

# **Risk and Safety in Engineering**

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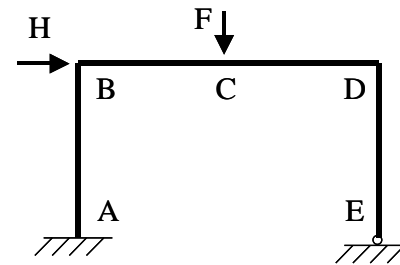
# Contents of Today's Lecture

- Introduction to structural systems reliability
- General systems reliability analysis
- Mechanical modelling of systems
- Reliability analysis for structural systems
- Risk based assessment of structural robustness

# Introduction to structural systems reliability

Until now we have focused on the reliability of individual failure modes

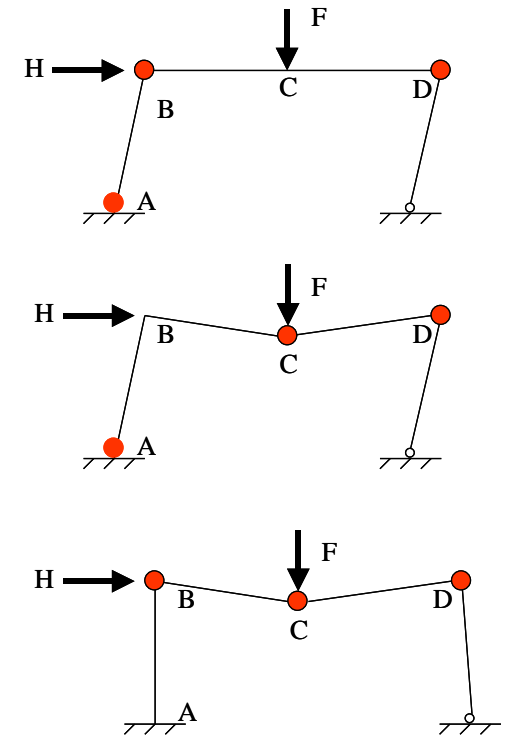
- Reliability analyses of components



However, generally structural systems only fail if two or more failure modes/components fail.

This problem complex is addressed by the theory of

- Structural systems reliability analysis



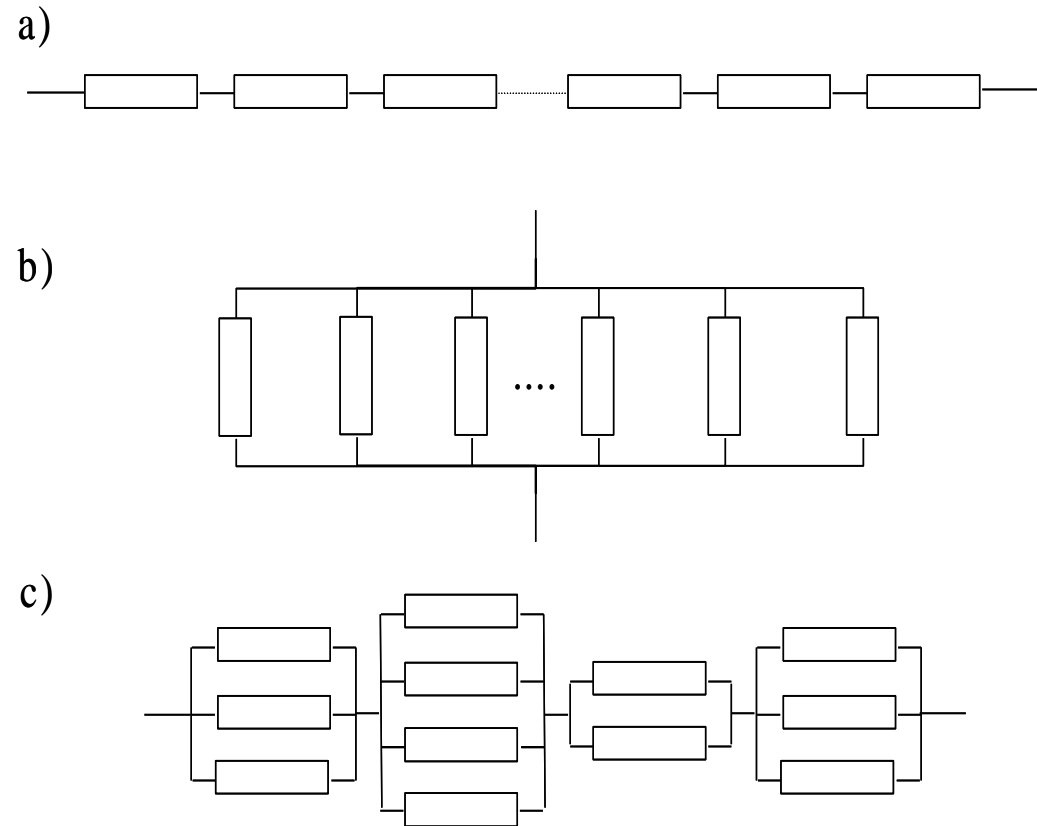
# General systems reliability analysis

## Probabilistic characteristics of systems

Block diagrams are normally used in the representation of systems in structural systems reliability analysis

Each component in the block diagrams represent one failure mode for the structure

- a) series system
- b) parallel system
- c) mixed system



# General systems reliability analysis

## Uncorrelated components

The failure probability of a **series system** may be determined by

$$P_F = 1 - P_S = 1 - \prod_{i=1}^n (1 - P(F_i))$$

The failure probability of a **parallel system** may be determined by

$$P_F = \prod_{i=1}^n P(F_i)$$

# General systems reliability analysis

## Correlated components

If the individual components of the system have linear and Normal distributed safety margins

The failure probability of a **series system** may be determined by

$$P_F = 1 - P_S = 1 - \Phi_n(\boldsymbol{\beta}, \boldsymbol{\rho})$$

The failure probability of a **parallel system** may be determined by

$$P_F = \Phi_n(-\boldsymbol{\beta}, \boldsymbol{\rho})$$

# General systems reliability analysis

## Simple bounds on systems reliability

The failure probability of a **series system** may be bounded by

$$\max_{i=1}^n \{P(F_i)\} \leq P_F \leq 1 - \prod_{i=1}^n (1 - P(F_i))$$

Full correlation                      Uncorrelated

The failure probability of a **parallel system** may be bounded by

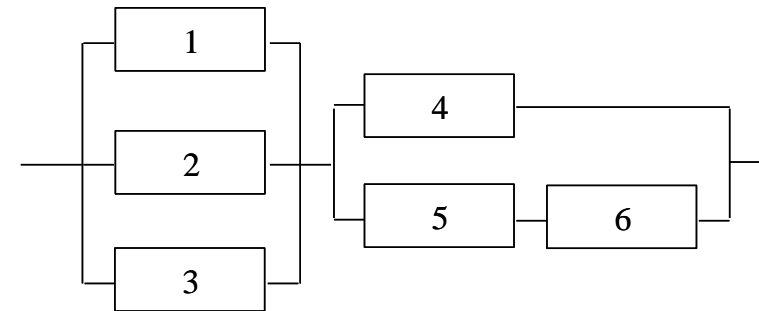
$$\prod_{i=1}^n P(F_i) \leq P_F \leq \min_{i=1}^n \{P(F_i)\}$$

Uncorrelated                      Full correlation

# General systems reliability analysis

## Example

We consider a structural system for which failure is represented by the following block diagram



The components have the following failure probabilities

$$P(F_1) = P(F_2) = P(F_4) = 1 \cdot 10^{-2}$$

$$P(F_3) = P(F_5) = P(F_6) = 1 \cdot 10^{-5}$$

The components may be correlated

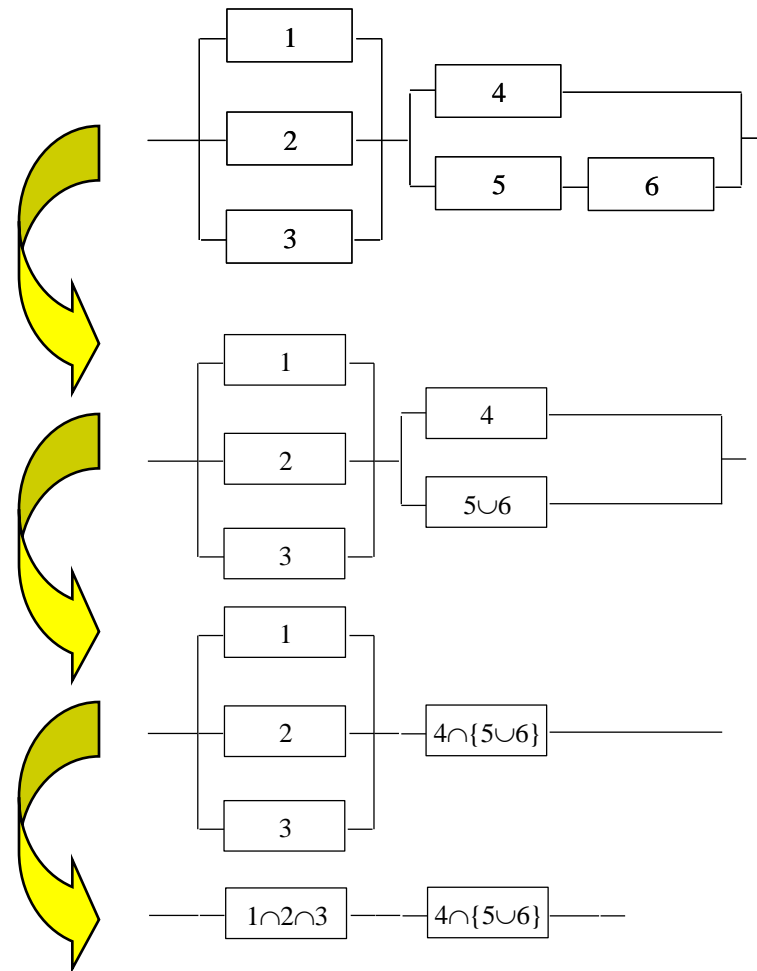


# General systems reliability analysis

## Example

How can we in a simplified manner analyse such a mixed system of series and parallel systems in combination

We can reduce it into sub-systems sequentially:  
either into series systems or parallel systems



# Systems reliability analysis

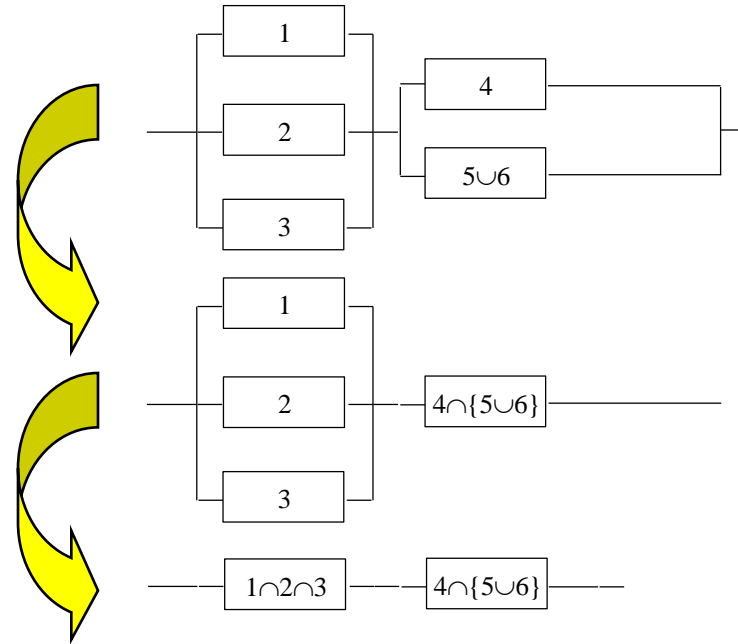
## Example

If we assume uncorrelated components we have

$$P(5 \cup 6) = 1 - (1 - 1 \cdot 10^{-5})^2 = 2 \cdot 10^{-5}$$

$$P(4 \cap \{5 \cup 6\}) = 1 \cdot 10^{-2} \times 2 \cdot 10^{-5} = 2 \cdot 10^{-7}$$

$$P(1 \cap 2 \cap 3) = (1 \cdot 10^{-2})^2 (1 \cdot 10^{-5}) = 1 \cdot 10^{-9}$$



$$P_{S, \rho=0} = P(\{1 \cap 2 \cap 3\} \cup \{4 \cap \{5 \cup 6\}\}) = 1 - (1 - 2 \cdot 10^{-7})(1 - 1 \cdot 10^{-9}) = 2.01 \cdot 10^{-7}$$

# Systems reliability analysis

## Example

If we assume correlated components we have

$$P(5 \cup 6) = \max(1 \cdot 10^{-5}, 1 \cdot 10^{-5}) = 1 \cdot 10^{-5}$$

$$P(4 \cap \{5 \cup 6\}) = \min(1 \cdot 10^{-2}, 1 \cdot 10^{-5}) = 1 \cdot 10^{-5}$$

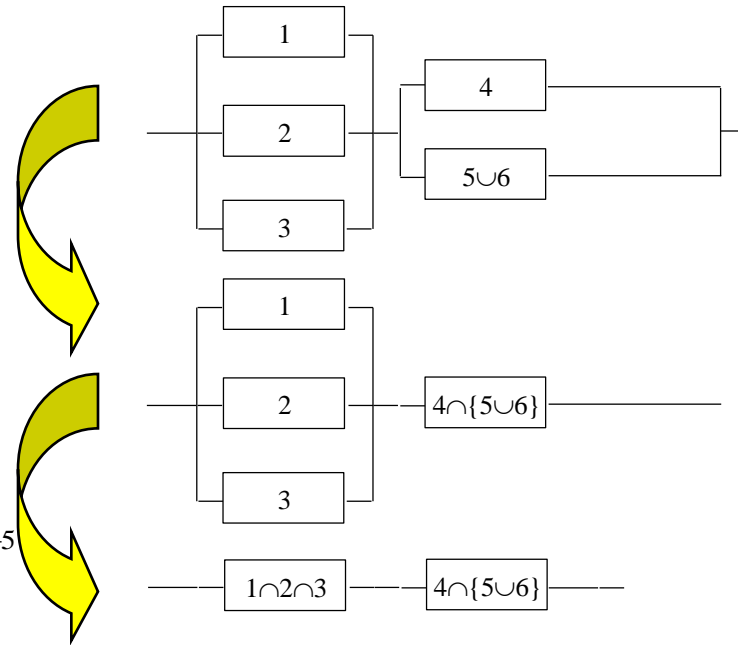
$$P(1 \cap 2 \cap 3) = \min(1 \cdot 10^{-2}, 1 \cdot 10^{-2}, 1 \cdot 10^{-5}) = 1 \cdot 10^{-5}$$

$$P_{S, \rho=1} = P(\{1 \cap 2 \cap 3\} \cup \{4 \cap \{5 \cup 6\}\}) = \max(1 \cdot 10^{-5}, 1 \cdot 10^{-5})$$

$$P_{S, \rho=1} = 1 \cdot 10^{-5}$$

The simple bounds are

$$2.01 \cdot 10^{-7} \leq P_S \leq 1 \cdot 10^{-5}$$



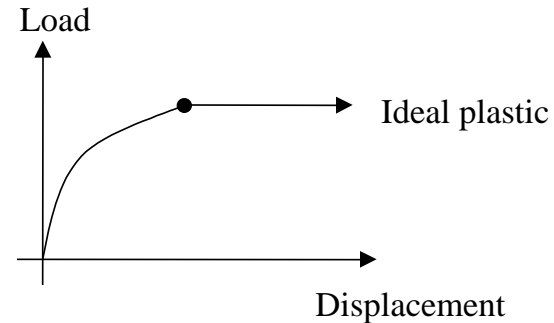
# General systems reliability analysis

## Mechanical modelling of structural systems

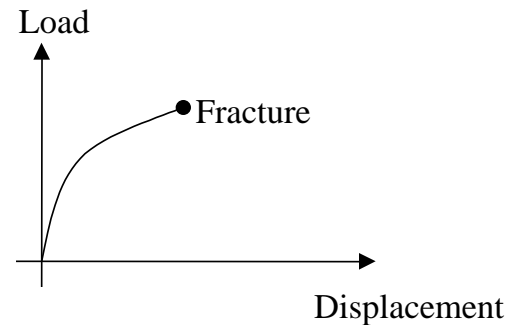
The behaviour of structural failure modes after failure is important for the safety of the system

Two extreme cases are

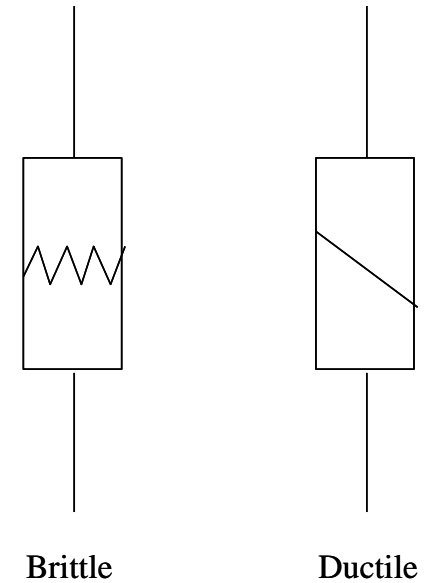
- ductile components
- brittle components



Ductile behaviour



Brittle behaviour



# General systems reliability analysis

## Parallel systems with ductile components

Assume a parallel system with  $n$  ductile components

The second order statistics of the strength are then given by

$$\mu_{R_S} = \sum_{i=1}^n \mu_{R_i} \quad \sigma_{R_S}^2 = \sum_{i=1}^n \sigma_{R_i}^2$$

Furthermore we have that the strength is Normal distributed using the **central limit theorem**

If  $\mu_{R_1} = \mu_{R_2} = \dots = \mu_{R_n} = \mu$  and  $\sigma_{R_1} = \sigma_{R_2} = \dots = \sigma_{R_n} = \sigma$  then we have:

$$COV = \frac{\sigma}{\sqrt{n} \cdot \mu}$$

## Why is robustness an issue?

- Despite modernization of design codes the engineering profession is still facing problems in terms of
  - collapsing structures and building
  - steady increase of insured damages

# Why is robustness an issue?

- Examples of collapses

Bad Reichenhall  
Germany, 2006



# Why is robustness an issue?

- Examples of collapses

Siemens arena  
Denmark, 2003

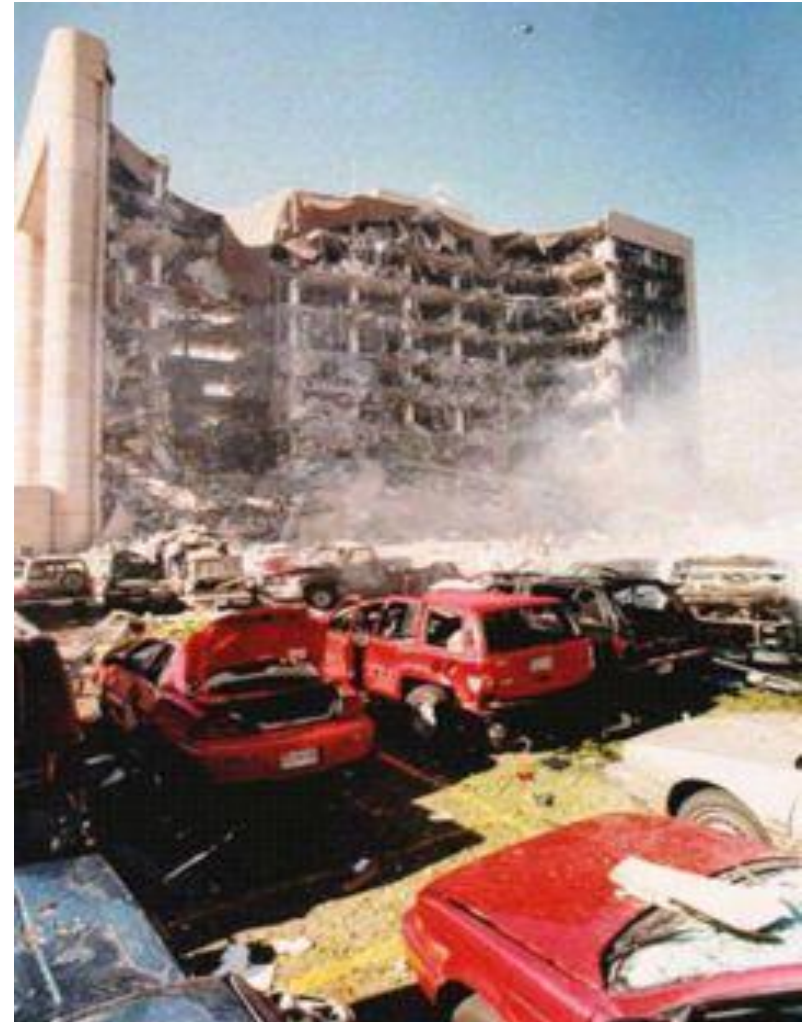




# Why is robustness an issue?

- Examples of collapses

Oklahoma City bombing  
USA, 1995



# Why is robustness an issue?

- Examples of collapses

World Trade Center  
USA, 2001



# Why is robustness an issue?

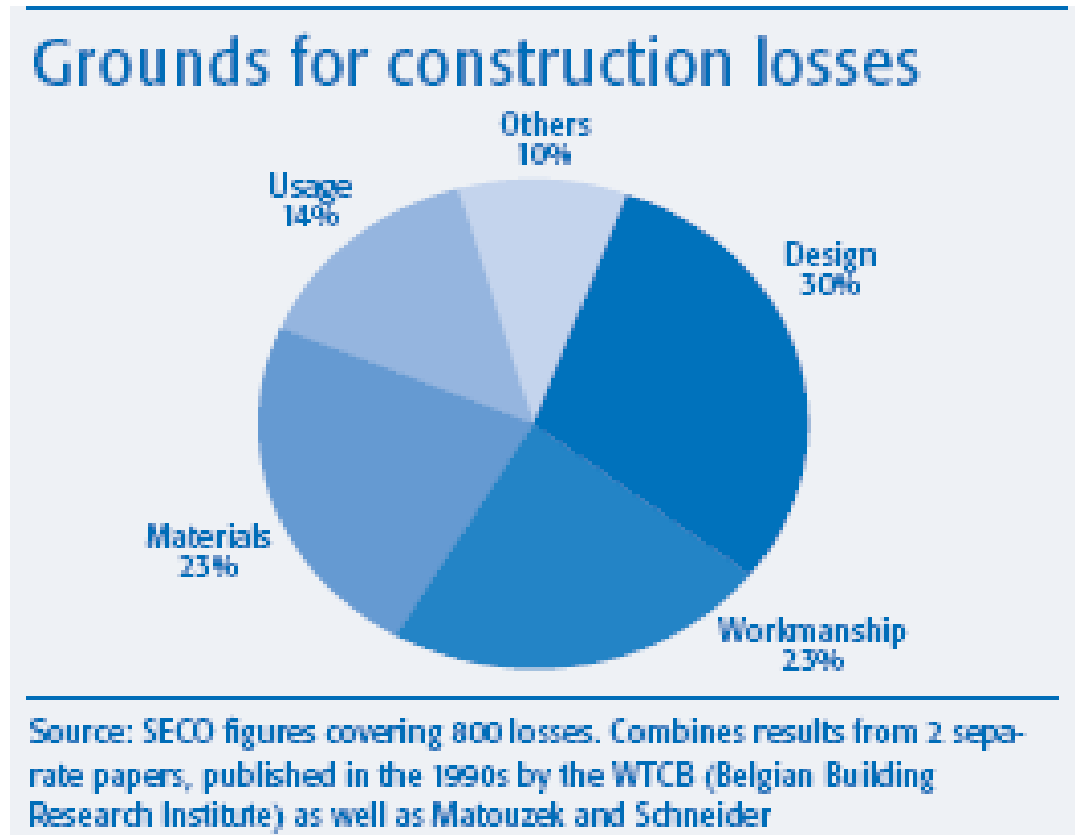
- Examples of collapses

Charles de Gaulle  
France, 2004



## Why is robustness an issue?

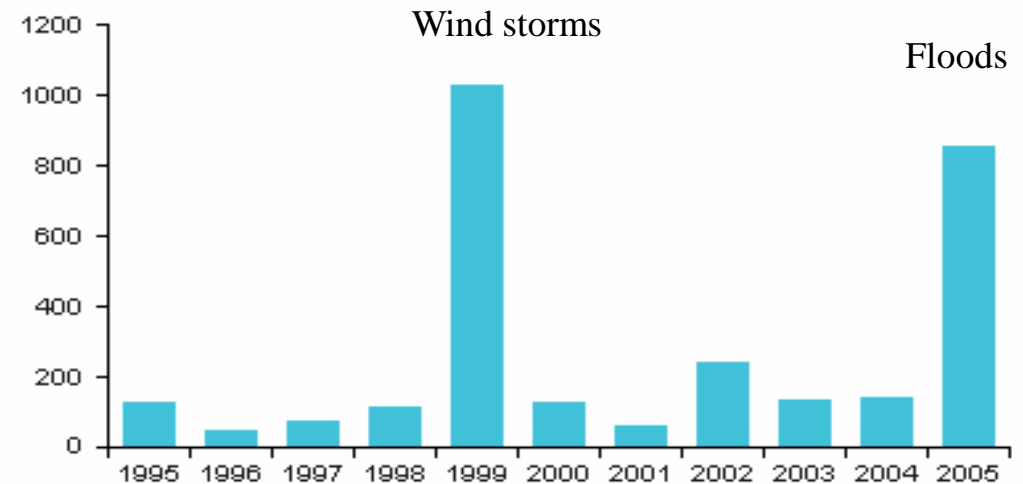
- Losses due to building failures



# Why is robustness an issue?

- Insured losses due to building failures

IRV Interkantonaler  
Rückversicherungs-  
verband, Switzerland



Quelle: Schadenstatistik VKF

# What is understood as robustness?

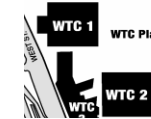
Structural Standards	The consequences of structural failure are not disproportional to the effect causing the failure [2].
Software Engineering	The ability...to react appropriately to abnormal circumstances (i.e., circumstances “outside of specifications”). A system may be correct without being robust [17].
Product Development and QC	The measure of the capacity of a production process to remain unaffected by small but deliberate variations of internal parameters so as to provide an indication of the reliability during normal use.
Ecosystems	The ability of a system to maintain function even with changes in internal structure or external environment [18].
Control Theory	The degree to which a system is insensitive to effects that are not considered in the design [19].
Statistics	A robust statistical technique is insensitive against small deviations in the assumptions [20].
Design Optimization	A robust solution in an optimization problem is one that has the best performance under its worst case (max-min rule) [21].
Bayesian Decision Making	By introducing a wide class of priors and loss functions, the elements of subjectivity and sensitivity to a narrow class of choices, are both reduced [22]
Language	The robustness of language...is a measure of the ability of human speakers to communicate despite incomplete information, ambiguity, and the constant element of surprise [23].

## What are the attributes of robustness?

- Design codes have so far focused on inherent properties of the structures (components)
  - redundancy
  - ductility
  
- More recently focus has been directed to:
  - system performance (removal of members)
  - structural ties

# What are the attributes of robustness?

The material loss cost consequences due to the collapse of the two WTC towers only comprised  $\frac{1}{4}$  of the total costs due to damaged or lost material



It seems relevant to include consequences in the robustness equation !

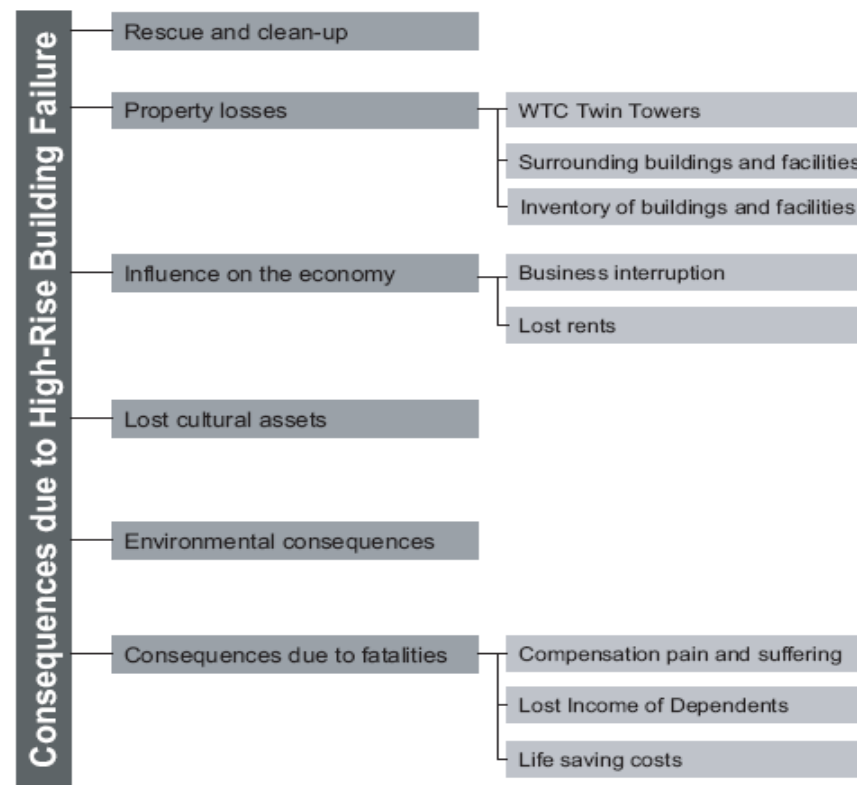
and these are scenario dependent !





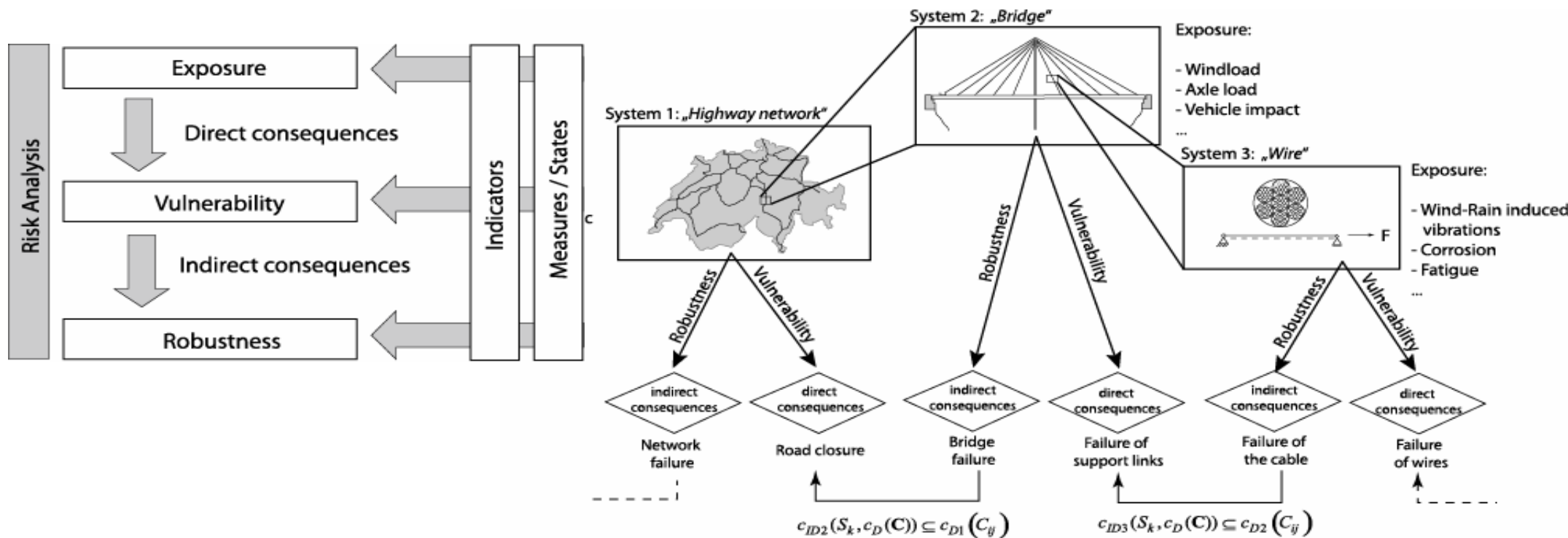
# Which are the attributes of robustness?

- The system definition is important because it defines the consequences following structural failures



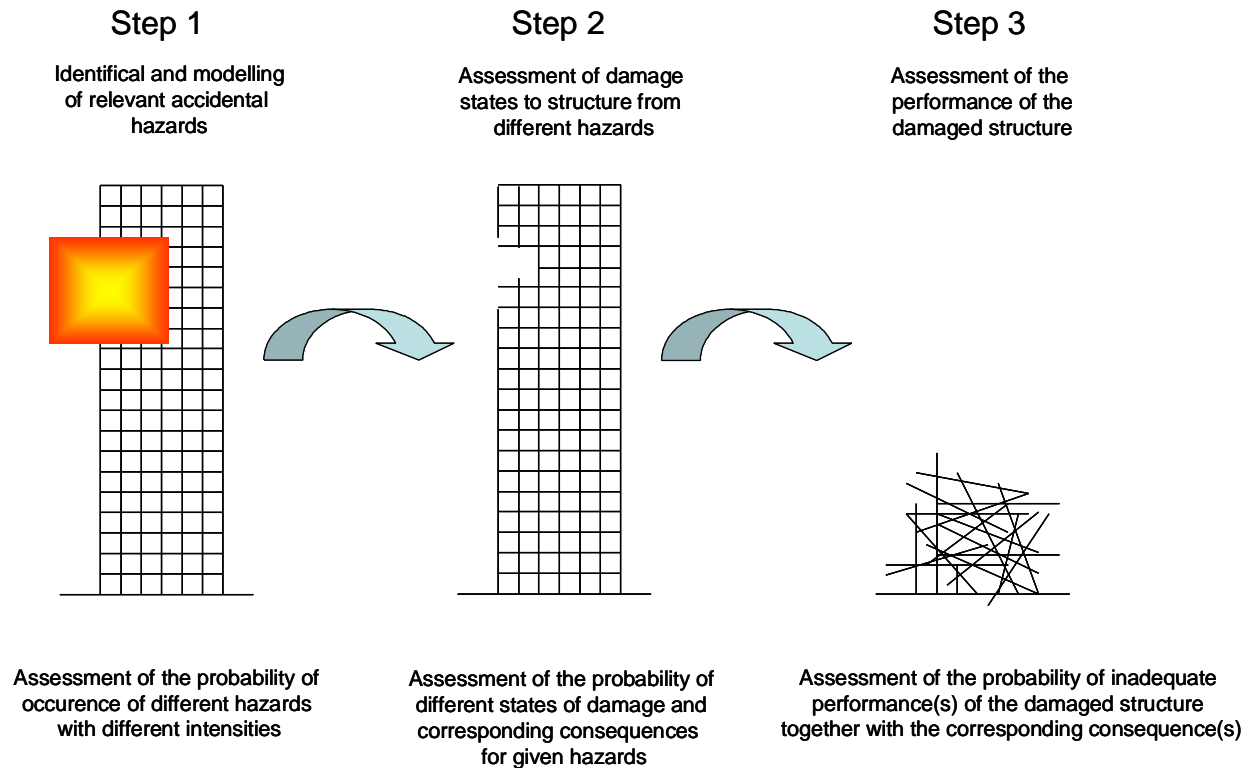
# How to frame robustness?

- Engineered systems have certain characteristics of generic nature – concept developed in the JCSS






# How to frame robustness?

- This concept is also the idea behind the Eurocodes



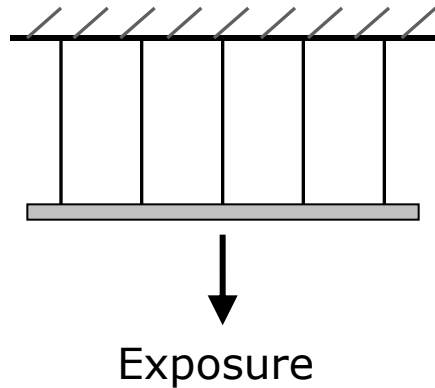
# How to frame robustness?

Scenario representation	Physical characteristics	Indicators	Potential consequences
<p><b>Exposure</b></p> 	<ul style="list-style-type: none"> <li>Flood</li> <li>Ship impact</li> <li>Explosion/Fire</li> <li>Earthquake</li> <li>Vehicle impact</li> <li>Wind loads</li> <li>Traffic loads</li> <li>Deicing salt</li> <li>Water</li> <li>Carbon dioxide</li> </ul>	<ul style="list-style-type: none"> <li>Use/functionality</li> <li>Location</li> <li>Environment</li> <li>Design life</li> <li>Societal importance</li> </ul>	
<p><b>Vulnerability</b></p> 	<ul style="list-style-type: none"> <li>Yielding</li> <li>Rupture</li> <li>Cracking</li> <li>Fatigue</li> <li>Wear</li> <li>Spalling</li> <li>Erosion</li> <li>Corrosion</li> </ul>	<ul style="list-style-type: none"> <li>Design codes</li> <li>Design target reliability</li> <li>Age</li> <li>Materials</li> <li>Quality of workmanship</li> <li>Condition</li> <li>Protective measures</li> </ul>	<p><b>Direct consequences</b></p> <ul style="list-style-type: none"> <li>Repair costs</li> <li>Temporary loss or reduced functionality</li> <li>Small number of injuries/fatalities</li> <li>Minor socio-economic losses</li> <li>Minor damages to environment</li> </ul>
<p><b>Robustness</b></p> 	<ul style="list-style-type: none"> <li>Loss of functionality</li> <li>partial collapse</li> <li>full collapse</li> </ul>	<ul style="list-style-type: none"> <li>Ductility</li> <li>Joint characteristics</li> <li>Redundancy</li> <li>Segmentation</li> <li>Condition</li> <li>control/monitoring</li> <li>Emergency preparedness</li> </ul>	<p><b>Indirect consequences</b></p> <ul style="list-style-type: none"> <li>Repair costs</li> <li>Temporary loss or reduced functionality</li> <li>Mid to large number of injuries/fatalities</li> <li>Moderate to major socio-economic losses</li> <li>Moderate to major damages to environment</li> </ul>

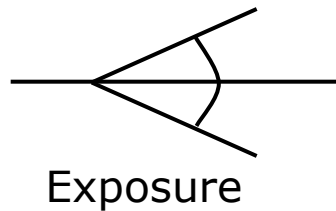
# Assessing robustness – a risk based framework

- **Desirable properties of a robustness measure**
  - **Applicable to general systems**
  - **Allows for ranking of alternative systems**
  - **Provides a criterion for identifying acceptable robustness**

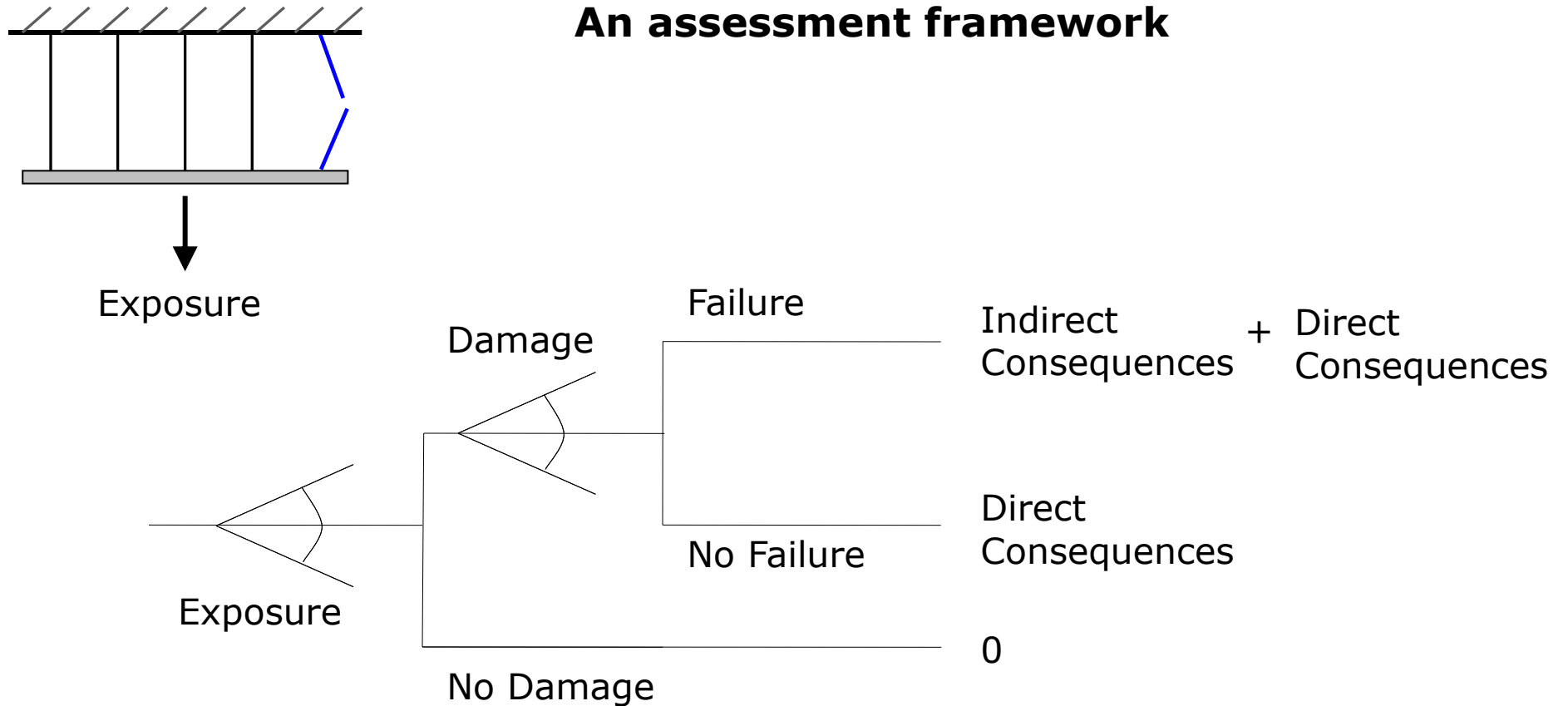
# Assessing robustness – a risk based framework



## An assessment framework

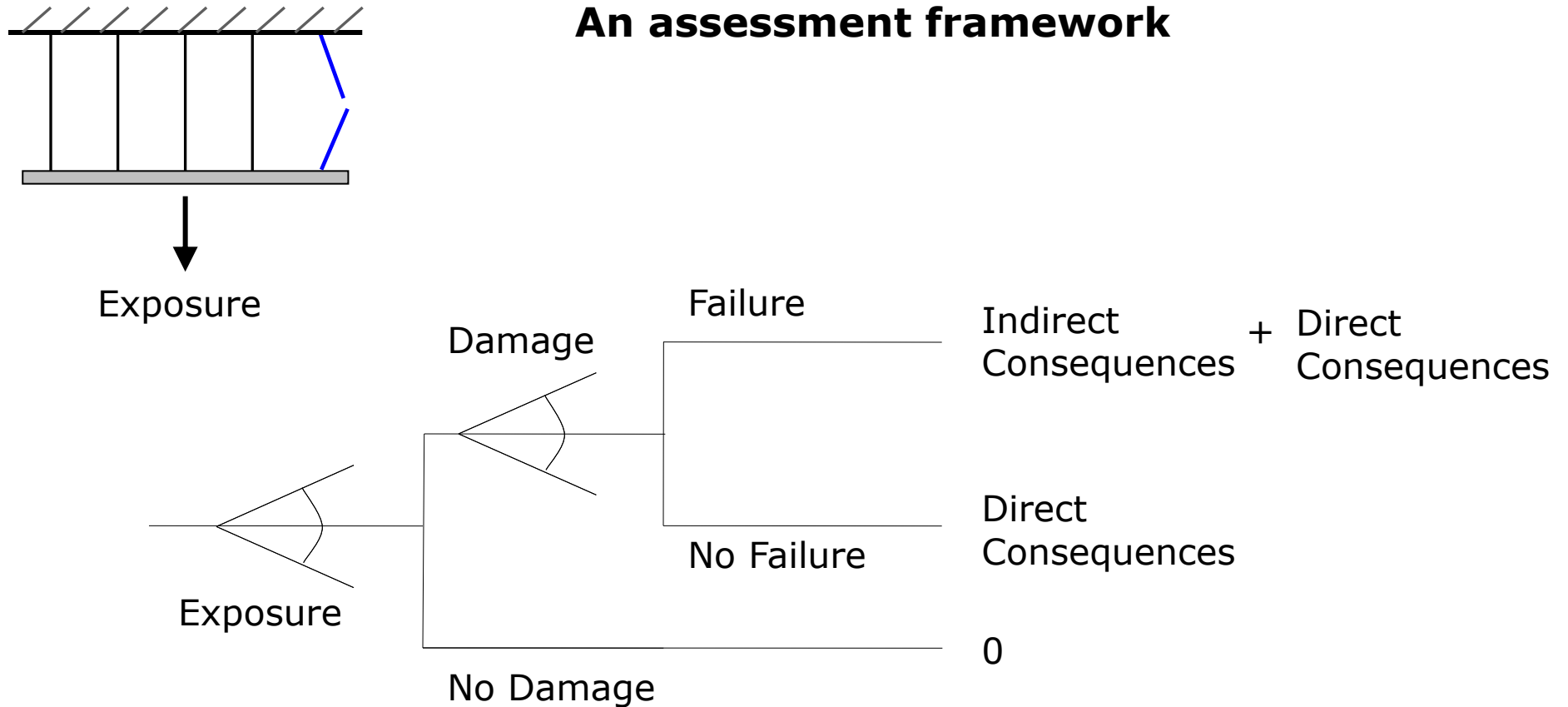


# Assessing robustness – a risk based framework





# Assessing robustness – a risk based framework



**An index of robustness:** 
$$I_{Rob} = \frac{\text{Direct Risk}}{\text{Direct Risk} + \text{Indirect Risk}}$$

# Assessing robustness – a risk based framework

## Features of the proposed index

$$I_{\text{Rob}} = \frac{\text{Direct Risk}}{\text{Direct Risk} + \text{Indirect Risk}}$$

- Assumes values between zero and one
- Measures relative risk only
- Dependent upon the probability of damage occurrence
- Dependent upon consequences