Risk and Safety in Engineering

Loads and Resistance

Jochen Köhler

Contents of this Lecture

- Probabilistic Modelling of Loads
- Time Variability
- Probabilistic Modelling of Resistances
- Probabilistic Modelling of Model Uncertainties
- The Joint Committee on Structural Safety Probabilistic Model Code

Probabilistic Modelling of Loads

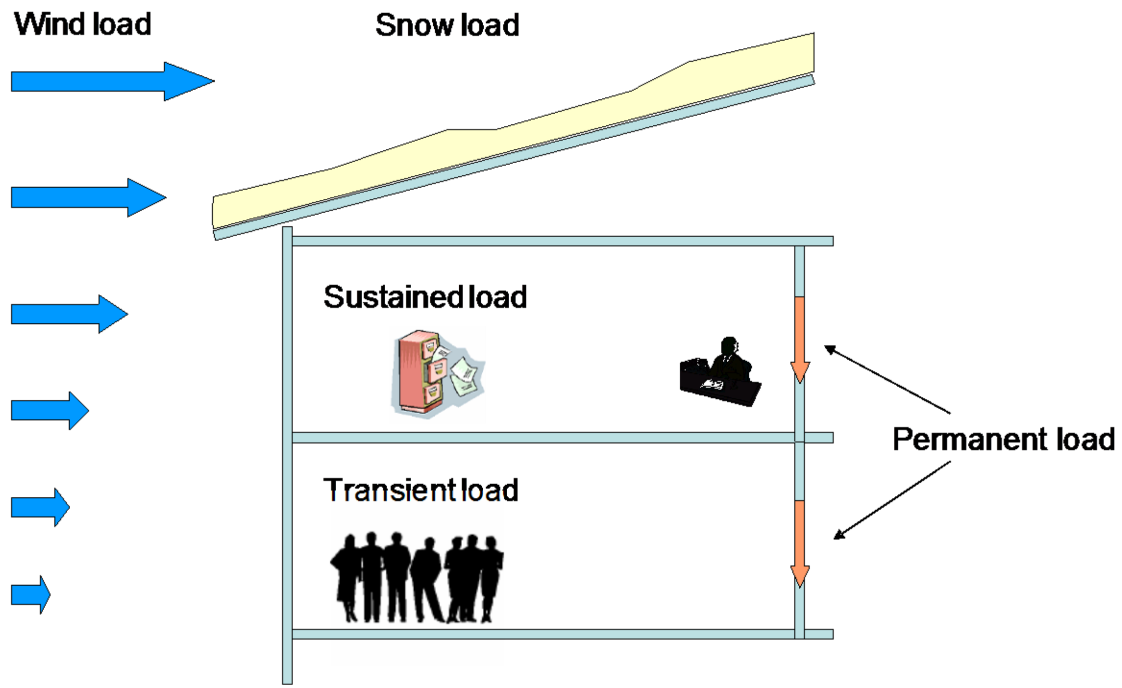
- Loads on Structures

Loads are uncertain due to:

- Random variations in space and time

- Model uncertainties

- Statistical uncertainties



Probabilistic Modelling of Loads

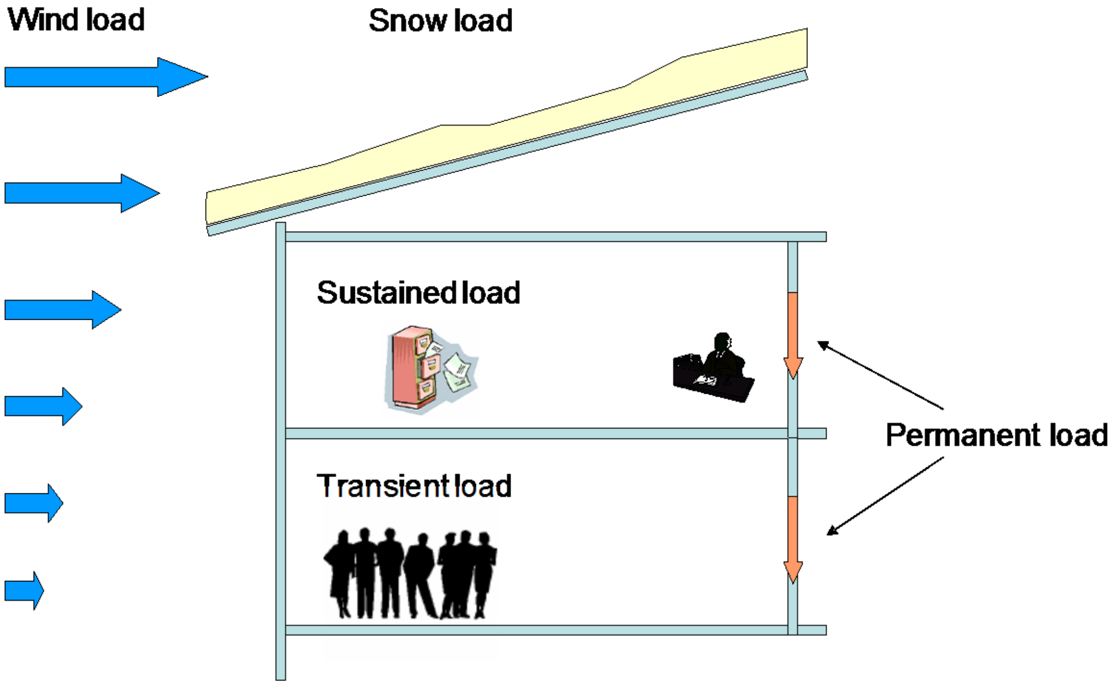
- Loads on Structures

It is often useful to characterize loads as:

- Permanent or variable

- Fixed or free

- Static or dynamic



Probabilistic Modelling of Loads

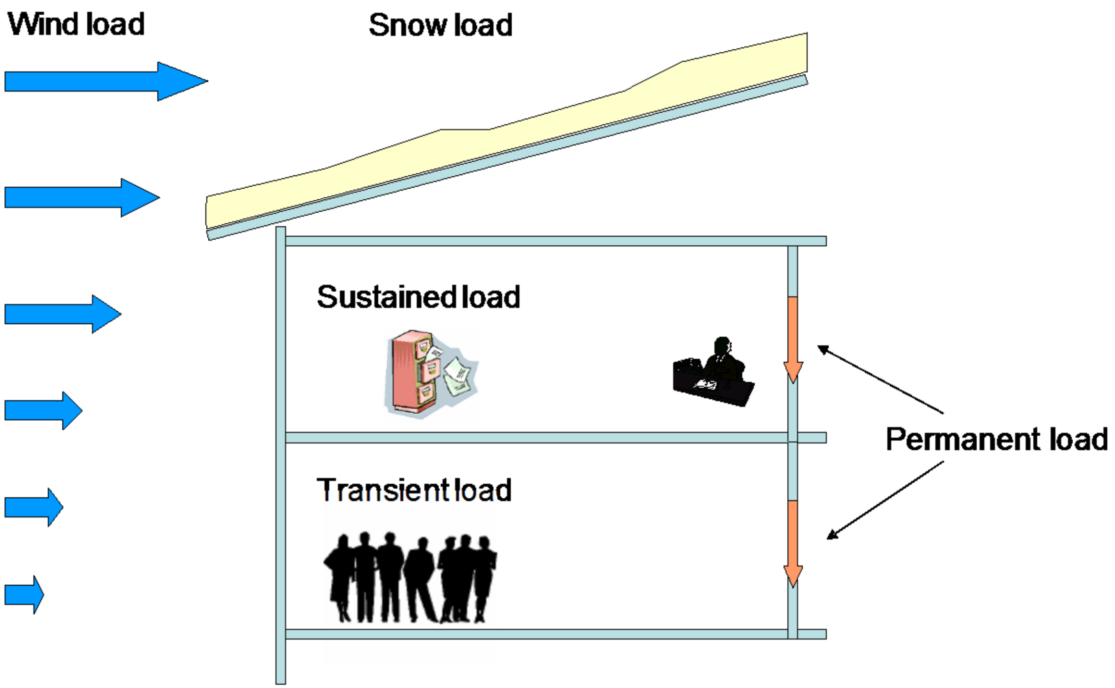
- ## Loads on Structures

The probabilistic modelling includes the following steps:

- specifying the definition of the random variables used to represent the uncertainties in the loading

- selecting a suitable distribution type to represent the random variable

- assigning the distribution parameters of the selected distribution.



Probabilistic Modelling of Loads

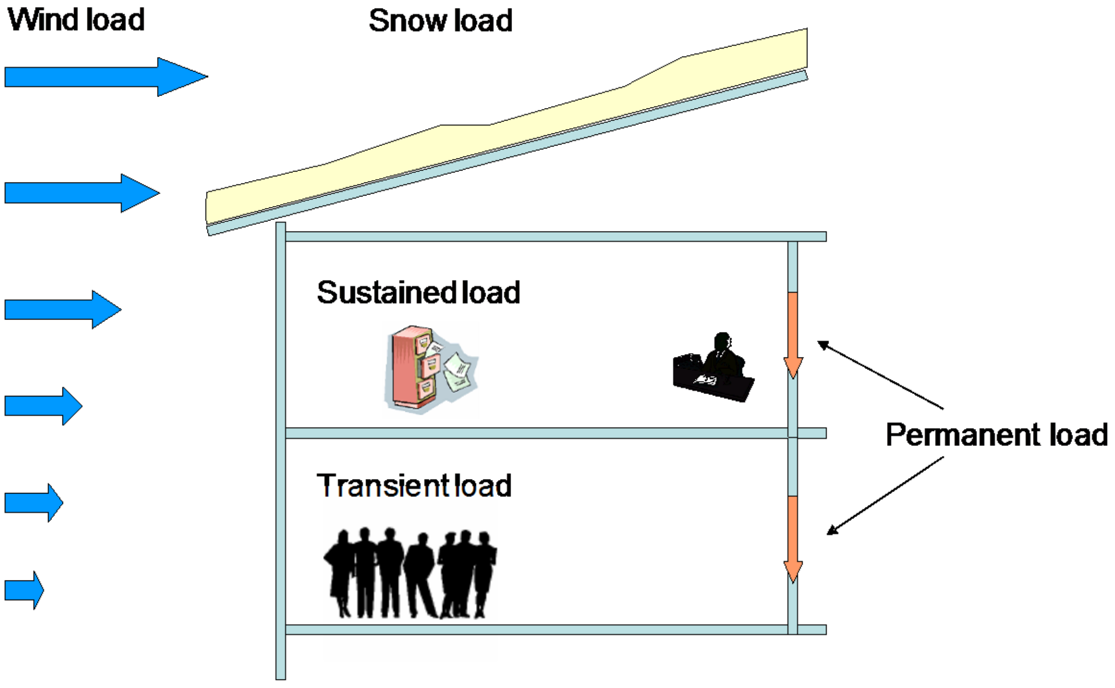
- **Loads on Structures**

Permanent loads:

$$G = \int_V \gamma \, dV$$

Density

Material	COV
Construction Steel	0.01
Concrete	0.04
Timber - sawn beam or strut - laminated beam, planed	 0.12 0.10



Probabilistic Modelling of Loads

- **Loads on Structures**

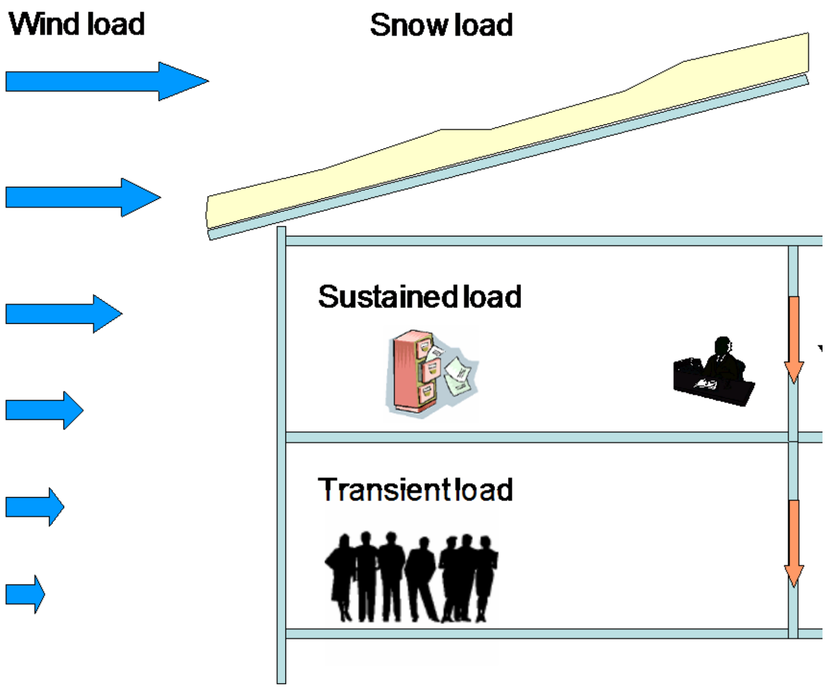
Live floor loads:

$$W(x,y) = m + V + U(x,y)$$

m **is the overall mean value for a given use category**

V **is a zero mean random variable**

U(x,y) **is a zero mean random field**



Probabilistic Modelling of Loads

- ## Loads on Structures

Wind loads

$$w = c_a c_g c_r \overline{Q}_{ref} = c_a c_e \overline{Q}_{ref}$$

Smaller rigid structures

$$w = c_d c_a c_e \overline{Q}_{ref}$$

Taller flexible structures

$$Q = \frac{1}{2}\rho U^2$$

Q_{ref}: 10 min mean U

c_r: roughness factor
c_g: gust factor
c_a: aero-dynamic shape factor
c_d: dynamic factor
c_e: exposure factor
ρ: 1.25 kg/m^3

Probabilistic Modelling of Loads

- ## Loads on Structures

Wind loads

***c_r*: roughness factor**
***c_g*: gust factor**
***c_a*: aero-dynamic shape factor**
***c_d*: dynamic factor**
***c_e*: exposure factor**
ρ: 1.25 kg/m³

Variable	Type	V
Q_{ref}	Gumbel	0.20 - 0.30
c_r	Lognormal	0.10 - 0.20
c_a - coefficient pressure coefficient force	Lognormal Lognormal	0.10 - 0.30 0.10 - 0.15
c_g	Lognormal	0.10 - 0.15
c_d	Lognormal	0.10 - 0.20

Wind load can be assumed to be Log-Normal distributed

$$V_w^2 \cong V_{c_d}^2 + V_{c_a}^2 + V_{c_r}^2 + V_{Q_{ref}}^2$$ **Dynamic sensitive**

$$V_w^2 \cong V_{c_a}^2 + V_{c_r}^2 + V_{Q_{ref}}^2$$ **Rigid**

Probabilistic Modelling of Loads

- ## Loads on Structures

Snow loads

$$S_r = S_g \; r \; k^{\frac{h}{h_r}}$$

$$S_g = d \cdot \rho(d)$$

S_r: Snow load on roof
S_g: Snow load on ground
r: ground to roof conversion factor
k: location factor (1.25 coastal, 1,5 inland)
h: altitude in meters
h_r: reference altitude (300 meters)
ρ: 1.25 kg/m^3

Typically the snow load is modelled by a Gamma or a Gumbel distribution

Stochastic Processes

Random variables represent random events, e.g. properties of objects

If we look at random events over time we speak about random processes to represent these.

Very often we speak implicitly about random processes in structural engineering

Examples:

- Earthquake with a return period of 475 years
- 100 yearly flood
- Maximum loads during the lifecycle of a structure

Stochastische Prozesse

- In many engineering problems we need to be able to describe the random variations in time more specifically:

  The occurrences of events at random discrete points in time (rock-fall, earthquakes, accidents, queues, failures, etc.)
  - Poisson process, exponential and Gamma distribution

  The random values of events occurring continuously in time (wind pressures, wave loads, temperatures, etc.)
  - Continuous random processes (Normal process)

Discrete event of flood

Continuous stress variations due to waves

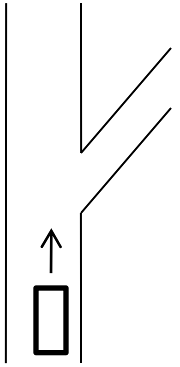
Bernoulli Trial and Binomialdistribution

- A sequence of experiments with two possible exclusive outcomes is called Bernoulli Trial.
- The outcomes are typically called success and failure.

Example:

Cars in a street; leave road: failure, probability p

straight: success, probability 1- p



Probability that two of five cars leave the road?

Probability that none(of 5) leaves: $P[Y=0]=(1-p)(1-p)...(1-p)=(1-p)^5$

Probability that two (of 5) leave: $P[Y=2]=k\cdot p^2(1-p)^{5-2}$

Binomialdistribution: $$p_Y(y)=\binom{n}{y}p^y(1-p)^{n-y}$$

k = number of possibilities 2 of 5

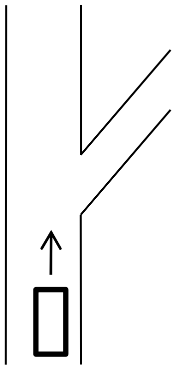
Bernoulli Trial and Binomialdistribution

- A sequence of experiments with two possible exclusive outcomes is called Bernoulli Trial.
- The outcomes are typically called success and failure.

Example:

Cars in a street; leave road: failure, probability p

straight: success, probability 1- p



Probability that two of five cars leave the road?

Probability that none(of 5) leaves: $P[Y=0]=(1-p)(1-p)\ldots(1-p)=(1-p)^5$

Probability that two (of 5) leave: $P[Y=2]=k\cdot p^2(1-p)^{5-2}$

Binomialdistribution:

$$p_Y(y)=\binom{n}{y}p^y(1-p)^{n-y}$$

k = number of possibilities 2 of 5

$$p_Y(2)=\binom{5}{2}0.3^2(1-0.3)^{5-2}=0.3087$$

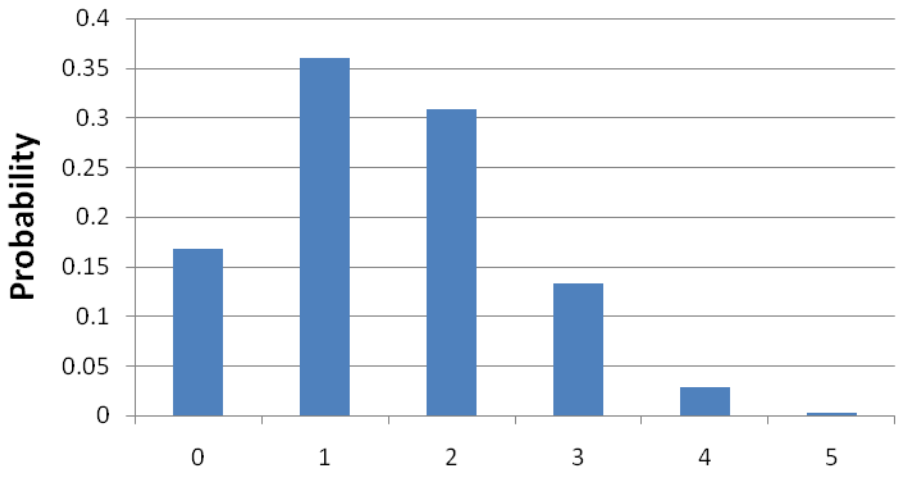
Bernoulli Trial and Binomialdistribution

- Example:

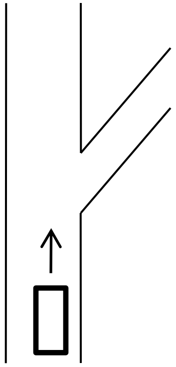
Cars in a street; leave road: failure, probability $p = 0.3$

- Binomialdistribution:

$$p_Y(y) = \binom{n}{y} p^y (1-p)^{n-y}$$



$p = 0.3; \quad n = 5$



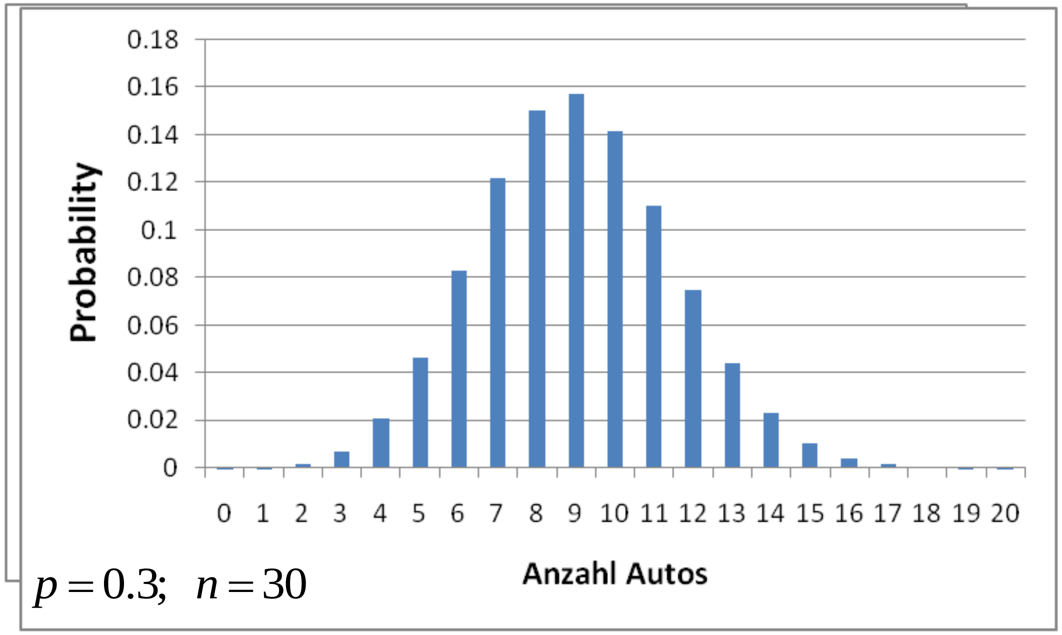
Bernoulli Trial and Binomialdistribution

- Example:

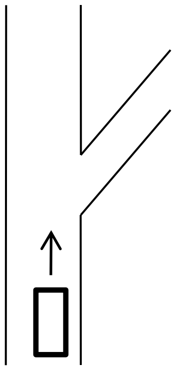
Cars in a street; leave road: failure, probability $p = 0.3$

-
- Binomialdistribution:

$$p_Y(y) = \binom{n}{y} p^y (1-p)^{n-y}$$



$p = 0.3; \quad n = 30$



$$E[Y] = np$$

$$Var[Y] = np(1-p)$$

Poissondistribution - Poissonprocess

In practice the number of trials is often not possible to estimate **(very large** or unknown**)**

$$n \to \infty \qquad p \to 0 \qquad np = u \to const. \qquad p = u/n$$

$$p_Y(y) = \binom{n}{y} (v/n)^y \left(1 - (v/n)\right)^{n-y} \xrightarrow[n\to\infty]{\text{im Limit}} \frac{u^y e^{-u}}{y!}$$

$$p_X(x) = \frac{u^x e^{-u}}{x!}$$

Poissondistribution

Poissondistribution - Poissonprocess

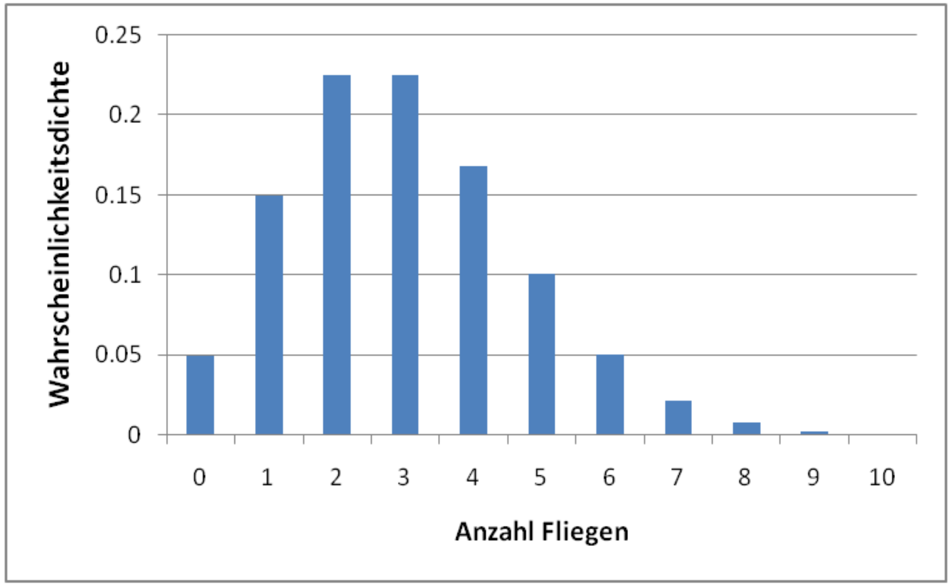
Poissondistrbution:

$$p_X(x) = \frac{u^x e^{-u}}{x!}$$

Moments:

$$E[X] = u \qquad Var[X] = u$$

Frequency u = 3 per Minute.



Poissondistribution - Poissonprocess

What about the number of event**s in** 10 Minutes:
3 per Minute -> 30 per 10 Minutes

u can be skaled over time: $u = vt$

Poissondistribution: $p_X(x) = \frac{u^x e^{-u}}{x!} = \frac{(vt)^x e^{-vt}}{x!}$

The sequence of events that might be described with a Poisson Distribution is called Poisson Process.

$$E[X] = u = vt$$

$$Var[X] = u = vt$$

Poissondistribution - Poissonprocess

- **The Poisson counting process is one of the most commonly applied families of probability distributions applied in reliability theory**

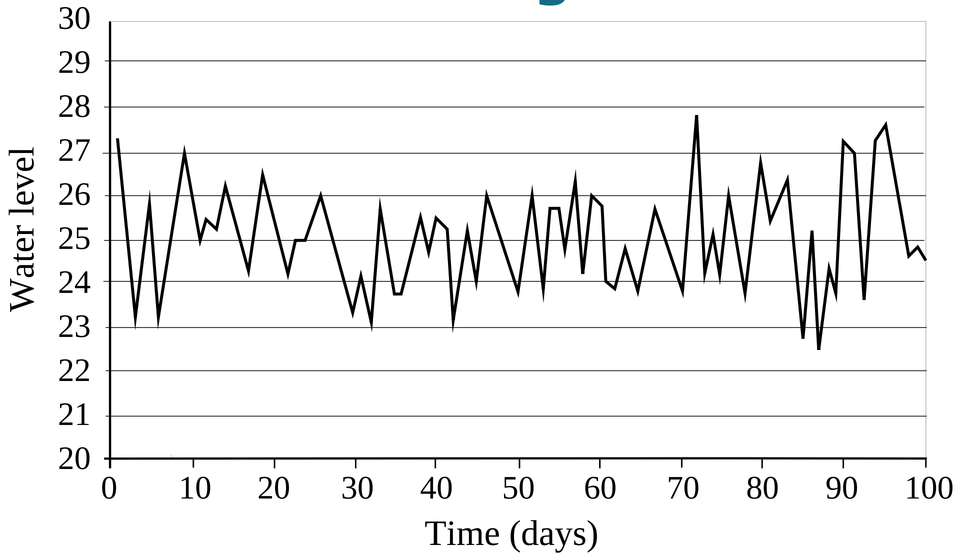
  Requirements:

  1) the probability of one event in the interval ($t,t+\Delta t$[is asymptotically proportional to Δt.
  2) the probability of more than one event in the interval ($t,t+\Delta t$[is 0 *for* $\Delta t \rightarrow 0$.
  3) events in disjoint intervals are mutually independent.

Random Processes

- **Continuous random processes**

  A continuous random process is a random process which has realizations continuously over time and for which the realizations belong to a continuous sample space.



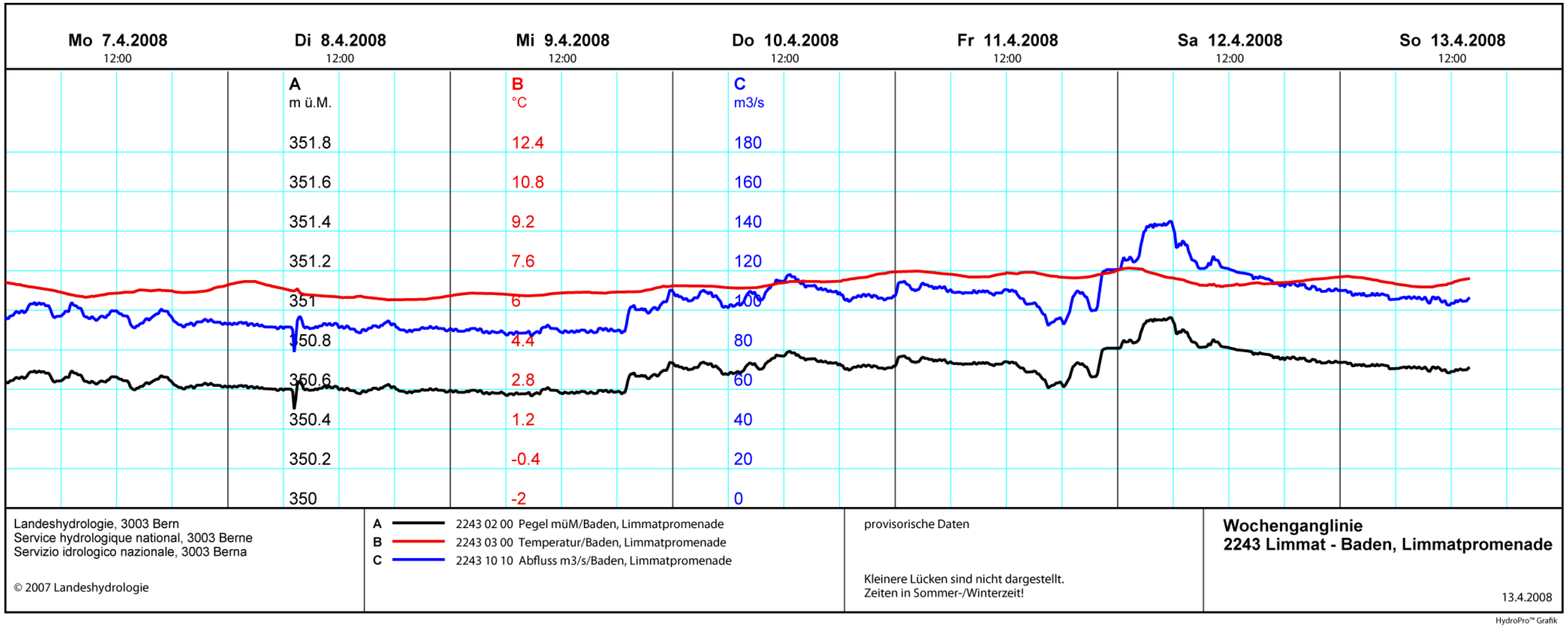
Variations of;
water levels
wind speed
rain fall
.
.
.

Realization of continuous scalar valued random process

Random Processes

Realisation of a continuous process

– Water level.



Random Processes

- **Continuous random processes**

The mean value of the possible realizations of a random process is given as:

$$\mu_X(t) = E\left[X(t)\right] = \int_{-\infty}^{\infty} x \; f_X(x\,;t)dx$$

Function of time !

The correlation between realizations at any two points in time is given as:

$$R_{XX}(t_1,t_2) = E\left[X(t_1)X(t_2)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 \; x_2 \;\; f_{XX}(x_1,x_2;t_1,t_2)dx_1 dx_2$$

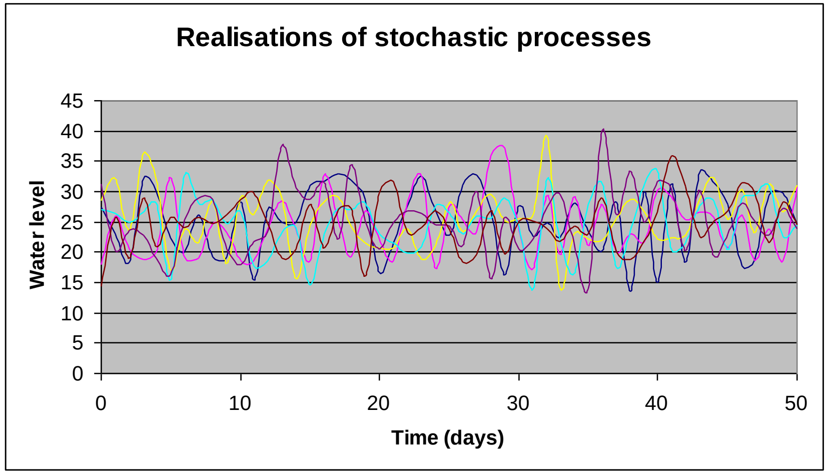
Auto-correlation function – refers to a scalar valued random process

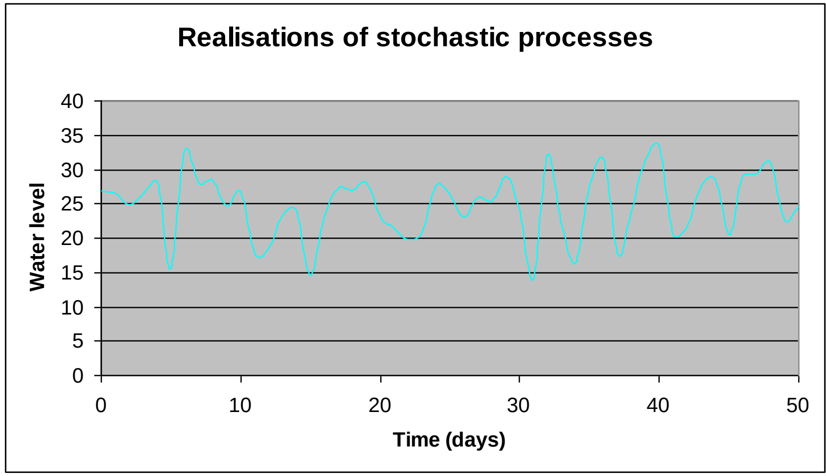
Random Processes

- Properties

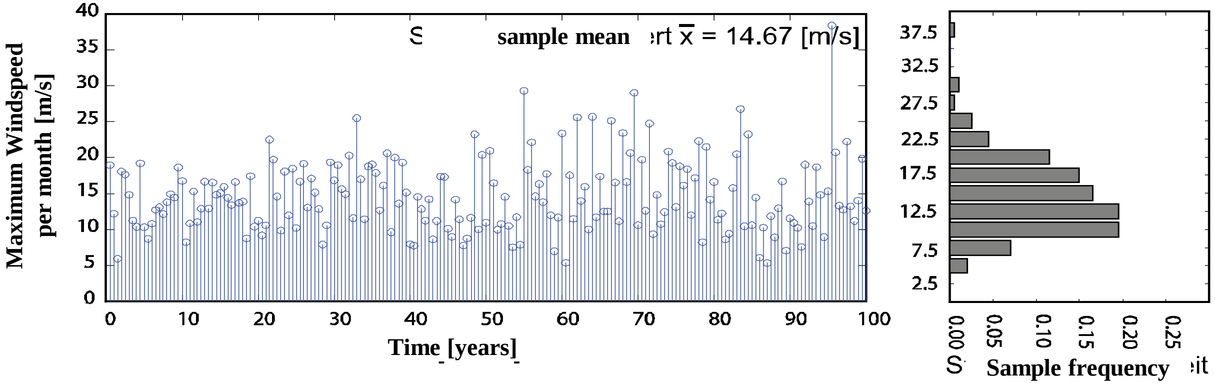
A random process is said to be ***strictly stationary*** **if all its moments are invariant to a shift in time.**

A random process is said to be ***strictly ergodic*** **if it is strictly stationary and in addition all its moments may be determined on the basis of one realization of the process.**

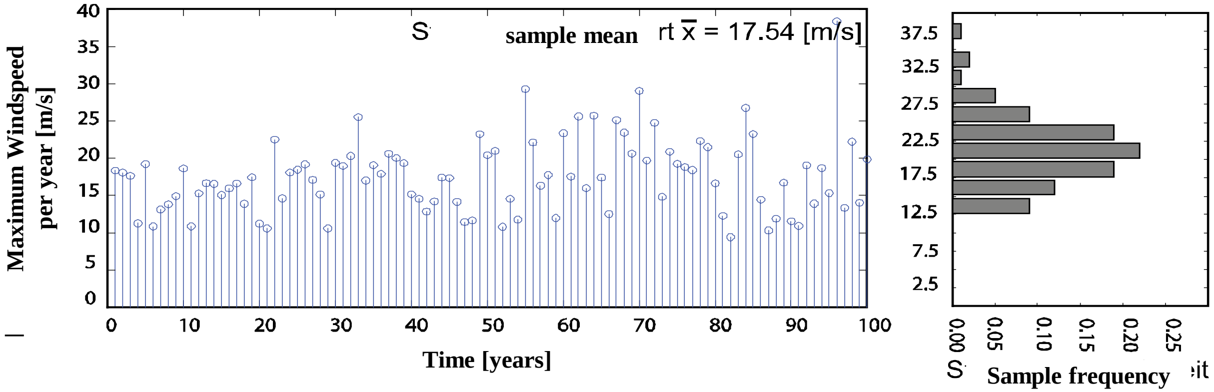




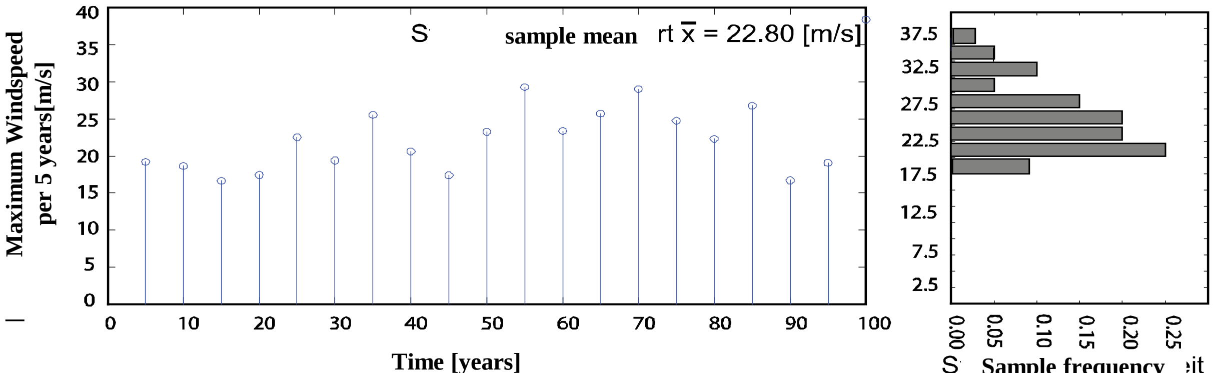
Time dependency – e.g. Windspeed:



Observed monthly extreme

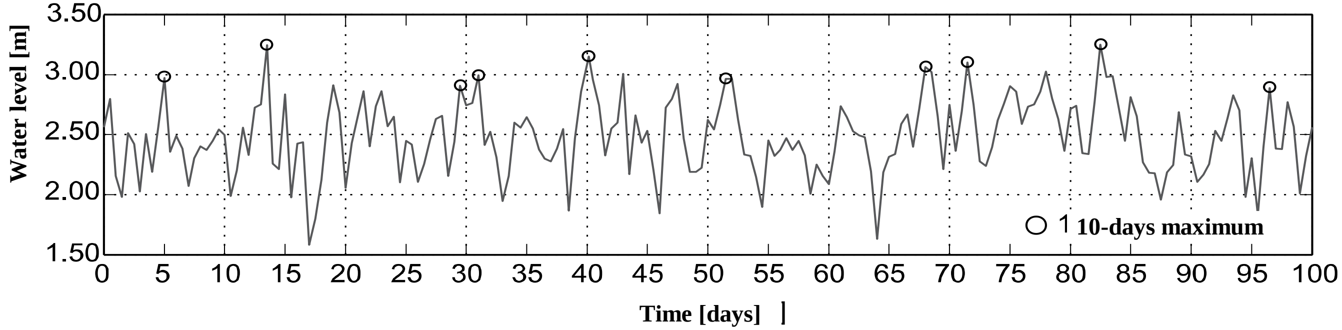


Observed yearly extreme

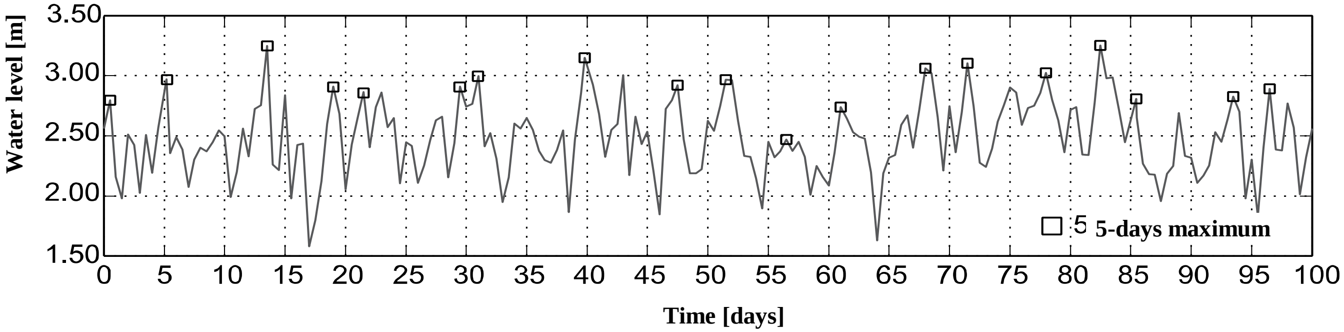


Observed 5 year extreme

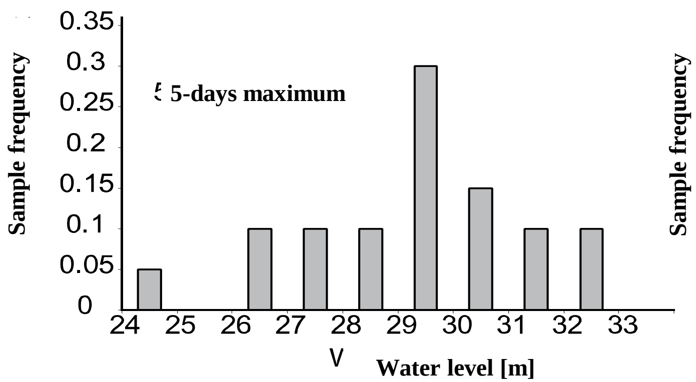
Time dependency – e.g. Water level:

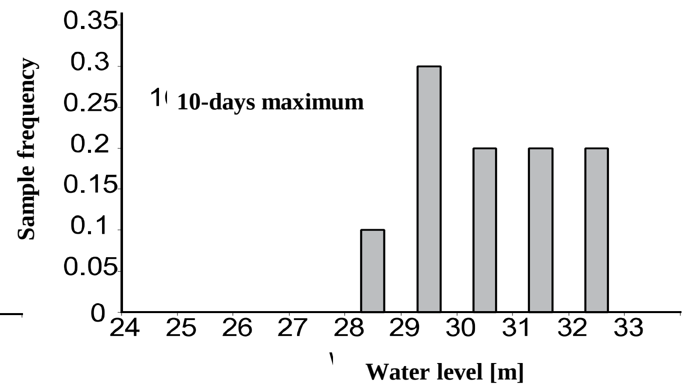


Observed extremes
10 days



Observed extremes
5 days





Time dependency – Extreme value distributions:

Assuming the extreme values within a period T of an ergodic random process $X(t)$ are independent and follow the probability distribution

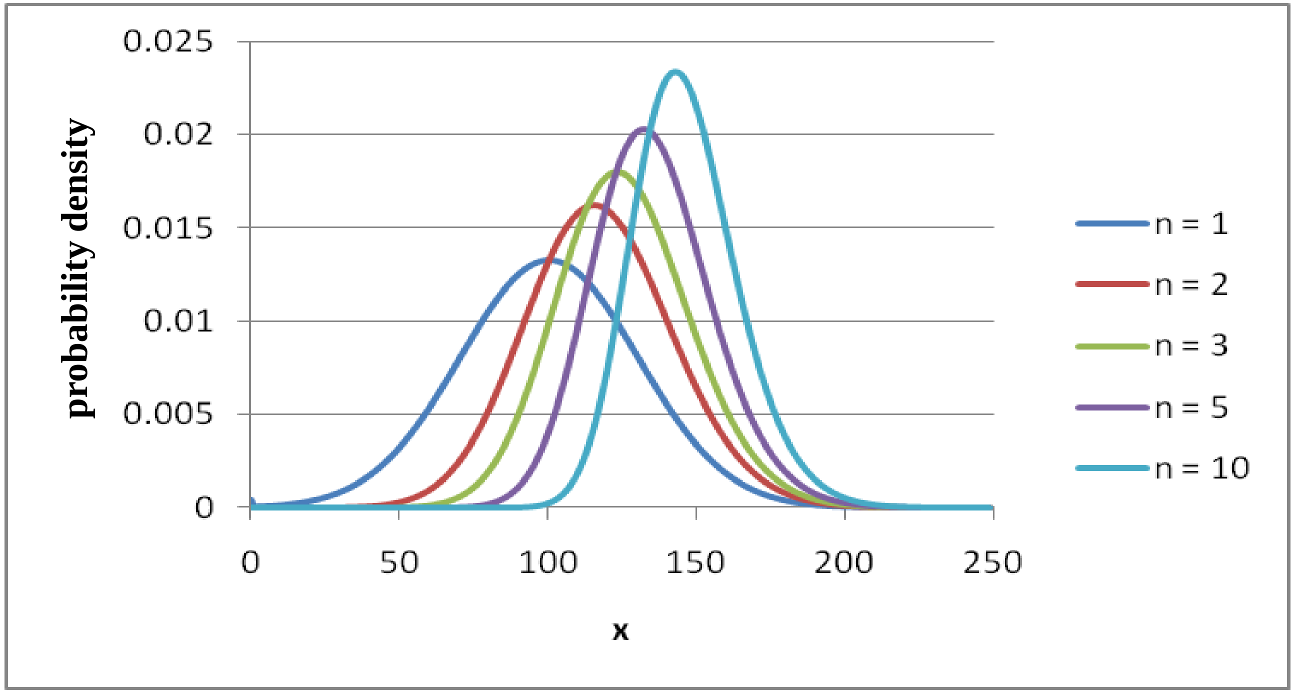
$$F_{X,T}^{\max}(x)$$

Then, the extreme values of the same process within the period

$$n \cdot T$$

Are following:

$$F_{X,nT}^{\max}(x) = \left(F_{X,T}^{\max}(x)\right)^n$$



Extreme Value Distributions

Extreme Type I – Gumbel Max

When the upper tail of the probability density function falls off exponentially (exponential, Normal and Gamma distribution) then the maximum in the time interval *T* is said to be Type I extreme distributed

$$f_{X,T}^{\max}(x) = \alpha \exp(-\alpha(x-u) - \exp(-\alpha(x-u)))$$

$$F_{X,T}^{\max}(x) = \exp(-\exp(-\alpha(x-u)))$$

$$\mu_{X_T^{\max}} = u + \frac{\gamma}{\alpha} = u + \frac{0.577216}{\alpha}$$

$$\sigma_{X_T^{\max}} = \frac{\pi}{\alpha\sqrt{6}}$$

For increasing time intervals the variance is constant but the mean value increases as:

$$\mu_{X_{nT}^{\max}} = \mu_{X_T^{\max}} + \frac{\sqrt{6}}{\pi}\sigma_{X_T^{\max}} \ln(n)$$

Extreme Value Distributions

Extreme Type II – Frechet Max

When a probability density function is downwards limited at zero and upwards falls off in the form

$$F_X(x) = 1 - \beta(\frac{1}{x})^k$$

then the maximum in the time interval *T* is said to be Type II extreme distributed

$$F_{X,T}^{\max}(x) = \exp(-\left(\frac{u}{x}\right)^k)$$

$$f_{X,T}^{\max}(x) = \frac{k}{u}\left(\frac{u}{x}\right)^{k+1} \exp(-\left(\frac{u}{x}\right)^k)$$

Mean value and standard deviation

$$\mu_{X_T^{\max}} = u\Gamma(1-\frac{1}{k})$$

$$\sigma_{X_T^{\max}}^2 = u^2\left[\Gamma(1-\frac{2}{k}) - \Gamma^2(1-\frac{1}{k})\right]$$

Extreme Value Distributions

Extreme Type III – Weibull Min

When a probability density function is downwards limited at ε and the lower tail falls off towards ε in the form

$$F(x) = c(x-\varepsilon)^k$$

then the minimum in the time interval *T* is said to be Type III extreme distributed

$$F_{X,T}^{\min}(x) = 1 - \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right)$$

$$f_{X,T}^{\min}(x) = \frac{k}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{k-1}\exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right)$$

Mean value and standard deviation

$$\mu_{X_T^{\min}} = \varepsilon + (u-\varepsilon)\Gamma(1+\frac{1}{k})$$

$$\sigma_{X_T^{\min}}^2 = (u-\varepsilon)^2\left[\Gamma(1+\frac{2}{k}) - \Gamma^2(1+\frac{1}{k})\right]$$

Return Period

The return period for extreme events T_R may be defined as:

$$T_R = n \cdot T = \frac{1}{(1 - F_{X,T}^{\max}(x))}$$

Example:

Let us assume that - according to the cumulative distribution function of the annual maximum traffic load - the annual probability that a truck load larger than 100 ton is equal to 0.02 – then the return period of such heavy truck events is:

$$T_R = n \cdot T = \frac{1}{0.02} \Rightarrow n = \frac{1}{1 \cdot 0.02} = 50 \text{ years}$$

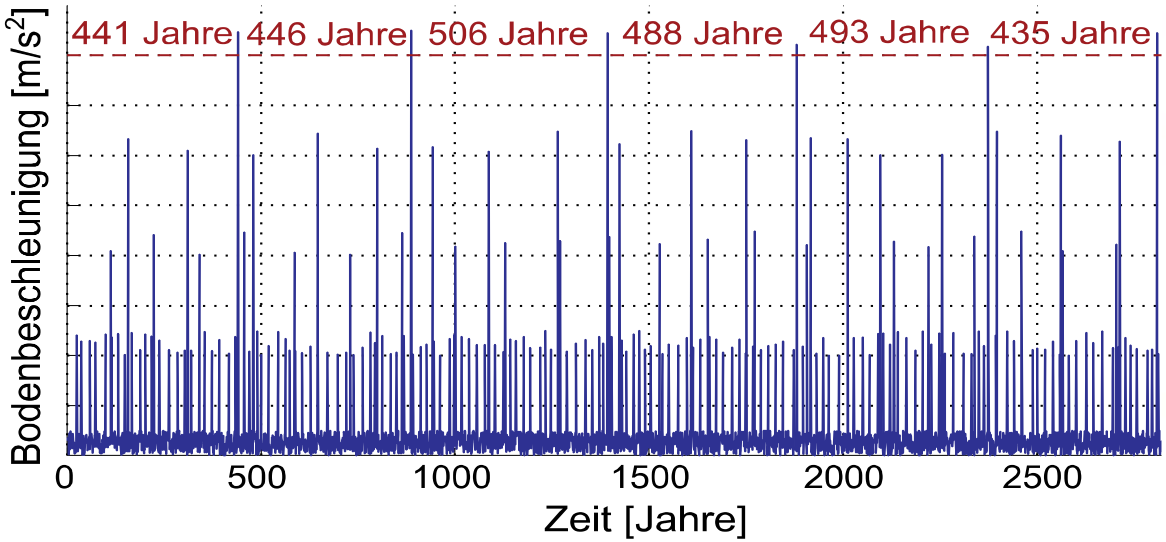
Example

A Earthquake hazard map represents the ground acceleration in (m/s^2) with a return period of 475 years.

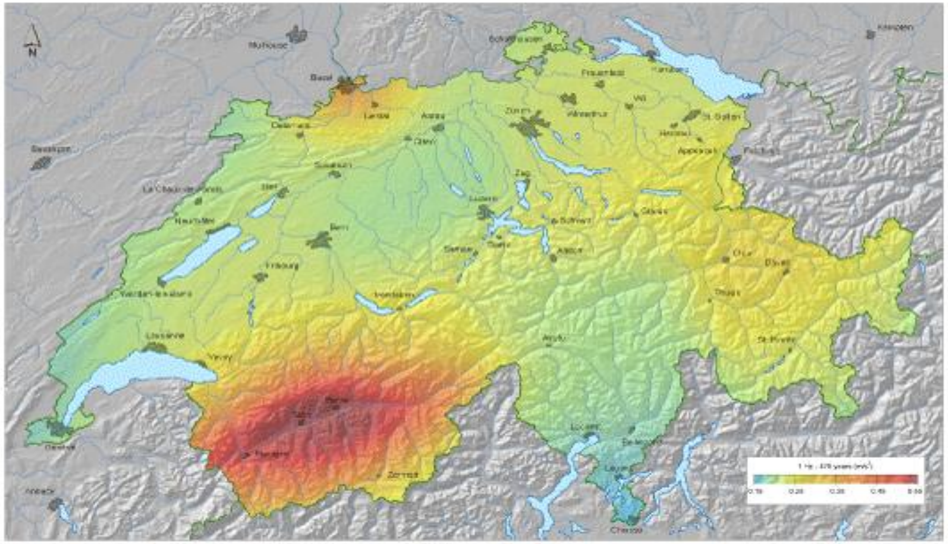
Tages-Anzeiger
Tuesday, 07. April 2009

Exercise

A Earthquake hazard map represents the ground acceleration in (m/s^2) with a return period of 475 years.



Mittlere Wiederkehrperidode = 475 Jahre



Earthquake Hazard in Switzerland: Horizontal Groundacceleration (m/s^2), 10% Probability of excess in 50 Years

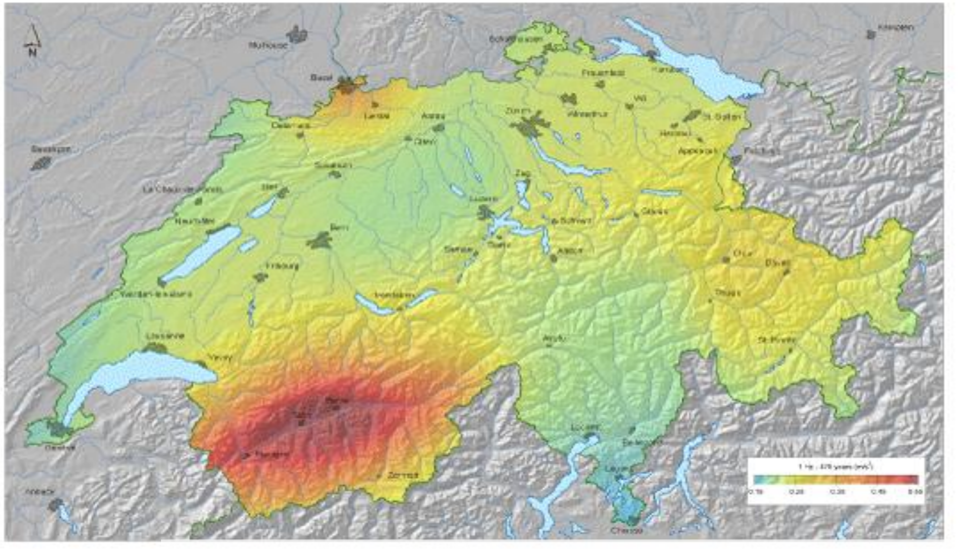
www.earthquake.ethz.ch

Excercise

A Earthquake hazard map represents the ground acceleration in (m/s^2) with a return period of 475 years.

a) Show that Return period 475 Years is equal to 10% Probability of excess in 50 Years

b) How large is the probability that an earthquake with that return period happens within 475 years?



Earthquake Hazard in Switzerland: Horizontal Groundacceleration (m/s^2), 10% Probability of excess in 50 Years
www.earthquake.ethz.ch

Homogenious Poisson Process

Solution

a) **Show that Return period 475 Years is equal to 10% Probability of excess in 50 Years**

yearly occurrence probability: $p = \frac{1}{T} = \frac{1}{475}$

mean time between two successive events: $E[T] = \frac{1}{p} = \frac{1}{\frac{1}{475}} = 475$

The time between two successive events for Poisson processes is exponential distributed.

$$E[T] = \frac{1}{\nu} = 475 \qquad \nu = \frac{1}{E[T]} = \frac{1}{475}$$

$$P\left[T \leq 50\, Jahre\right] = 1 - e^{-\nu(t)\cdot t} = 1 - e^{-\frac{1}{475}\cdot 50} = 0.1$$

Solution

b) How large is the probability that an earthquake with that return period happens within 475 years?

$$P\left[T \leq 475\right] = 1 - e^{-\nu(t)\cdot t} = 1 - e^{-\frac{1}{475}475} = 1 - \frac{1}{e} = 0.63$$

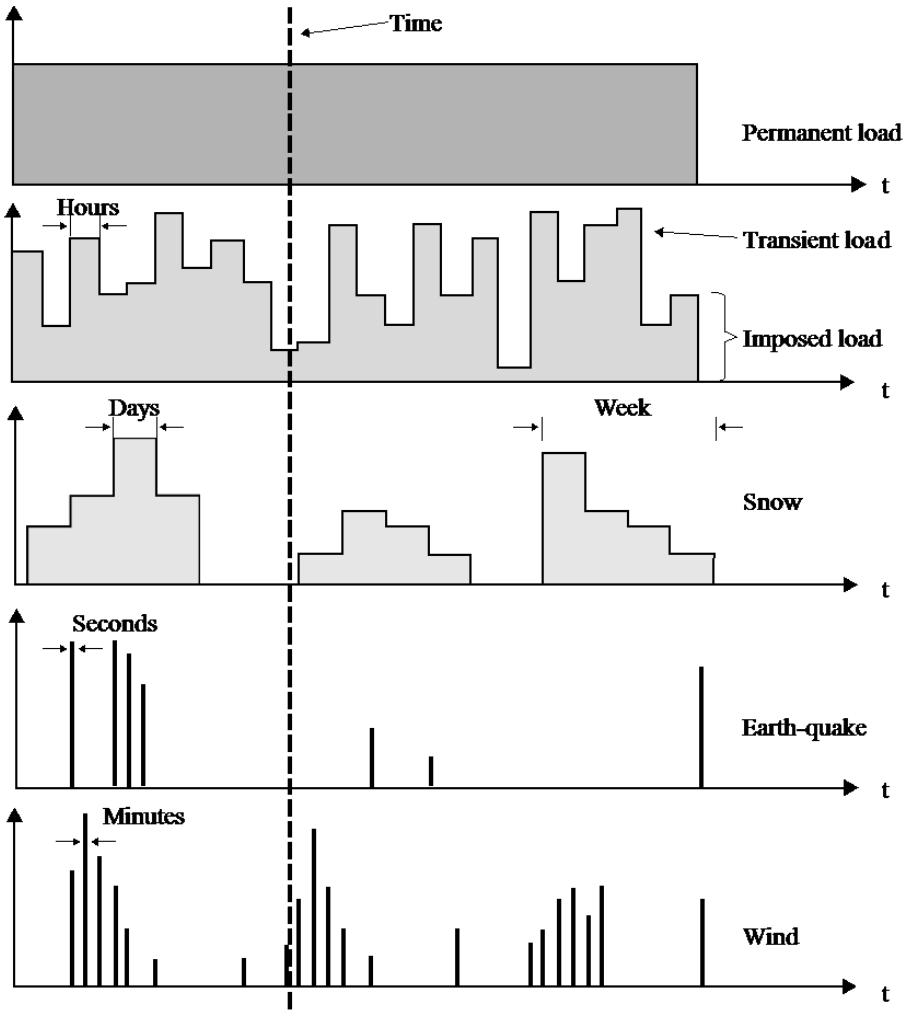
Probabilistic Modelling of Loads

- ## Loads on Structures

  Combination of loads

  We are interested in the maximum of a sum of load effects from different loads

$$X_{max}(T) = \max_{T}\{X_1(t) + X_2(t) + \ldots + X_n(t)\}$$



Probabilistic Modelling of Loads

- **Loads on Structures**

  Combination of loads

  Turkstra's load combination rule

  We take the max of the following combinations

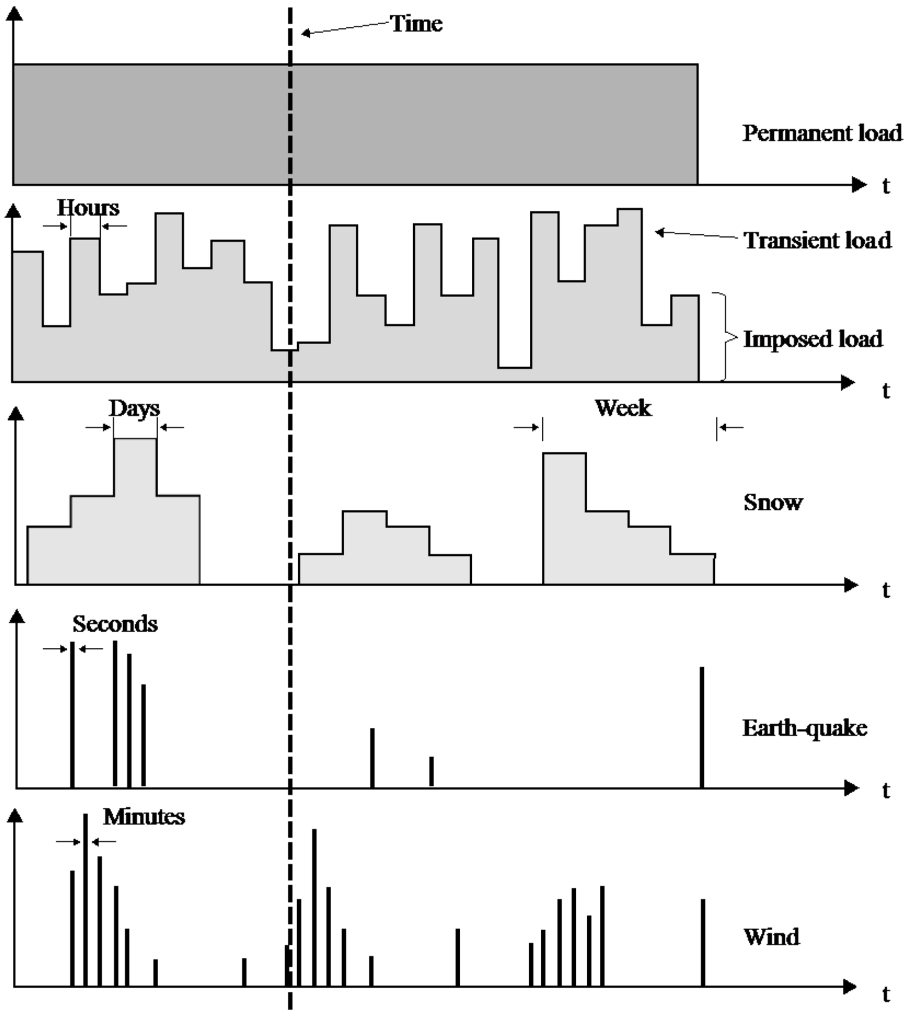
$$Z_1 = \max_T\{X_1(t)\} + X_2(t^*) + X_3(t^*) + \ldots + X_n(t^*)$$

$$Z_2 = X_1(t^*) + \max_T\{X_2(t)\} + X_3(t^*) + \ldots + X_n(t^*)$$

$$\vdots$$

$$Z_n = X_1(t^*) + X_2(t^*) + X_3(t^*) + \ldots + \max_T\{X_n(t)\}$$

$$X_{max}(T) \approx \max_i\{Z_i\}$$



Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

In structural engineering resistances include the following uncertainties

- Geometrical uncertainties
- Material characteristics
- Model uncertainties

} Random variation in time and space

The steps in the modelling process are:

- define the random variables used to represent the uncertainties in the resistances
- select a suitable distribution type to represent the random variable
- to assign the distribution parameters of the selected distribution.

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Concrete compressive strength

$$f_c = \alpha(t,\tau) f_{co}^{\lambda}$$

f_{co}:	**28 day compressive strength**
$\alpha(t,\tau)$:	**spatial stress and loading time function**
λ:	**conversion factor between in-situ concrete strength and cylinder compressive strength**

The concrete compressive strength can be assumed to be Log-Normal distributed with a coefficient of variation equal to 15%

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Reinforcement steel yield strength

$$f_s = X_1 + X_2 + X_3$$

X_1 normal distributed random variable representing the variation in the mean of different mills.

X_2 normal distributed zero mean random variable, which takes into account the variation between batches

X_3 normal distributed zero mean random variable, which takes into account the variation within a batch.

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Reinforcement steel yield strength

Variable	**Type**	$E[X]$	$\sigma_x\,[MPa]$	V_x
X_1	Normal	μ	19	-
X_2	Normal	0	22	-
X_3	Normal	0	8	-
A	-	A_{nom}	-	0.02

μ: nominal steel grade + two standard deviations of X_1

Yield stress depends on diameter of reinforcement bars

$$\mu(d) = \mu(0.87 + 0.13\exp(-0.08d))^{-1}$$

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Structural steel yield strength

Description	**Variable**	**Type**	$E[X]$	V_X
Yield stress	f_y	Lognormal	$f_{y\ sp} \alpha\ e^{-uV_{fy}} - C$	0.07
ultimate stress	f_u	Lognormal	$B\ E[f_u]$	0.04
modulus of elasticity	E	Lognormal	E_{sp}	0.03
Poisson's ratio	v	Lognormal	v_{sp}	0.03
ultimate strain	ε_u	Lognormal	$\varepsilon_{u\ sp}$	0.06

Distribution characteristics

	f_y	f_u	E	v	ε_u
f_y	1	0.75	0	0	-0.45
f_u		1	0	0	-0.60
E			1	0	0
v	Symmetry			1	0
ε_u					1

Dependencies

Probabilistic Modelling of Resistances

- ## Model uncertainties

  Model uncertainties relate engineering model results with actual structural behaviour

$$X = \Xi \cdot X_{\mathrm{mod}}$$

$$\xi = \frac{x_{\mathrm{mod}}}{x_{\mathrm{exp}}}$$

X:	**true value**
Ξ:	**model uncertainty**
X_{mod}:	**model value**
x_{exp}:	**experimentally obtained value**

Probabilistic Modelling of Resistances

- The JCSS Probabilistic Model Code (PMC)
  http://www.jcss.ethz.ch/publications/publications_pmc.html

  Part I : Basis of design
  Part II: Load models
  Part III: Resistance models
  Part IV: Examples

Probabilistic Modelling of Resistances

- **The JCSS PMC – Load Models**

2.00	GENERAL PRINCIPLES	05.2001
2.01	SELF WEIGHT	06.2001
2.02	LIVE LOAD	05.2001
2.06	LOAD IN CAR PARKS	05.2001
2.12	SNOW LOAD	05.2001
2.13	WIND LOAD	05.2001
2.15	WAVE LOAD	05.2006
2.17	EARTHQUAKE	09.2002
2.18	IMPACT LOAD	05.2001
2.20	FIRE	05.2001

Probabilistic Modelling of Resistances

- ## The JCSS PMC – Resistance models

3.00	GENERAL PRINCIPLES	03.2001
3.01	CONCRETE	05.2002
3.02	STRUCTURAL STEEL	03.2001
3.0*	REINFORCING STEEL	03.2001
3.04	PRESTRESSING STEEL	04.2005
3.05	TIMBER	05.2006
3.07	SOIL PROPERTIES	06.2002
3.09	MODELUNCERTAINTIES	03.2001
3.10	DIMENSIONS	03.2001
3.11	EXCENTRICITIES	03.2001