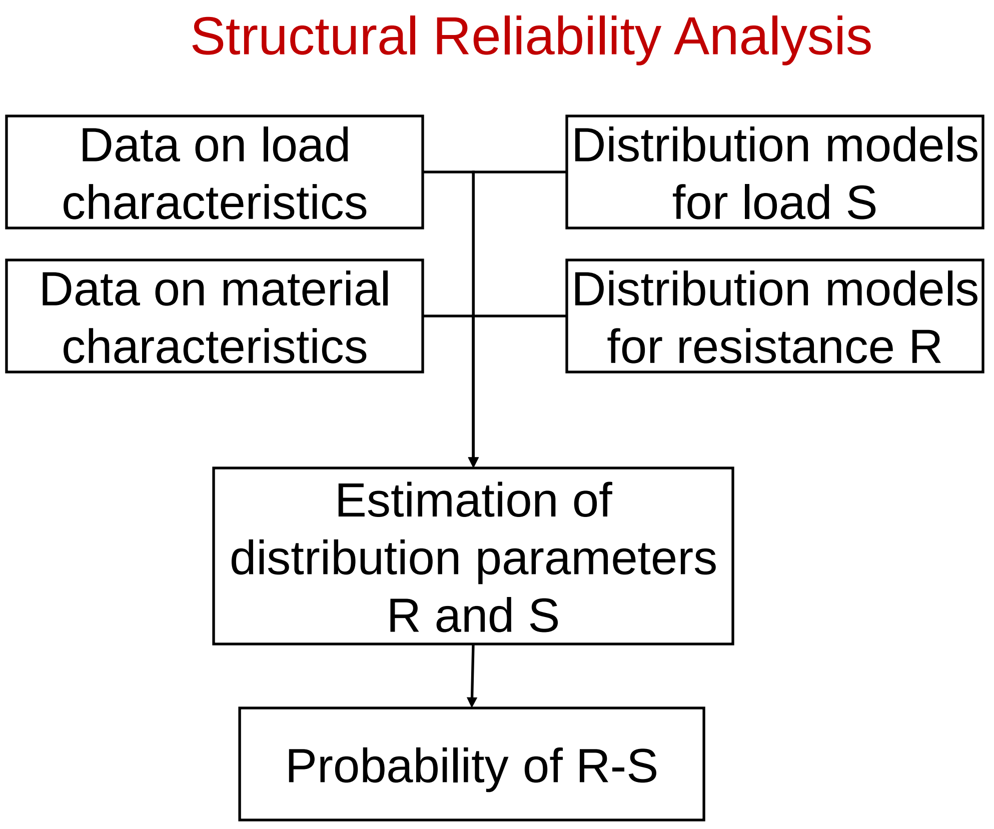
- the failure mechanism tends to be related to extreme load events exceeding the residual capacity of the component – not the direct effect of deterioration alone

For such systems a different approach is thus required, namely

- an individual modelling of both the resistance as a function of time and the loading as a function of time

Introduction to Classical Reliability Theory





Structural Reliability Analysis

Can be generally used for problems in Civil- and Environmental Engineering if the event of intrest can be represented by

"Demand"	exceeding	"Capacity"

e.g.

required water volume	exceeding	existing water volume
traffic volume	exceeding	road capacity
stress	exceeding	resistance

The "Failure" event is thereby defined by $\mathrm{F} = \{R \leq S\}$

Structural Reliability Analysis

The fundamental case:

The "Failure" event is defined by $\mathbf{F} = \{R \leq S\}$

The failure probability is defined as $\Pr(F) = \Pr(R \leq S)$

With known "stress" *s* $\Pr(F)$ is $\Pr(F|S = s) = \Pr(R \leq s) = F_R(s)$

According to the total probability theorem the failure probability for unknown *s:*

$$\Pr(F) = \int_S \Pr(F|S = s) f_S(s) ds = \int_S F_R(s) f_S(s) ds$$

or, similarly $\Pr(F) = 1 - \int_R F_S(r) f_R(r) dr$

Structural Reliability Analysis

Safety Margin: $M = R - S$

$$\Pr(F) = \Pr(M \leq 0) = F_M(0)$$

If R and S are Normal distributed:

$$E[M] = E[R] - E[S] \quad \leftrightarrow \quad \mu_M = \mu_R - \mu_S$$

$$Var[M] = Var[R] + Var[S] \quad \leftrightarrow \quad \sigma_M = \sqrt{{\sigma_R}^2 - \ {\sigma_S}^2}$$

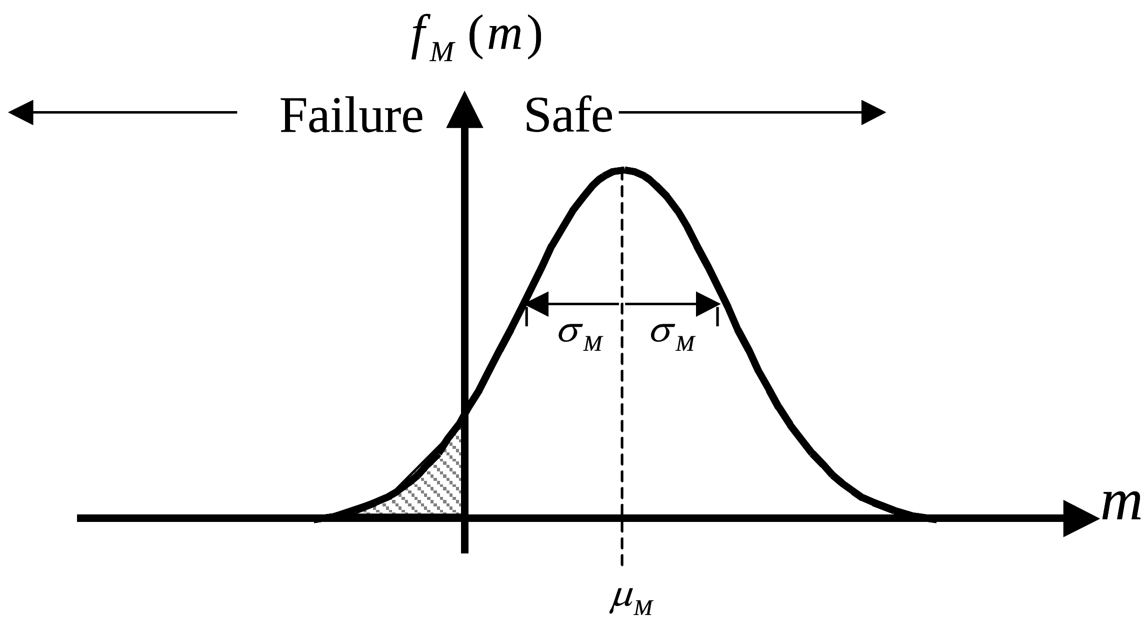
The failure probability is:

$$\Pr(F) = F_M(0) = \ \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right) = \Phi\left(-\frac{\mu_R - \mu_S}{\sqrt{{\sigma_R}^2 - {\sigma_S}^2}}\right)$$

Structural Reliability Analysis

The failure probability is:

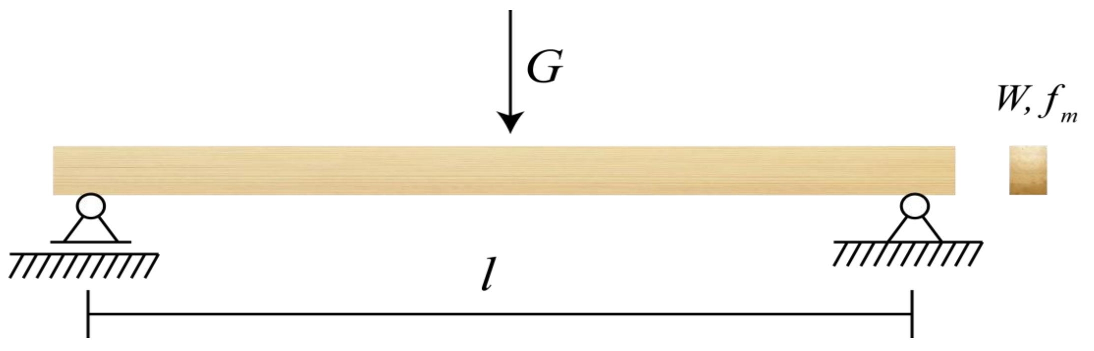
$$\Pr(F) = F_M(0) = \Phi\left(\frac{0-\mu_M}{\sigma_M}\right) = \Phi\left(-\frac{\mu_R-\mu_S}{\sqrt{{\sigma_R}^2-{\sigma_S}^2}}\right) = \Phi(-\beta)$$



Failure:

In general defined as: capacity < demand
resistance < load effect

e.g. a beam in bending:

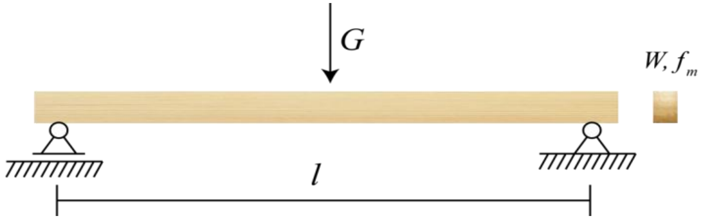


$$f_m W = r < s = \frac{G\ l}{4}$$

BUT - resistance and load effect are uncertain !!!

Assuming r and s are Normal distributed random variables with known mean values and standard deviations.

Probability of failure defined as:

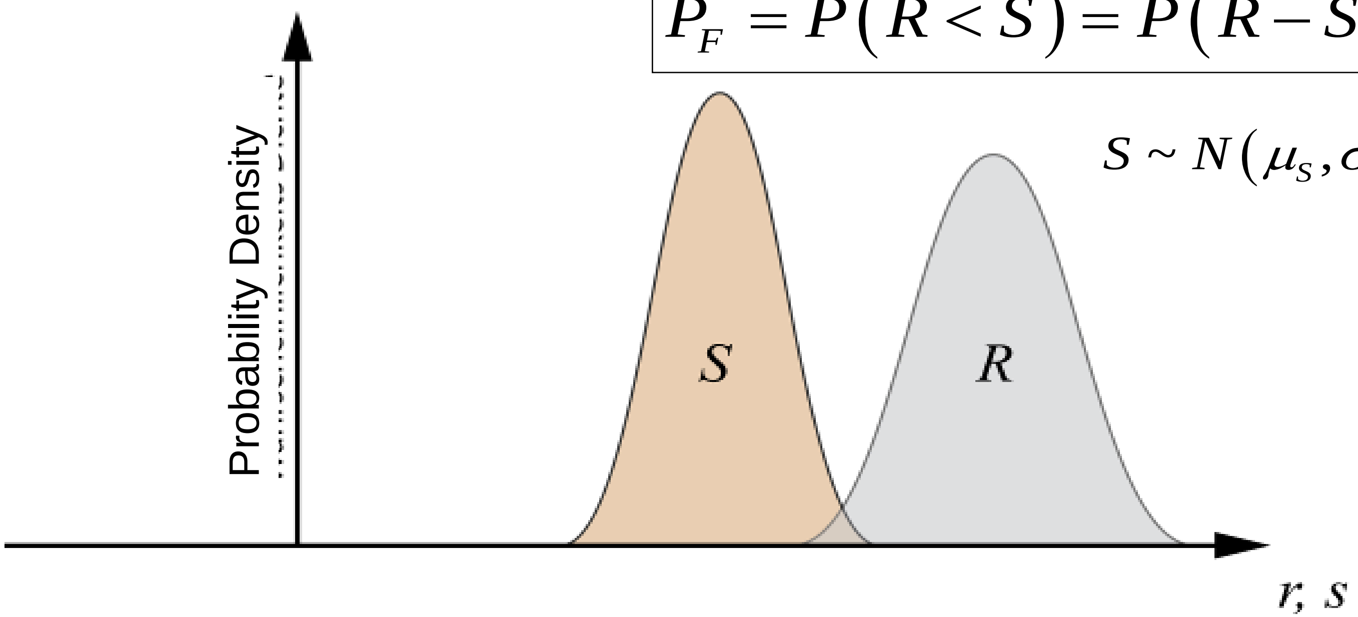


The probability, that resistance < load effect

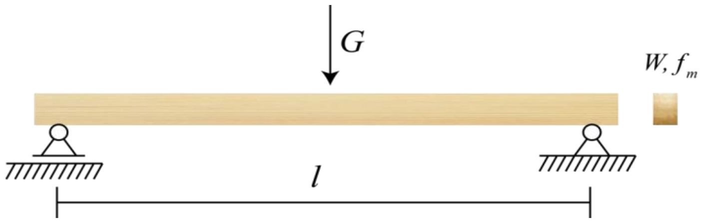
$$R = f_m W; \quad S = \frac{Gl}{4}$$

$$P_F = P(R < S) = P(R - S < 0)$$

$$S \sim N(\mu_S, \sigma_S) \quad R \sim N(\mu_R, \sigma_R)$$



Probability of failure defined as:



The probability, that resistance < load effect

$$R = f_m W; \quad S = \frac{Gl}{4}$$

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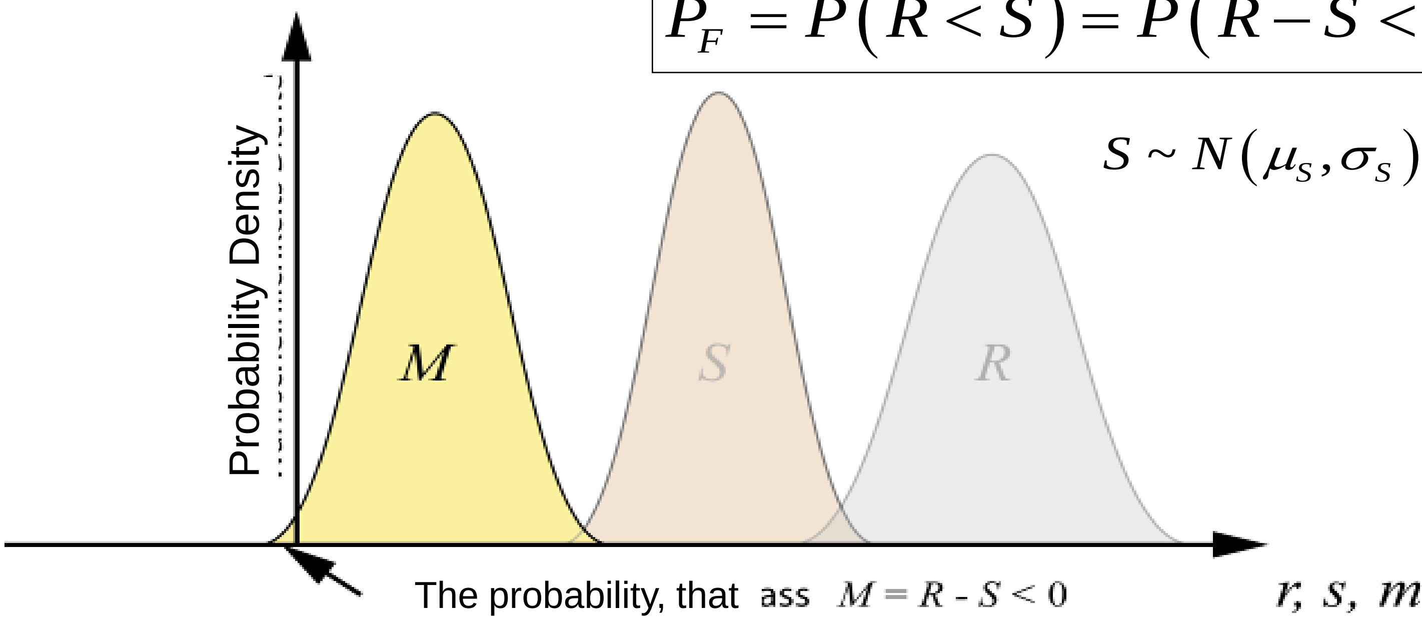


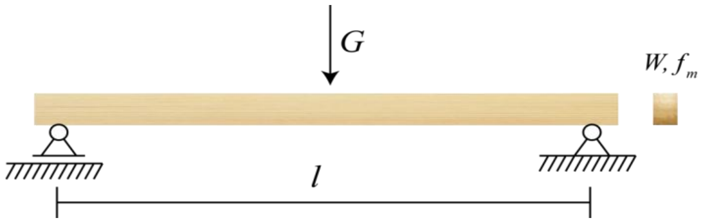
$$S \sim N(\mu_S, \sigma_S) \quad R \sim N(\mu_R, \sigma_R)$$

$$M \sim N(\mu_M, \sigma_M)$$

$$\mu_M = \mu_R - \mu_S$$

$$\sigma_M^2 = \sigma_R^2 + \sigma_S^2$$

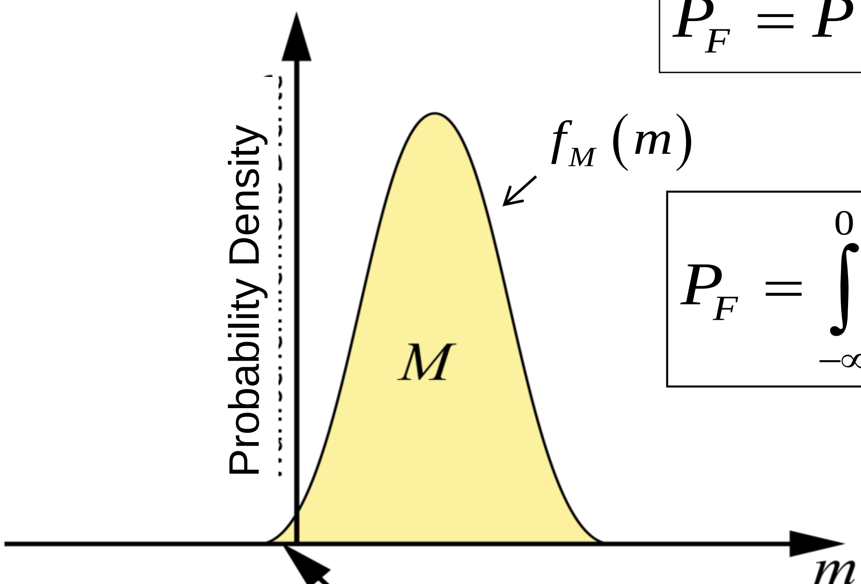
Probability of failure defined as:



The probability, that resistance < load effect

$$R = f_m W; \quad S = \frac{Gl}{4}$$

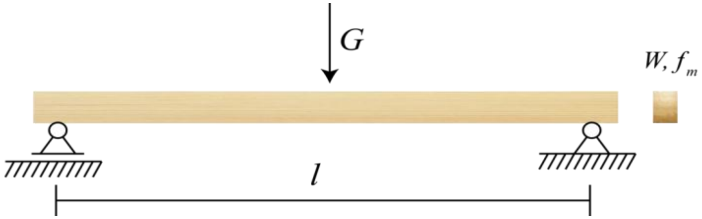
$$P_F = P(R < S) = P(R - S < 0) = P(M < 0)$$



$$P_F = \int_{-\infty}^{0} f_M(m)\, dm = \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right)$$

The probability, that ss $M = R - S < 0$

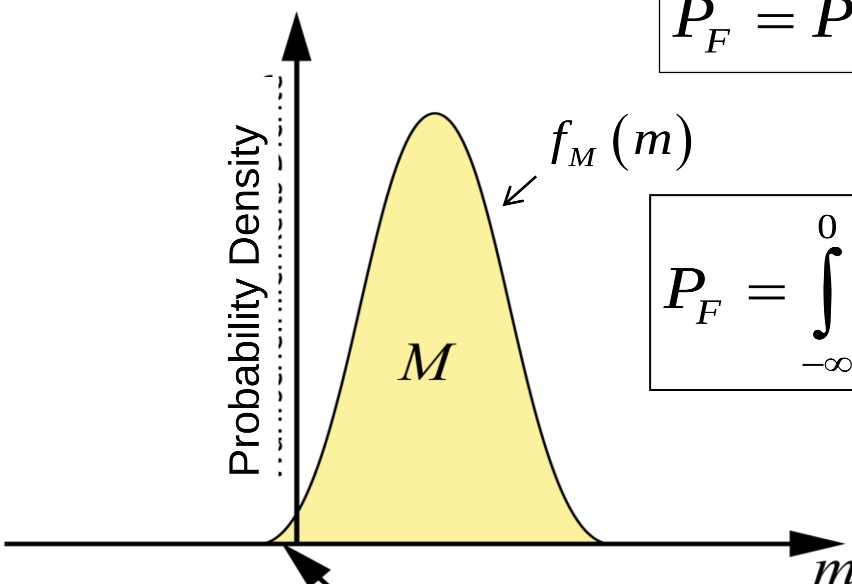
Probability of failure defined as:



The probability, that
resistance < load effect

$$R = f_m W; \quad S = \frac{Gl}{4}$$

$$P_F = P(R < S) = P(R - S < 0) = P(M < 0)$$



The probability, that ss $M = R - S < 0$

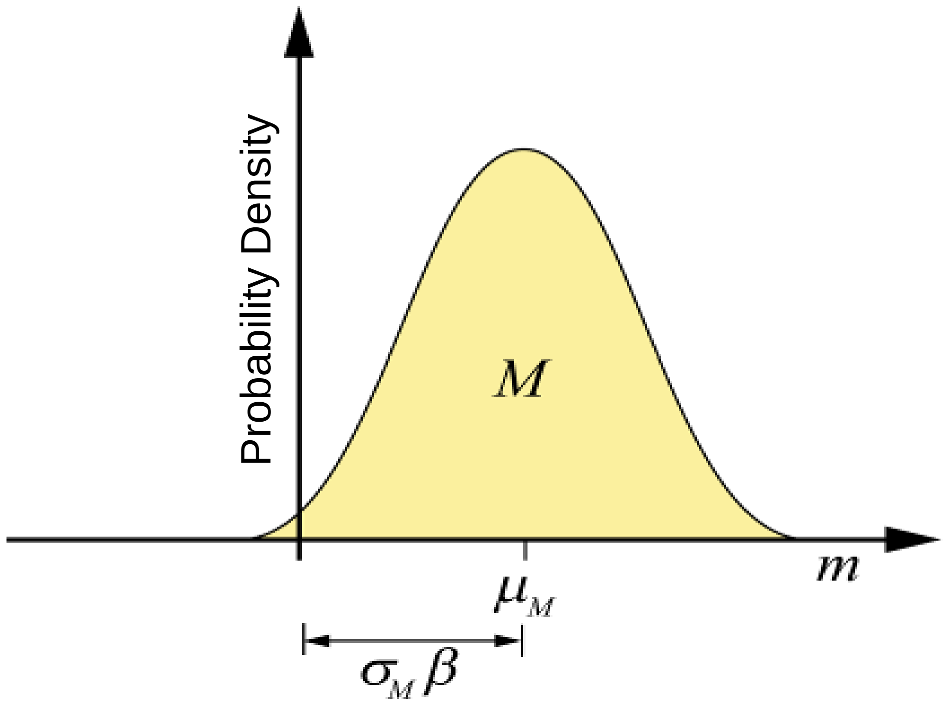
$$P_F = \int_{-\infty}^{0} f_M(m)\, dm = \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right) = \Phi(-\beta)$$

Reliability Index

Reliability Index defined as:

$$P_F = \Phi\left(-\frac{\mu_M}{\sigma_M}\right) = \Phi(-\beta)$$

Geometrical interpretation:

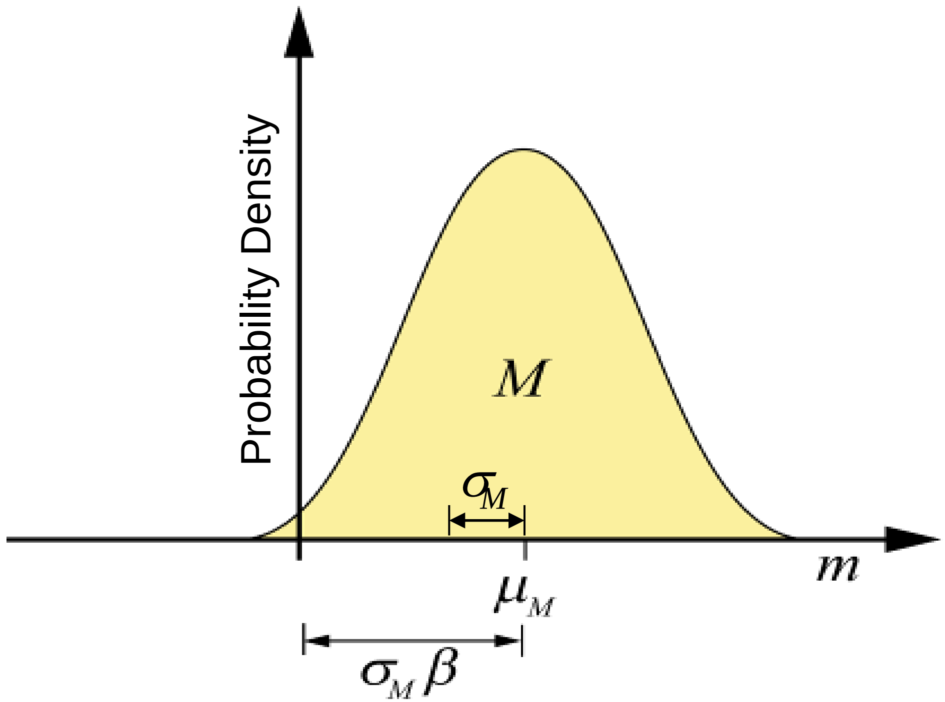


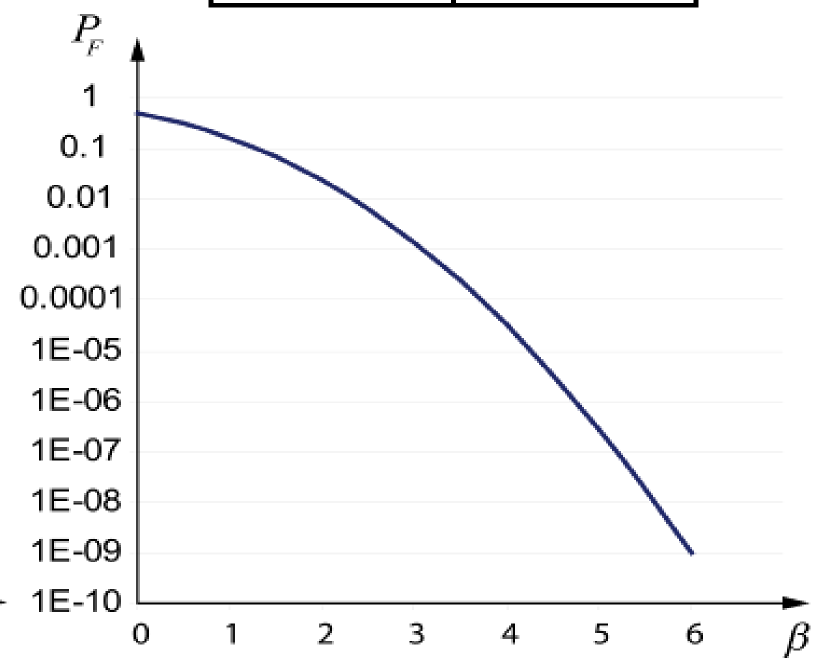
Reliability Index defined as:

$$P_F = \Phi\left(-\frac{\mu_M}{\sigma_M}\right) = \Phi(-\beta)$$

β	P_F
0	0.5
1	0.158655
2	0.02275
3	0.00135
4	3.17E-05
5	2.87E-07
6	9.87E-10
7	1.28E-12
8	6.22E-16

Geometrical interpretation:





Example – Reliability assessment:

- Timber beam: strength class C24, span l = 8 m, 16 x 16 cm.
- Load: 1 Person
- Simple probabilistic modelling:

<u>Bending strength</u> → normal distributed
The 5% fractile value is 24 Mpa, the assumed CoV (stdev/mean) is 0.25.

$$f_{m,05} = 24\,MPa;\; CoV = 0.25$$

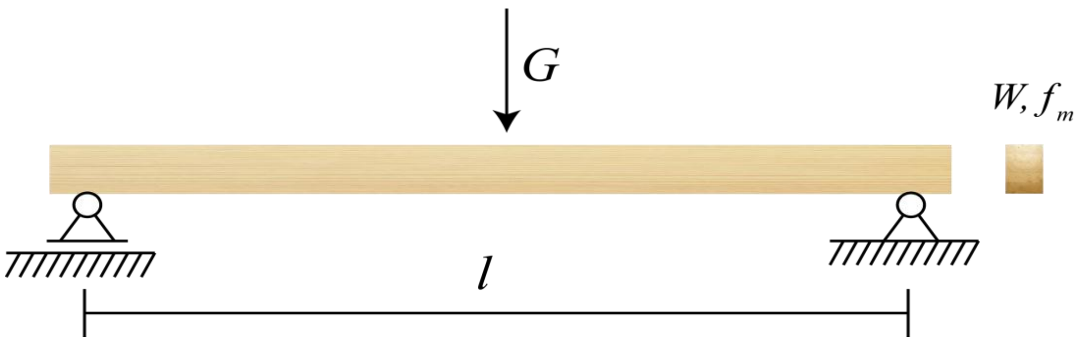
$$\Rightarrow \mu_{f_m} = f_{m,05} \Big/ \left(1 + \Phi^{-1}(0.05)\,CoV\right) = 40.8\,MPa$$

$$\sigma_{f_m} = 10.2\,MPa$$

<u>Person load</u> → normal distributed

$$\mu_G = 0.8\,kN$$

$$\sigma_G = 0.15\,kN$$



$$f_m W = r < s = \frac{G\;l}{4}$$

Example – Reliability assessment:

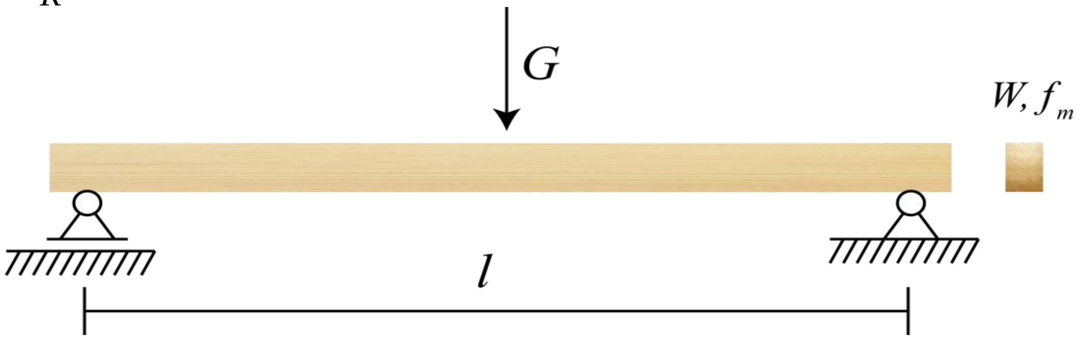
- Timber beam: strength class C24, span l = 8 m, 10 x 10 cm.
- Load: 1 Person
- Simple probabilistic modelling:

<u>Moment Resistance:</u>

$$\mu_{f_m} = 40.8\,MPa$$
$$\sigma_{f_m} = 10.2\,MPa$$
$$W = 0.1^3 m^3/6 = 0.00017 m^3$$
$$\mu_R = 6.8\,kNm$$
$$\sigma_R = 1.7\,kNm$$

<u>Person load effect:</u>

$$\mu_G = 0.8\,kN$$
$$\sigma_G = 0.15\,kN$$
$$\mu_S = 1.6\,kNm$$
$$\sigma_S = 0.3\,kNm$$



$$f_m W = r < s = \frac{G\ l}{4}$$

Example – Reliability assessment:

Moment Resistance:

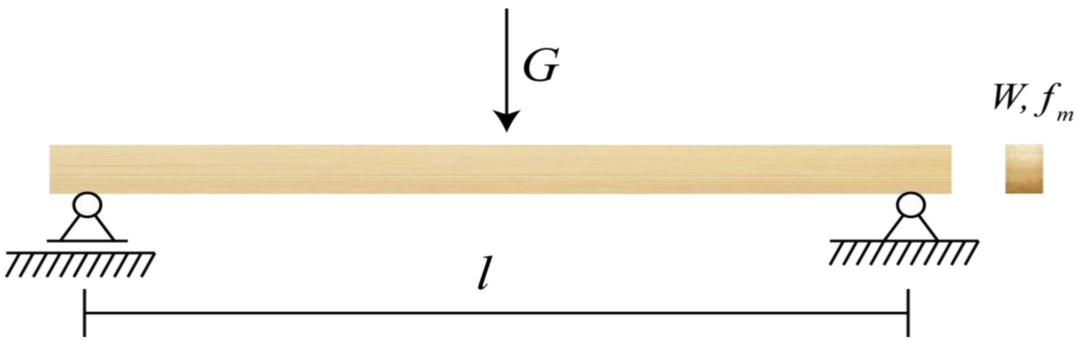
$$\mu_R = 6.8\,kNm$$
$$\sigma_R = 1.7\,kNm$$

Person load effect:

$$\mu_S = 1.6\,kNm$$
$$\sigma_S = 0.3\,kNm$$

Safety Margin M:

$$M = R - S$$
$$\mu_M = 6.8 - 1.6 = 5.2\,kNm$$
$$\sigma_M = \sqrt{1.7^2 + 0.3^2} = 1.73\,kNm$$



$$f_m W = r < s = \frac{G\,l}{4}$$

Example – Reliability assessment:

<u>Moment Resistance:</u>

$$\mu_R = 6.8\,kNm$$

$$\sigma_R = 1.7\,kNm$$

<u>Safety Margin M:</u>

$$M = R - S$$

$$\mu_M = 6.8 - 1.6 = 5.2\,kNm$$

$$\sigma_M = \sqrt{1.7^2 + 0.3^2} = 1.73\,kNm$$

<u>Person load effect:</u>

$$\mu_S = 1.6\,kNm$$

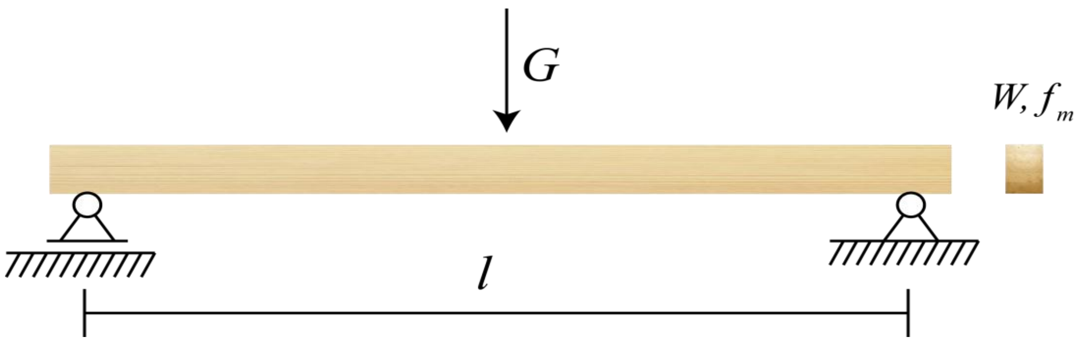
$$\sigma_S = 0.3\,kNm$$

<u>Reliability Index β:</u>

$$\beta = \frac{\mu_M}{\sigma_M} = 3.01$$

<u>Probability of failure:</u>

$$P_F = \Phi(-\beta) = 0.0013$$



$$f_m W = r < s = \frac{G\ l}{4}$$

Example – Reliability assessment:

- Failure probability and reliability index are estimates conditional on the information we have!

a. Let's assume that we know that the person weights 1 kN.

Moment Resistance:

$$\mu_R = 6.8\,kNm$$

$$\sigma_R = 1.7\,kNm$$

Person load effect:

$$g = 1\,kN$$

$$s = 2\,kNm$$

Safety Margin M:

$$M = R - s$$

$$\mu_M = 6.8 - 2 = 4.8\,kNm$$

$$\sigma_M = 1.7\,kNm$$

Reliability Index β:

$$\beta = \frac{\mu_M}{\sigma_M} = 2.82$$

Probability of failure:

$$P_F = \Phi(-\beta) = 0.0023$$

Structural Reliability Analysis

Linear Limit State Function with uncorrelated normal distributed random variables:

$$g(\mathbf{X}) = c + a_1X_1 + a_2X_2 + \cdots + a_nX_n = c + \mathbf{a}^T\mathbf{X}$$

$$E[g(\mathbf{X})] = c + a_1E[X_1] + a_2E[X_2] + \cdots + a_nE[X_n]$$
$$Var[g(\mathbf{X})] = a_1^2Var[X_1] + a_2^2Var[X_2] + \cdots + a_n^2Var[X_n]$$

$$\beta = \frac{E[g(\mathbf{X})]}{Var[g(\mathbf{X})]} = \frac{c+a_1E[X_1]+a_2E[X_2]+\cdots+a_nE[X_n]}{a_1^2Var[X_1]+a_2^2Var[X_2]+\cdots+a_n^2Var[X_n]}$$

$$\Pr(F) = \Phi(-\beta)$$