Risk and Safety

in

Engineering

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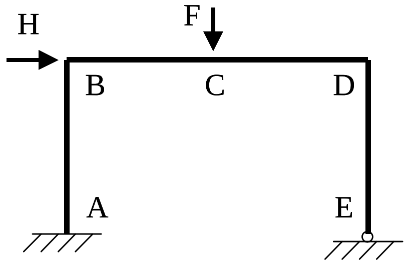
Contents of Today's Lecture

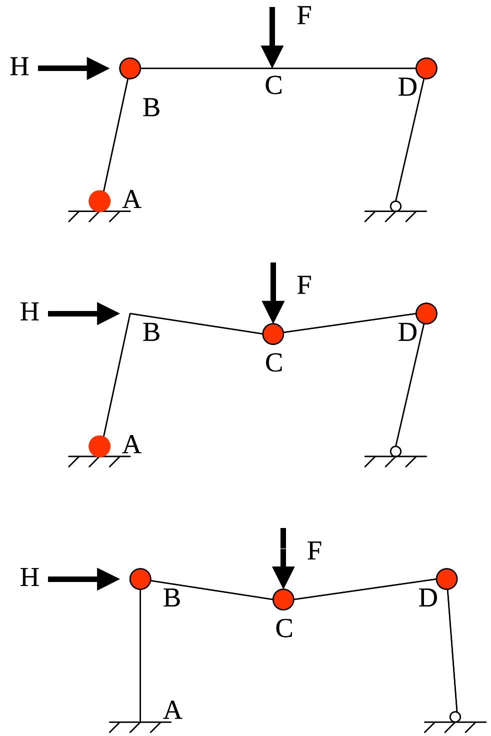
- Introduction to structural systems reliability
- General systems reliability analysis
- Mechanical modelling of systems
- Reliability analysis for structural systems
- Risk based assessment of structural robustness

Introduction to structural systems reliability

Until now we have focused on the reliability of individual failure modes



- Reliability analyses of components



However, generally structural systems only fail if two or more failure modes/components fail.

This problem complex is addressed by the theory of

- Structural systems reliability analysis

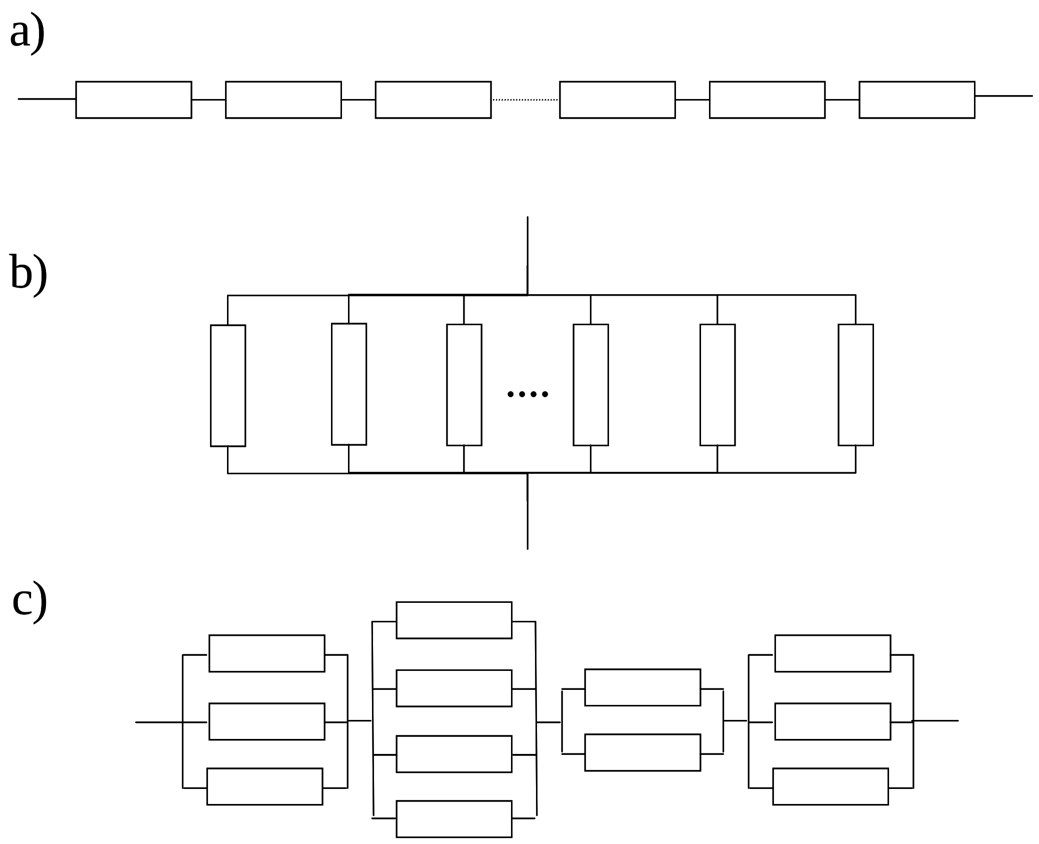
General systems reliability analysis

Probabilistic characteristics of systems

Block diagrams are normally used in the representation of systems in structural systems reliability analysis

Each component in the block diagrams represent one failure mode for the structure

a) series system
b) parallel system
c) mixed system



General systems reliability analysis

Uncorrelated components

The failure probability of a series system may be determined by

$$P_F = 1 - P_S = 1 - \prod_{i=1}^{n} (1 - P(F_i))$$

The failure probability of a parallel system may be determined by

$$P_F = \prod_{i=1}^{n} P(F_i)$$

General systems reliability analysis

Correlated components

If the individual components of the system have linear and Normal distributed safety margins

The failure probability of a **series system** may be determined by

$$P_F = 1 - P_S = 1 - \Phi_n(\boldsymbol{\beta}, \boldsymbol{\rho})$$

The failure probability of a **parallel system** may be determined by

$$P_F = \Phi_n(-\boldsymbol{\beta}, \boldsymbol{\rho})$$

General systems reliability analysis

Simple bounds on systems reliability

The failure probability of a series system may be bounded by

$$\max_{i=1}^{n}\left\{P(F_i)\right\} \leq P_F \leq 1-\prod_{i=1}^{n}(1-P(F_i))$$

Full correlation Uncorrelated

The failure probability of a parallel system may be bounded by

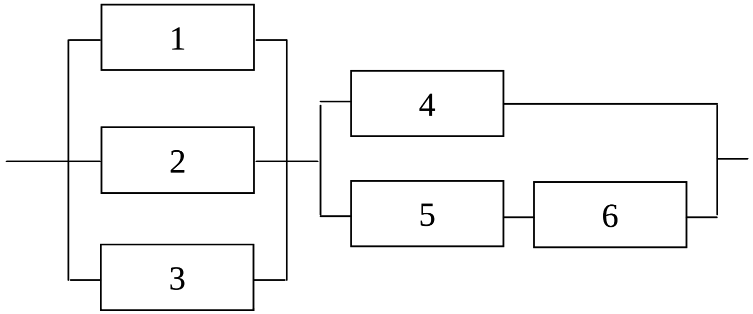
$$\prod_{i=1}^{n}P(F_i) \leq P_F \leq \min_{i=1}^{n}\left\{P(F_i)\right\}$$

Uncorrelated Full correlation

General systems reliability analysis

Example

We consider a structural system for which failure is represented by the following block diagram



The components have the following failure probabilities

$$P(F_1) = P(F_2) = P(F_4) = 1 \cdot 10^{-2}$$

$$P(F_3) = P(F_5) = P(F_6) = 1 \cdot 10^{-5}$$

The components may be correlated

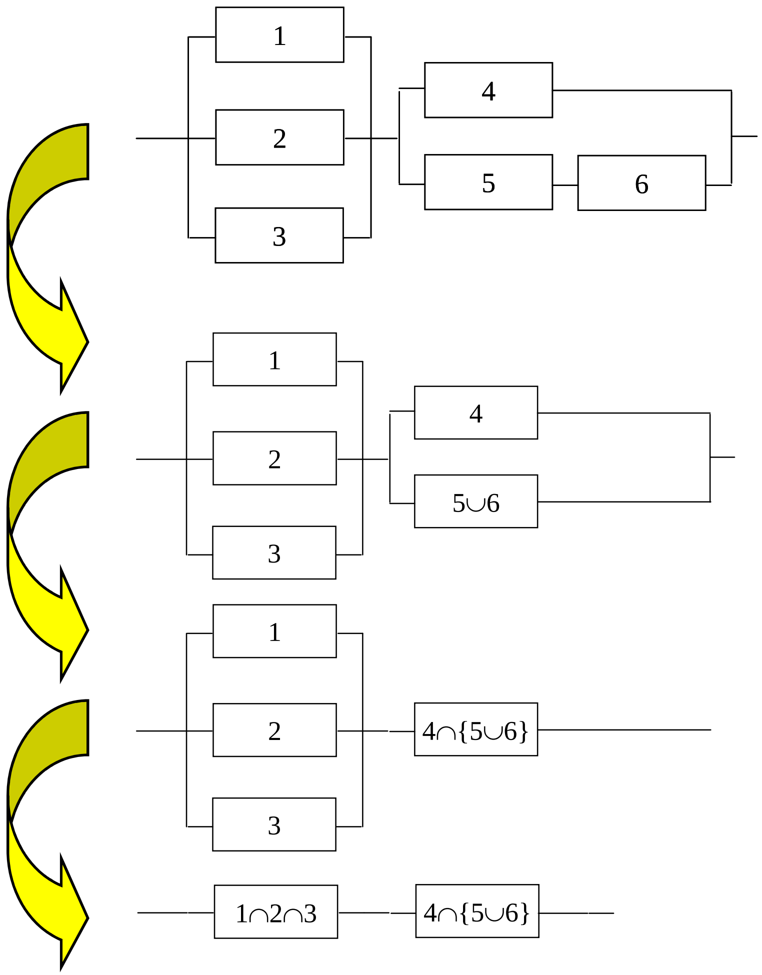
General systems reliability analysis

Example

How can we in a simplified manner analyse such a mixed system of series and parallel systems in combination

We can reduce it into sub-systems sequentially:

either into series systems or parallel systems



Systems reliability analysis

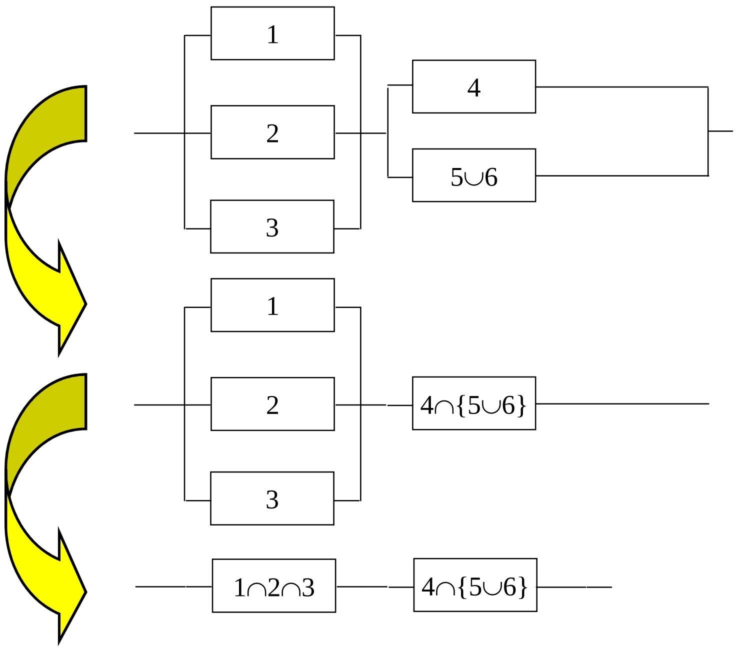
Example

If we assume uncorrelated components we have

$$P(5 \bigcup 6) = 1-(1-1\cdot 10^{-5})^2 = 2\cdot 10^{-5}$$

$$P(4 \bigcap \{5 \bigcup 6\}) = 1\cdot 10^{-2} \times 2\cdot 10^{-5} = 2\cdot 10^{-7}$$

$$P(1 \bigcap 2 \bigcap 3) = (1\cdot 10^{-2})^2 (1\cdot 10^{-5}) = 1\cdot 10^{-9}$$

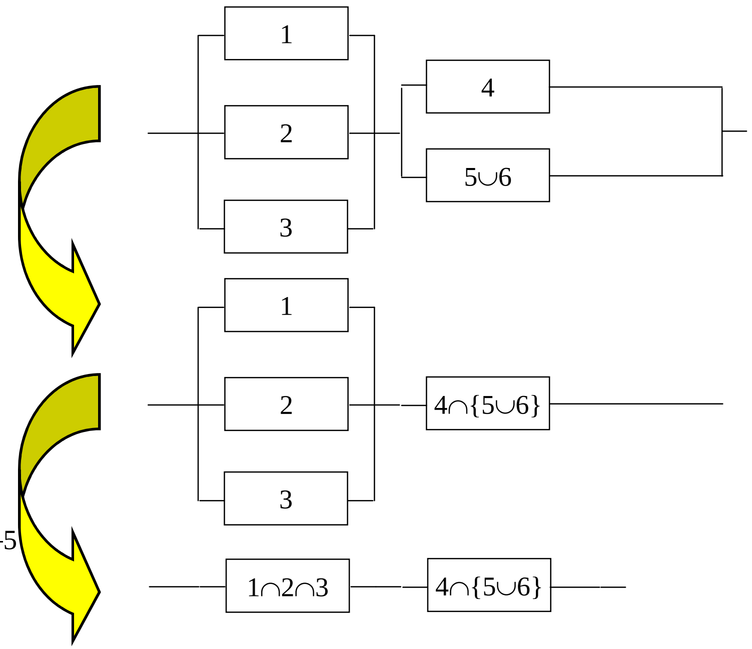


$$P_{S,\rho=0} = P(\{1 \bigcap 2 \bigcap 3\} \bigcup \{4 \bigcap \{5 \bigcup 6\}\}) = 1-(1-2\cdot 10^{-7})(1-1\cdot 10^{-9}) = 2.01\cdot 10^{-7}$$

Systems reliability analysis

Example

If we assume correlated components we have



$$P(5 \bigcup 6) = \max(1 \cdot 10^{-5}, 1 \cdot 10^{-5}) = 1 \cdot 10^{-5}$$

$$P(4 \bigcap \{5 \bigcup 6\}) = \min(1 \cdot 10^{-2}, 1 \cdot 10^{-5}) = 1 \cdot 10^{-5}$$

$$P(1 \bigcap 2 \bigcap 3) = \min(1 \cdot 10^{-2}, 1 \cdot 10^{-2}, 1 \cdot 10^{-5}) = 1 \cdot 10^{-5}$$

$$P_{S,\rho=1} = P(\{1 \bigcap 2 \bigcap 3\} \bigcup \{4 \bigcap \{5 \bigcup 6\}\}) = \max(1 \cdot 10^{-5}, 1 \cdot 10^{-5})$$

$$P_{S,\rho=1} = 1 \cdot 10^{-5}$$

The simple bounds are

$$2.01 \cdot 10^{-7} \leq P_S \leq 1 \cdot 10^{-5}$$

General systems reliability analysis

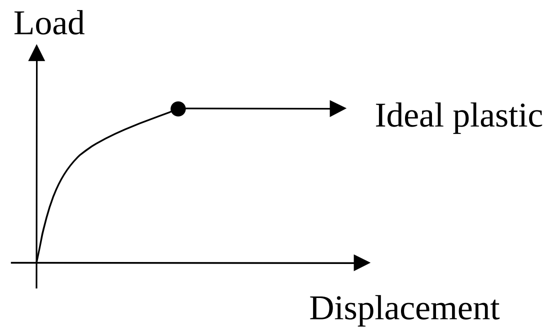
Mechanical modelling of structural systems

The behaviour of structural failure modes after failure is important for the safety of the system

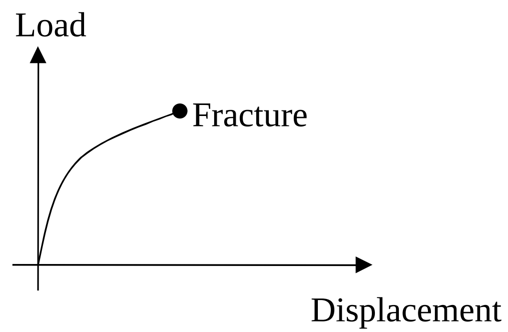
Two extreme cases are

- ductile components

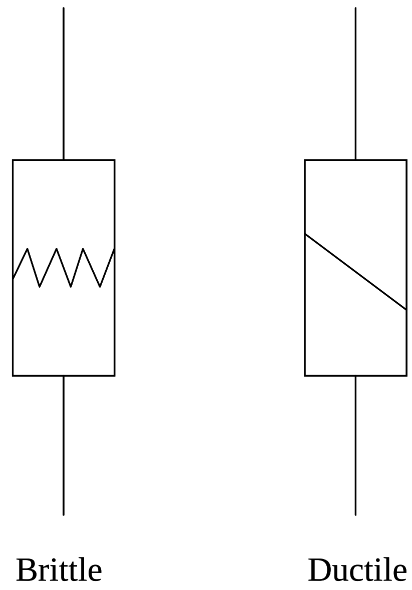
- brittle components



Ductile behaviour



Brittle behaviour



General systems reliability analysis

Parallel systems with ductile components

Assume a parallel system with n ductile components

The second order statistics of the strength are then given by

$$\mu_{R_S} = \sum_{i=1}^{n} \mu_{R_i} \qquad \sigma_{R_S}^2 = \sum_{i=1}^{n} \sigma_{R_i}^2$$

Furthermore we have that the strength is Normal distributed using the central limit theorem

If $\mu_{R_1} = \mu_{R_2} = \ldots = \mu_{R_n} = \mu$ and $\sigma_{R_1} = \sigma_{R_2} = \ldots = \sigma_{R_n} = \sigma$ then we have:

$$COV = \frac{\sigma}{\sqrt{n} \cdot \mu}$$

General systems reliability analysis

Parallel systems with brittle components

If the stengths of the n components are $r_1, r_2, ..., r_{n-1}, r_n$
and $r_1 \le r_2 \le .... \le r_{n-1} \le r_n$ then we have:

$$R_S = \max(nr_1, (n-1)r_2, ..., 2r_{n-1}, r_n)$$

and

$$\mu_{R_S} = n \cdot r_0 (1 - F_R(r_0))$$

$$\sigma^2_{R_S} = n \cdot r_0^2 F_R(r_0)(1 - F_R(r_0))$$

$$\text{where } r_0 = \text{value maximising } r(1 - F_R(r))$$

The uncertainty of the strength of parallel systems approaches zero for large *n*

$$COV = \frac{\sqrt{F_R(r_0)(1 - F_R(r_0)}}{\sqrt{n}(1 - F_R(r_0)}$$

General systems reliability analysis

Methods of structural systems reliability analysis

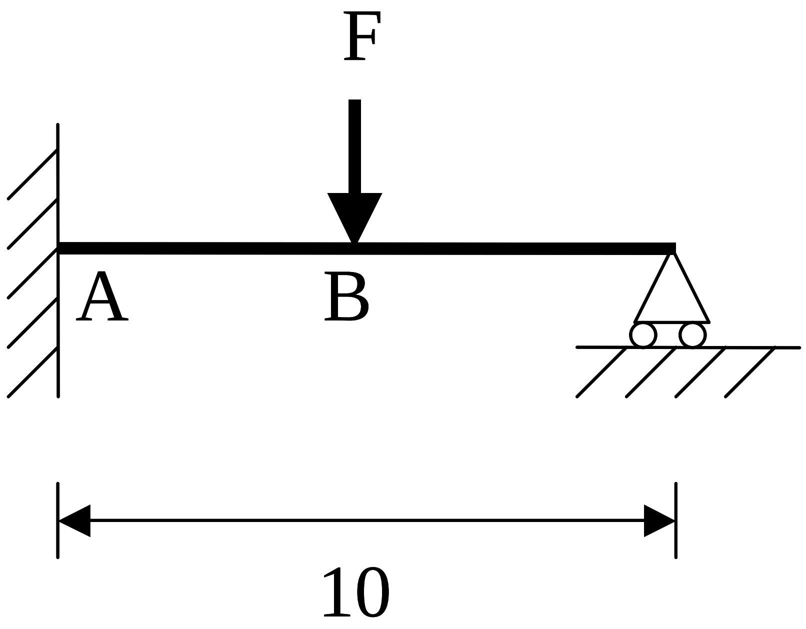
In principle two different approaches to reliability analysis of structural systems may be followed

namely the

- β-unzipping method

- fundamental mechanism method

we will consider an example

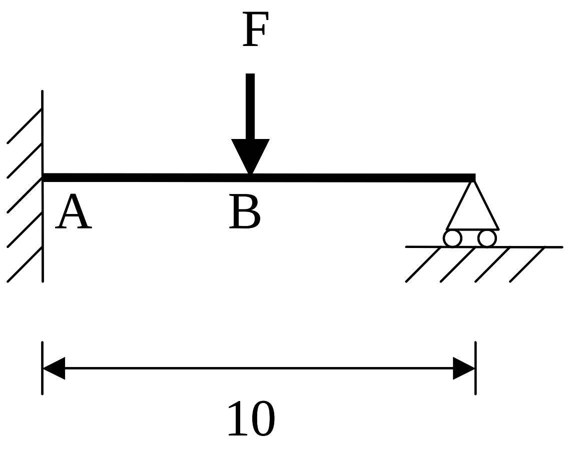


General systems reliability analysis

Example

The bending moment capacity *R* and the loading *F* on the beam structure are assumed to be normal distributed

Following the β-unzipping method failure of a structural system may be defined at different levels – where levels corresponds to the number of failed failure modes assumed to be associated with failure of the structure.



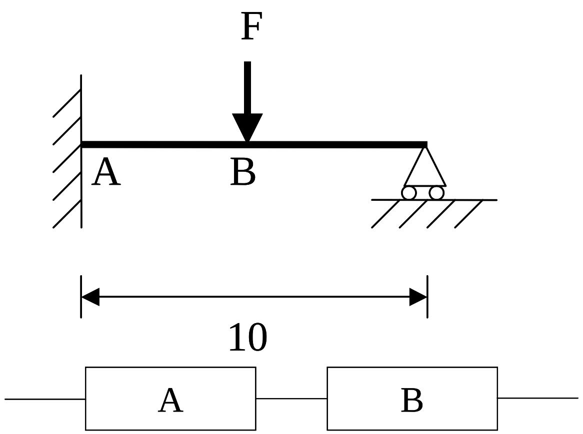
$$\mu_R = 300, \sigma_R = 30$$

$$\mu_F = 100, \sigma_R = 20$$

General systems reliability analysis

Example

Assuming that bending failures will occur at location A or location B the block diagram to be considered is a simple series system



The limit state functions for moment failure at locations A and B are easily established as

$$g_A(\mathbf{x}) = r + m_A = r - 1.875 \cdot f$$

$$g_B(\mathbf{x}) = r - m_B = r - 1.563 \cdot f$$

FORM analysis yields

$$P_{f,A} = 9.58 \cdot 10^{-3} \qquad P_{f,B} = 4.56 \cdot 10^{-4}$$

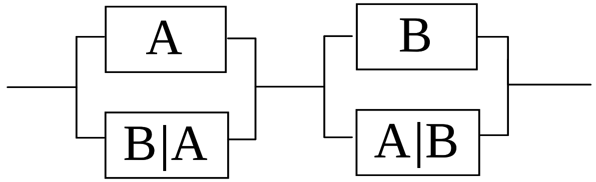
the simple bounds yields

$$9.58 \cdot 10^{-3} \le P_f \le 1 \cdot 10^{-2}$$

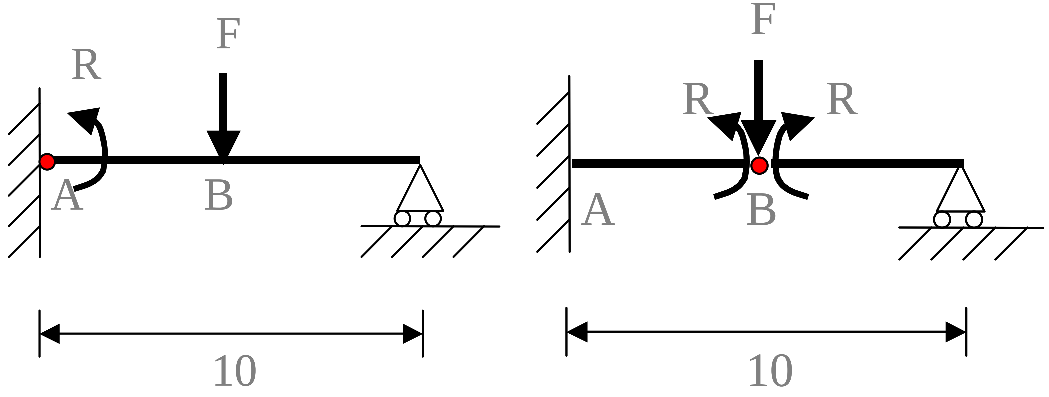
General systems reliability analysis

Example

If systems failure is defined by the event that two failure modes have failed the system to be considered is given by the mixed system



at the location of failures fictitious forces are introduced corresponding to the moment capacity



the limit state equations are found as:

$$g_{B|A}(\mathbf{x}) = r - m_{B|A} + 0.5 \cdot r = r - 2.5 \cdot f + 0.5 \cdot r$$

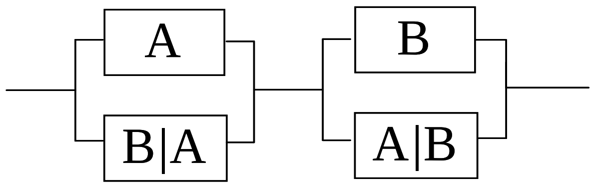
$$g_{A|B}(\mathbf{x}) = r - m_{A|B} + 2 \cdot r = 3 \cdot r - 5 \cdot f$$

FORM analysis yields:

$$P_{f,B|A} = 1.47 \cdot 10^{-3} \qquad P_{f,A|B} = 1.47 \cdot 10^{-3}$$

General systems reliability analysis

Example



We can now calculate the simple bounds for the parallel system as:

$$1.41\cdot 10^{-5} \leq P(A \cap B|A) \leq 9.58\cdot 10^{-3}$$

$$6.71\cdot 10^{-7} \leq P(B \cap A|B) \leq 1.47\cdot 10^{-3}$$

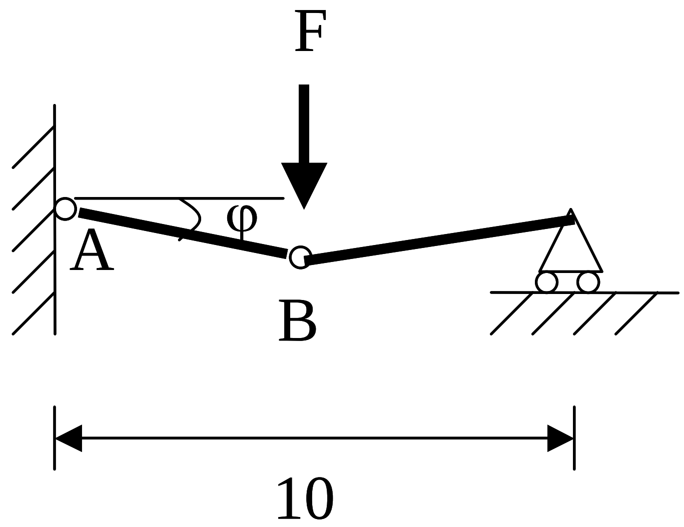
and finally the simple bounds for the series system as:

$$1.48\cdot 10^{-5} \leq P_f \leq 9.58\cdot 10^{-3}$$

General systems reliability analysis

Example

Following the **fundamental mechanism approach** failure of the considered structure is defined as the development of a collapse mechanism for the structure



Considering our simple example there is only one bending failure mechanism

$$g(\mathbf{x}) = A_I - A_E = r + 2 \cdot r - 5 \cdot f$$

which is readily analysed

$$P_f = 1.47 \cdot 10^{-3}$$

General systems reliability analysis

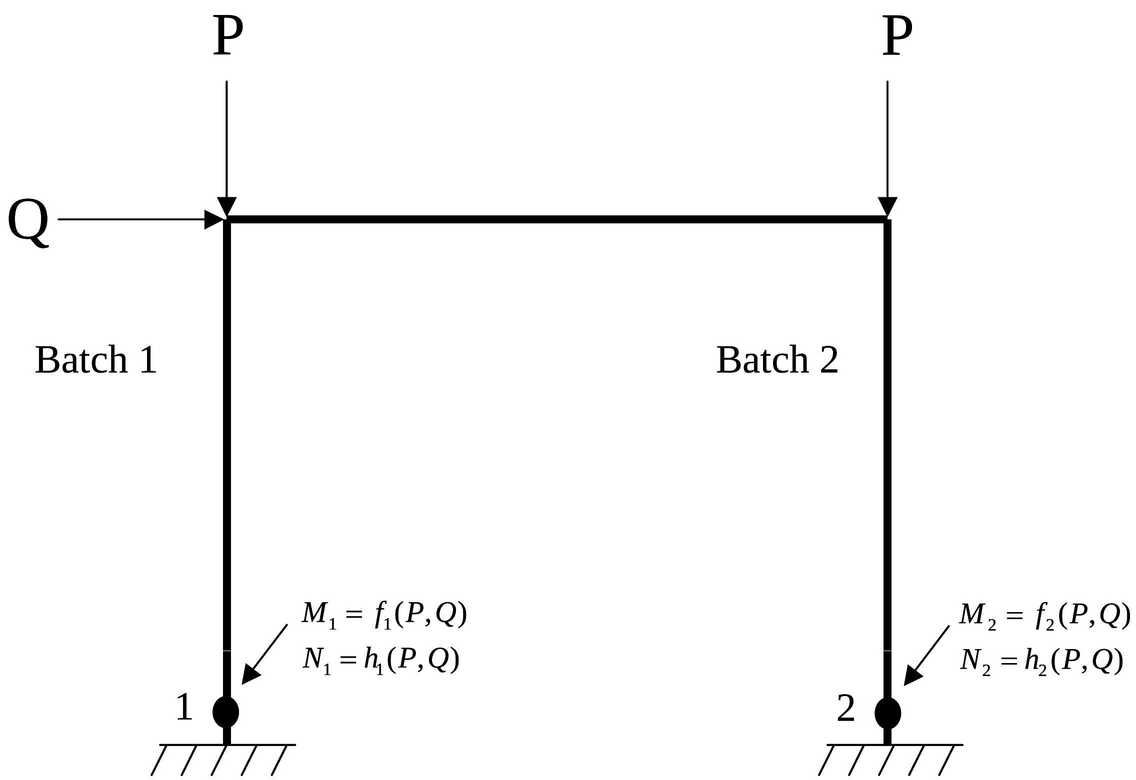
Aspects of correlation

Correlation is important when analysing structures

Correlation between failure modes in systems analysis is present due to the

- **Loading**

- **Materials**



Why is robustness an issue?

- **Despite modernization of design codes the engineering profession is still facing problems in terms of**

  - collapsing structures and building

  - steady increase of insured damages

Why is robustness an issue?

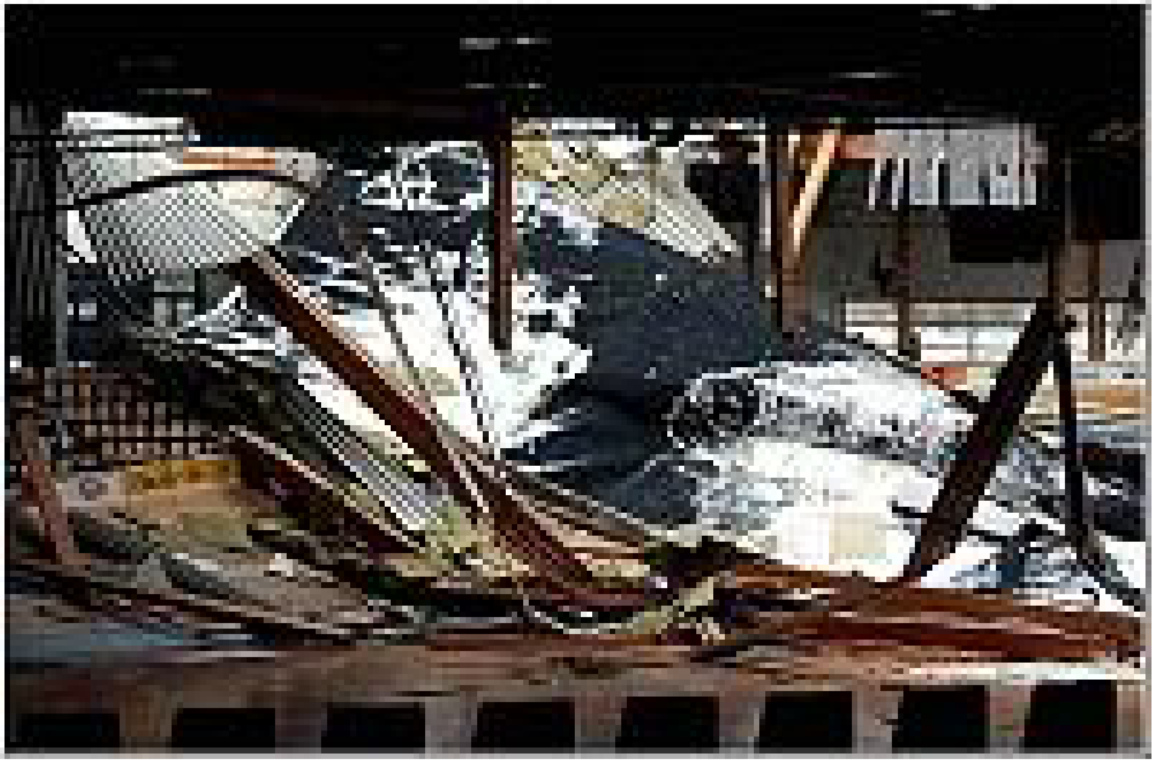
- **Examples of collapses**

Bad Reichenhall
Germany, 2006

Why is robustness an issue?

- **Examples of collapses**

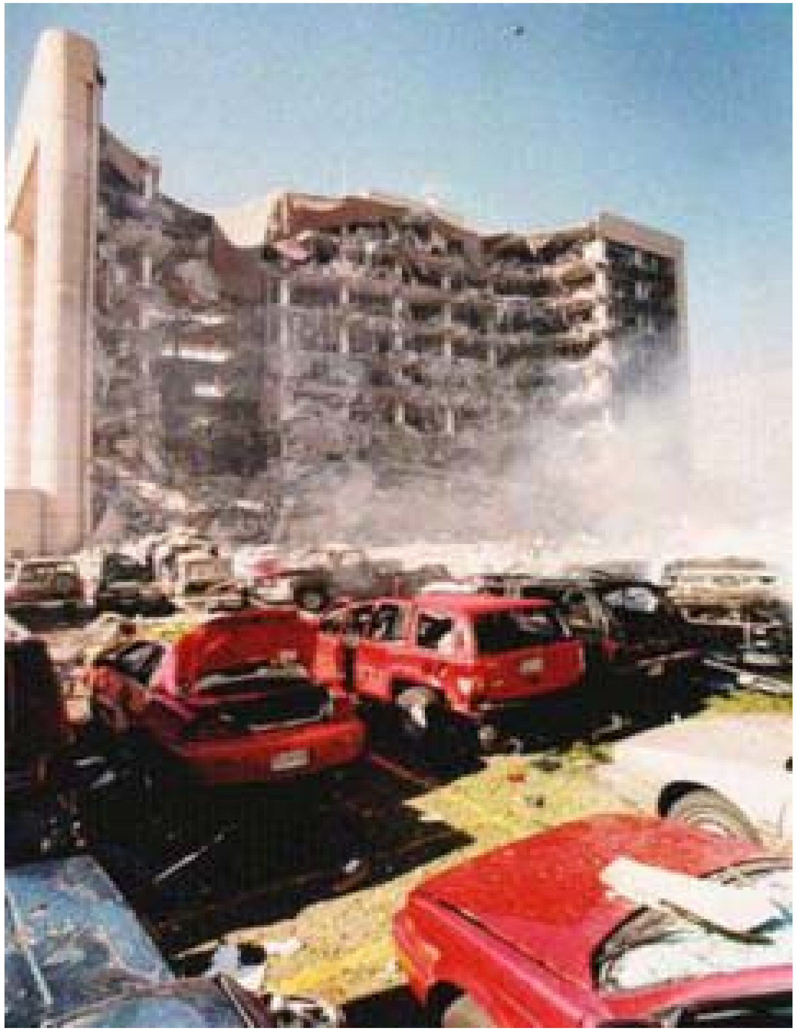
Siemens arena
Denmark, 2003



Why is robustness an issue?

- **Examples of collapses**

  Oklahoma City bombing
  USA, 1995



Why is robustness an issue?

- **Examples of collapses**

World Trade Center
USA, 2001

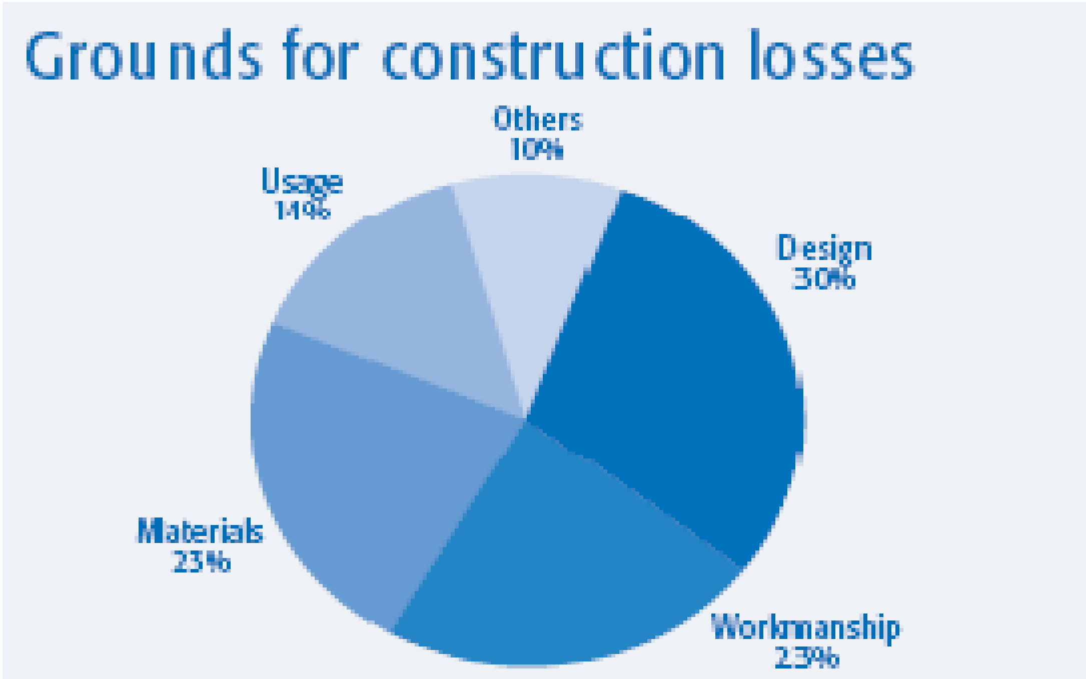
Why is robustness an issue?

- **Examples of collapses**

Charles de Gaulle
France, 2004

Why is robustness an issue?

- **Losses due to building failures**

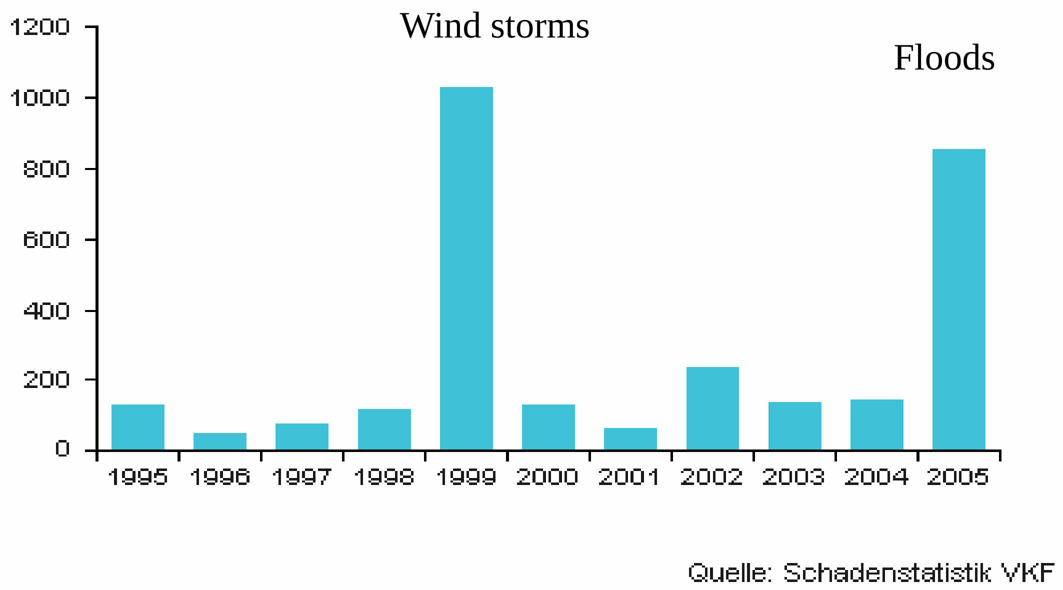


Source: SECO figures covering 800 losses. Combines results from 2 separate papers, published in the 1990s by the WTCB (Belgian Building Research Institute) as well as Matouzek and Schneider

Why is robustness an issue?

- **Insured losses due to building failures**

  IRV Interkantonaler Rückversicherungs-verband, Switzerland



What is understood as robustness?

Structural Standards	The consequences of structural failure are not disproportional to the effect causing the failure [2].
Software Engineering	The ability…to react appropriately to abnormal circumstances (i.e., circumstances "outside of specifications"). A system may be correct without being robust [17].
Product Development and QC	The measure of the capacity of a production process to remain unaffected by small but deliberate variations of internal parameters so as to provide an indication of the reliability during normal use.
Ecosystems	The ability of a system to maintain function even with changes in internal structure or external environment [18].
Control Theory	The degree to which a system is insensitive to effects that are not considered in the design [19].
Statistics	A robust statistical technique is insensitive against small deviations in the assumptions [20].
Design Optimization	A robust solution in an optimization problem is one that has the best performance under its worst case (max-min rule) [21].
Bayesian Decision Making	By introducing a wide class of priors and loss functions, the elements of subjectivity and sensitivity to a narrow class of choices, are both reduced [22]
Language	The robustness of language…is a measure of the ability of human speakers to communicate despite incomplete information, ambiguity, and the constant element of surprise [23].

What are the attributes of robustness?

- Design codes have so far focused on inherent properties of the structures (components)

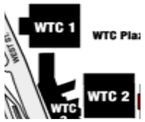
  - redundancy
  - ductility

- More recently focus has been directed to:

  - system performance (removal of members)
  - structural ties

What are the attributes of robustness?

The material loss cost consequences due to the collapse of the two WTC towers only comprised ¼ of the total costs due to damaged or lost material



It seems relevant to include consequences in the robustness equation !

and these are scenario dependent !

What are the attributes of robustness?

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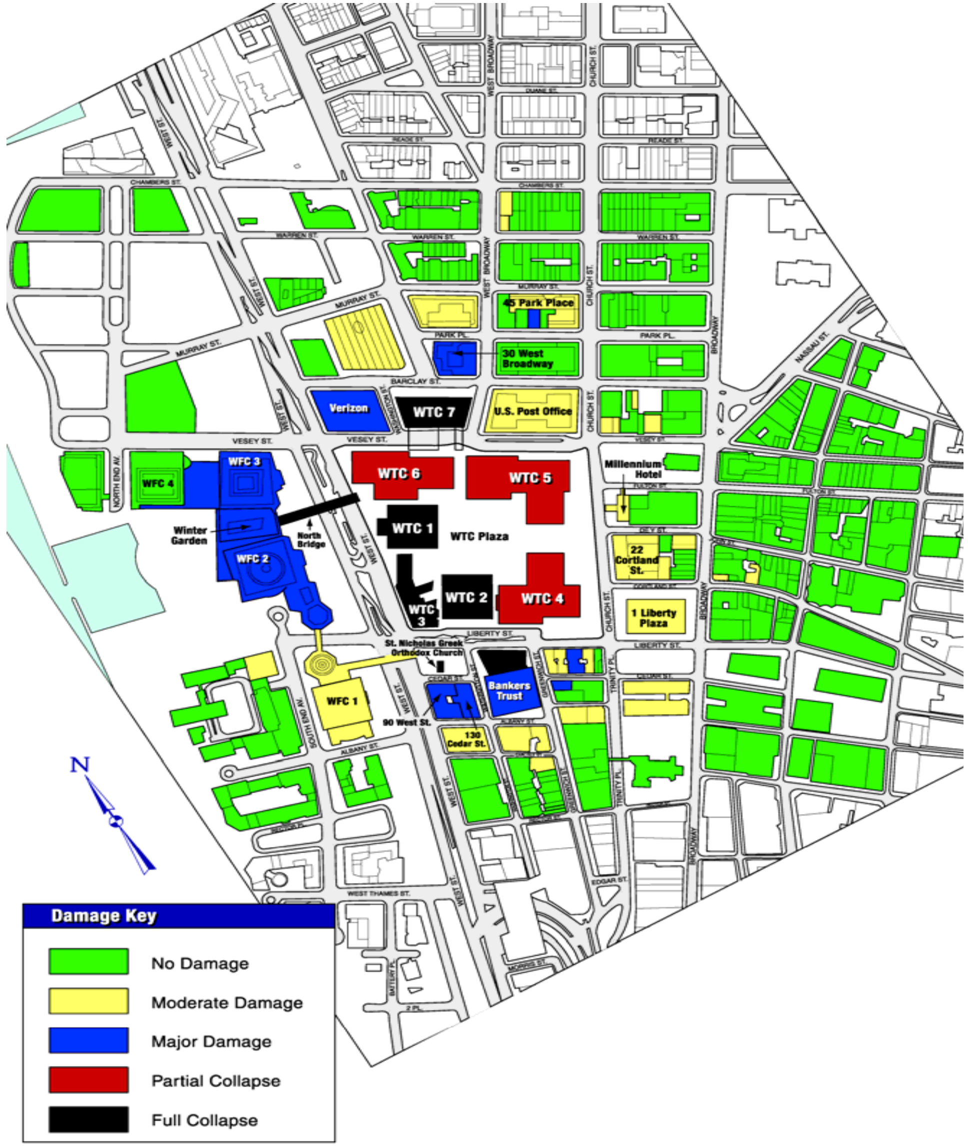
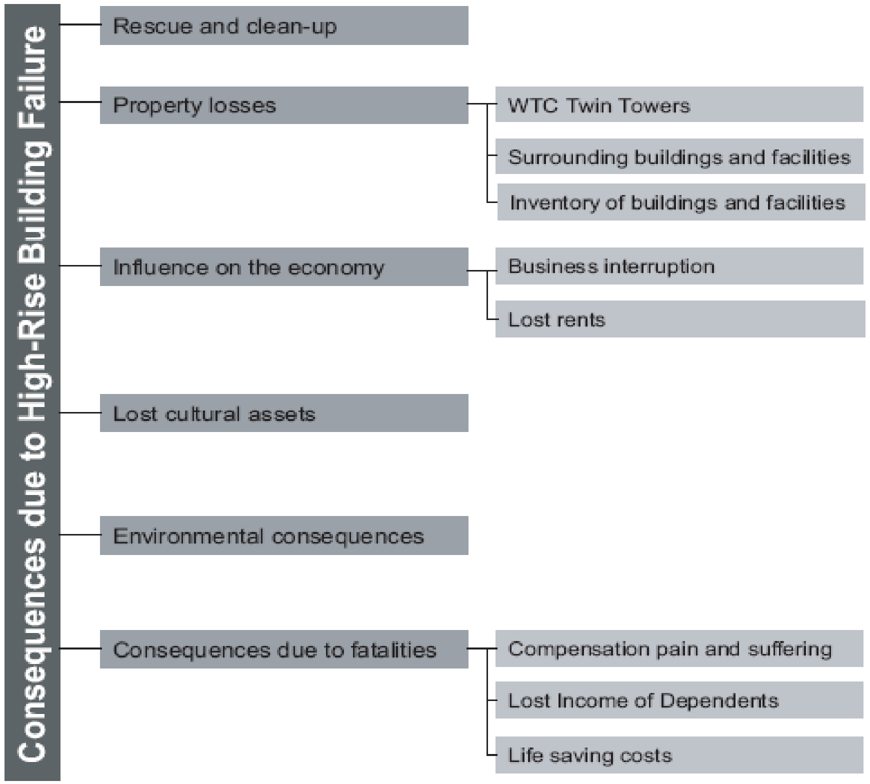


It seems relevant to include consequences in the robustness equation !

and these are scenario dependent !

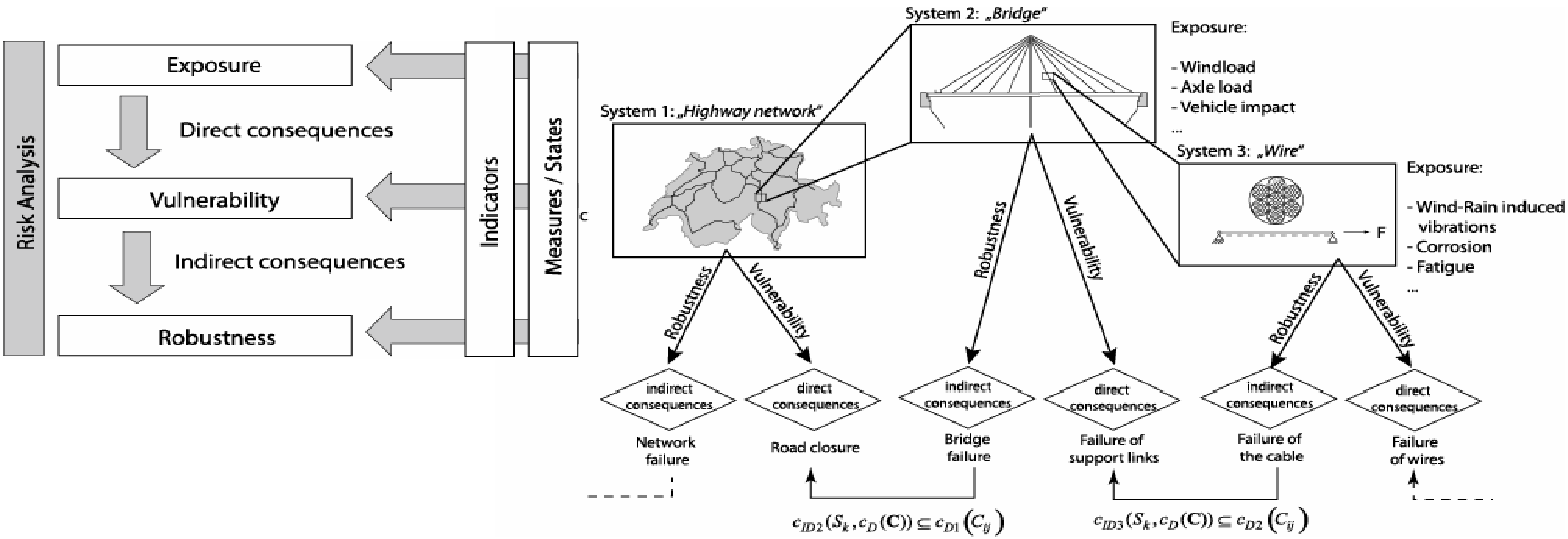
Which are the attributes of robustness?

- **The system definition is important because it defines the consequences following structural failures**



How to frame robustness?

- **Engineered systems have certain characteristics of generic nature – concept developed in the JCSS**



How to frame robustness?

- **This concept is also the idea behind the Eurocodes**

Step 1

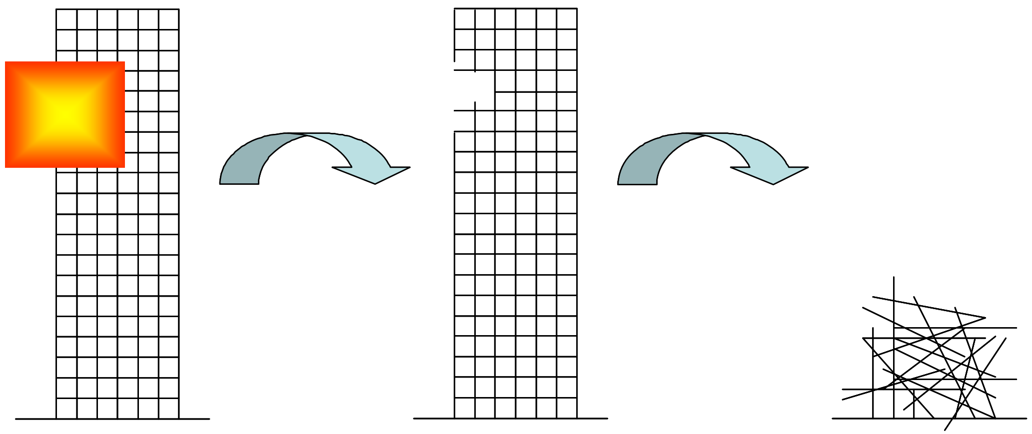
Identifical and modelling of relevant accidental hazards

Step 2

Assessment of damage states to structure from different hazards

Step 3

Assessment of the performance of the damaged structure

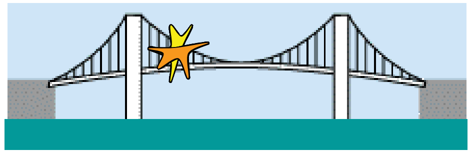
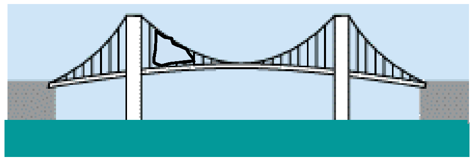
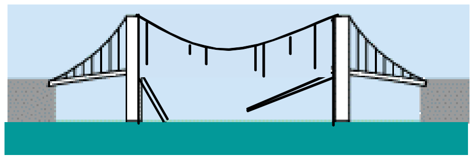


Assessment of the probability of occurence of different hazards with different intensities

Assessment of the probability of different states of damage and corresponding consequences for given hazards

Assessment of the probability of inadequate performance(s) of the damaged structure together with the corresponding consequence(s)

How to frame robustness?

Scenario representation	Physical characteristics	Indicators	Potential consequences
Exposure 	Flood Ship impact Explosion/Fire Earthquake Vehicle impact Wind loads Traffic loads Deicing salt Water Carbon dioxide	Use/functionality Location Environment Design life Societal importance	
Vulnerability 	Yielding Rupture Cracking Fatigue Wear Spalling Erosion Corrosion	Design codes Design target reliability Age Materials Quality of workmanship Condition Protective measures	**Direct consequences** Repair costs Temporary loss or reduced functionality Small number of injuries/fatalities Minor socio-economic losses Minor damages to environment
Robustness 	Loss of functionality partial collapse full collapse	Ductility Joint characteristics Redundancy Segmentation Condition control/monitoring Emergency preparedness	**Indirect consequences** Repair costs Temporary loss or reduced functionality Mid to large number of injuries/fatalities Moderate to major socio-economic losses Moderate to major damages to environment

Assessing robustness – a risk based framework

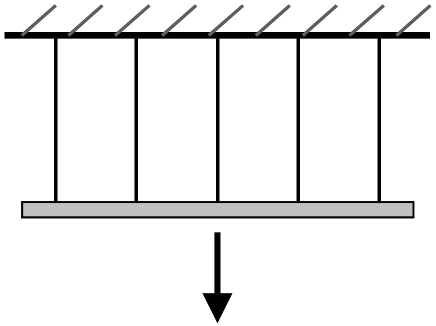
- Desirable properties of a robustness measure

  - Applicable to general systems

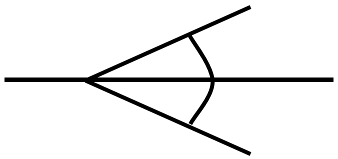
  - Allows for ranking of alternative systems

  - Provides a criterion for identifying acceptable robustness

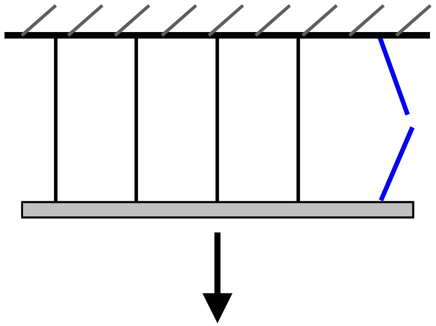
Assessing robustness – a risk based framework



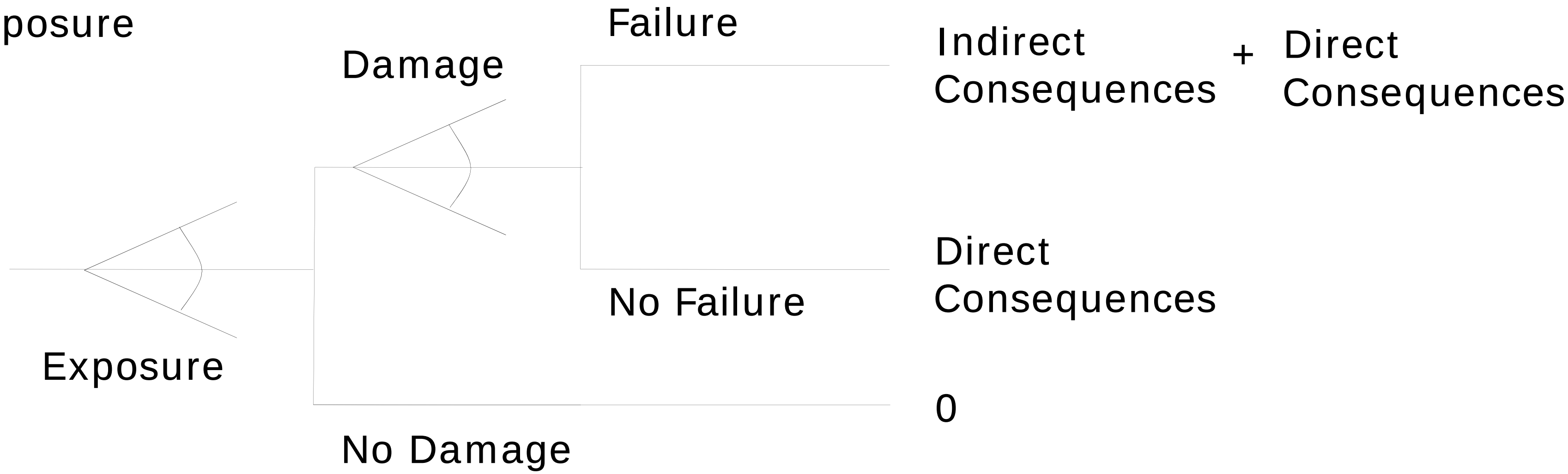
An assessment framework



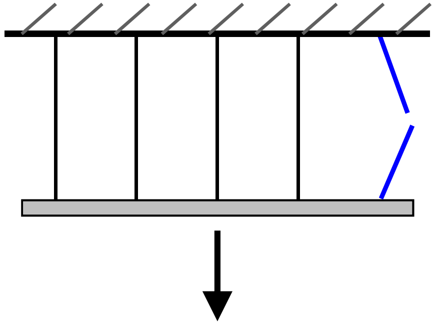
Assessing robustness – a risk based framework



An assessment framework

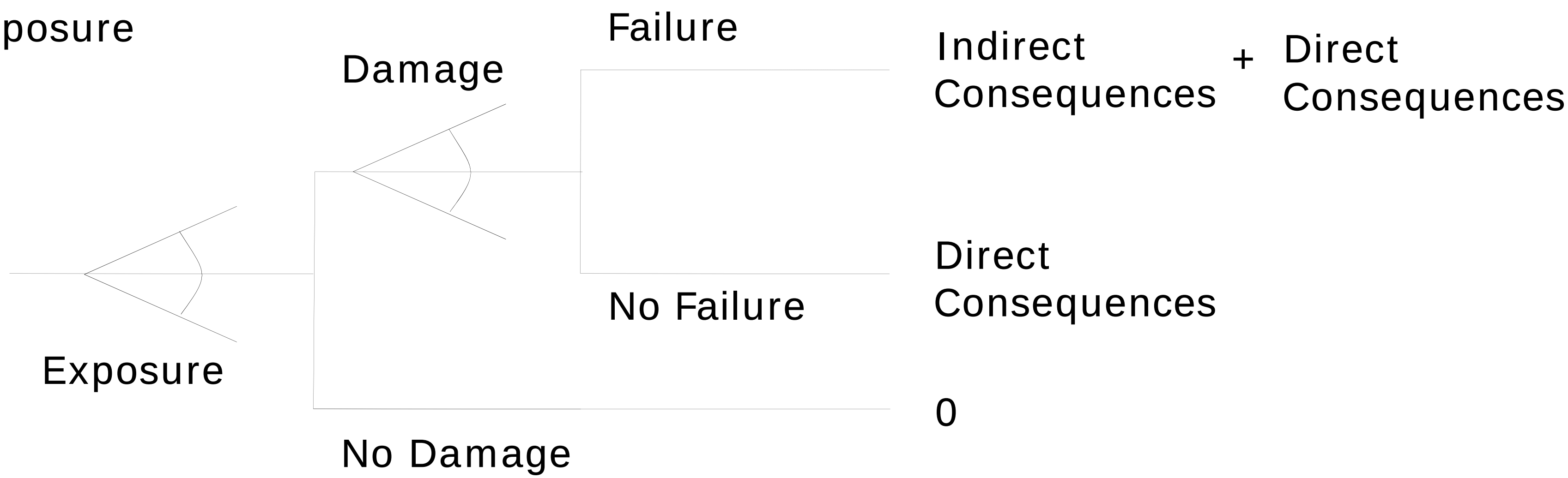


Assessing robustness – a risk based framework



Exposure

An assessment framework



An index of robustness: $I_{Rob} = \dfrac{\textbf{Direct Risk}}{\textbf{Direct Risk + Indirect Risk}}$

Assessing robustness – a risk based framework

Features of the proposed index

$$I_{Rob} = \frac{\text{Direct Risk}}{\text{Direct Risk} + \text{Indirect Risk}}$$

- Assumes values between zero and one

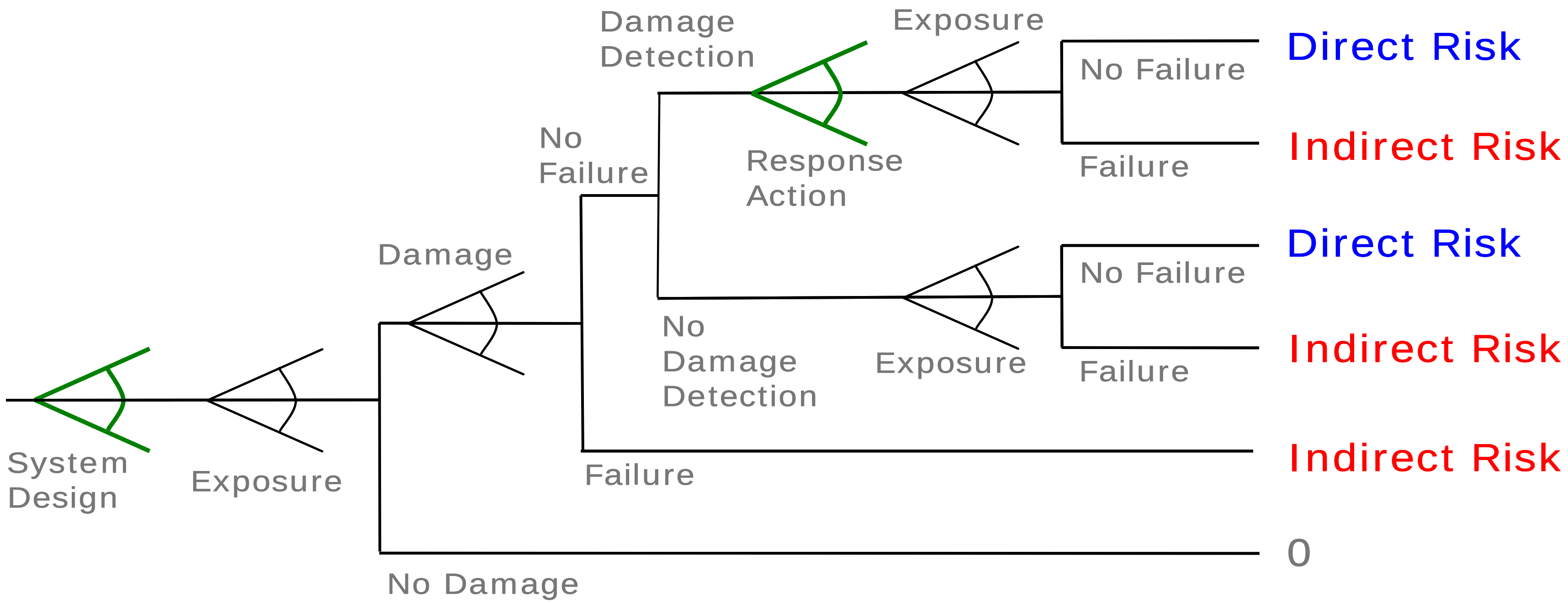
- Measures relative risk only

- Dependent upon the probability of damage occurrence

- Dependent upon consequences

Assessing robustness – a risk based framework

- **The framework easily facilitates decision analysis**
  - Choice of the physical system
  - Choice of inspection and repair
  - Choices to reduce consequences

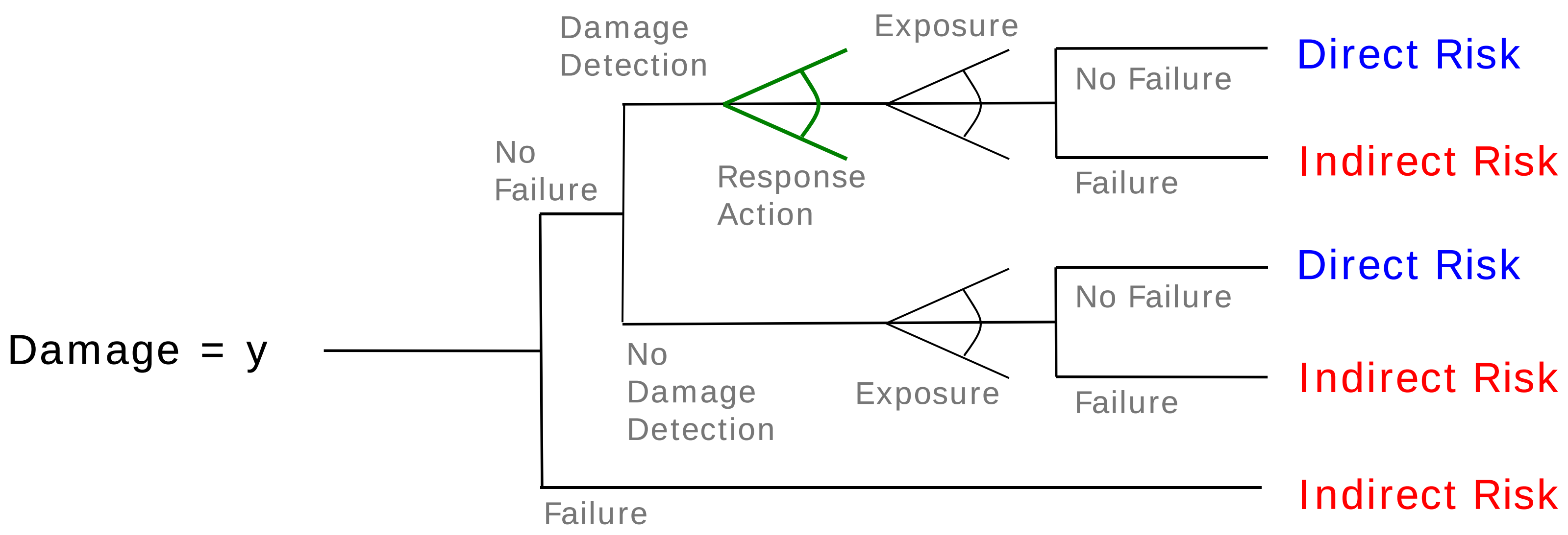


Assessing robustness – a risk based framework

- **“Conditional robustness” is a useful extension of the framework helpful for events such as terrorist attacks**

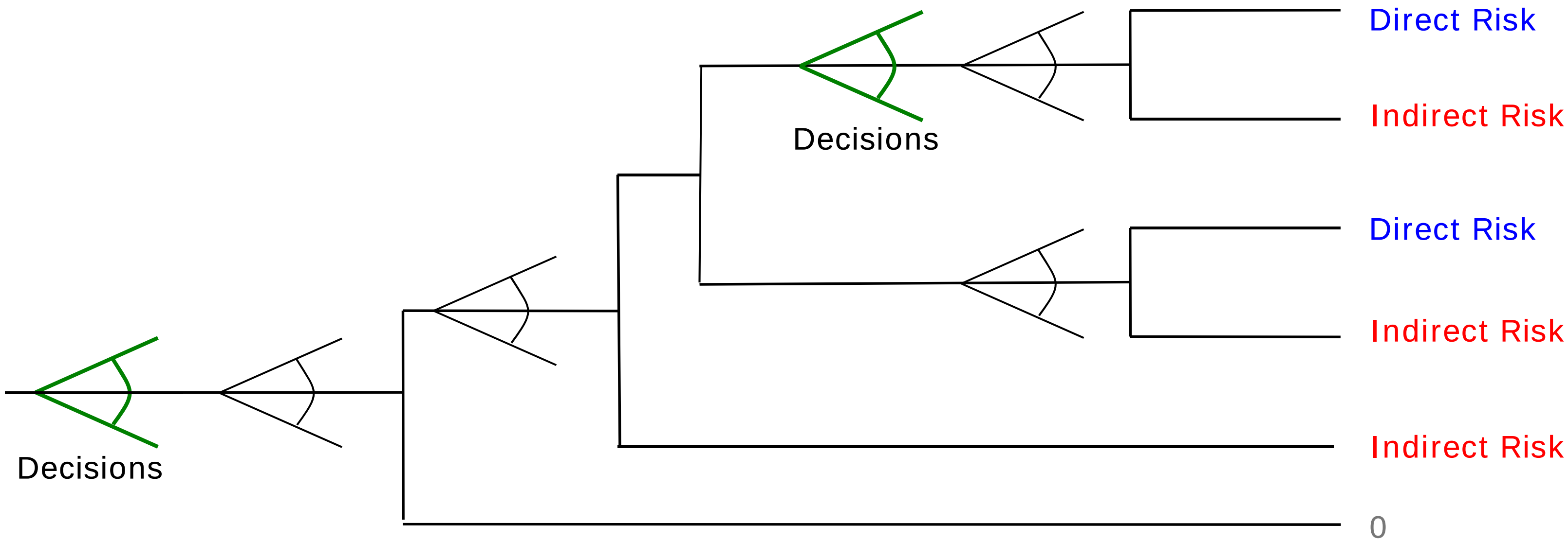
  - Helpful for communication, using a scenario event

  - Can be easily used to calculate (marginal) robustness



Assessing robustness – a risk based framework

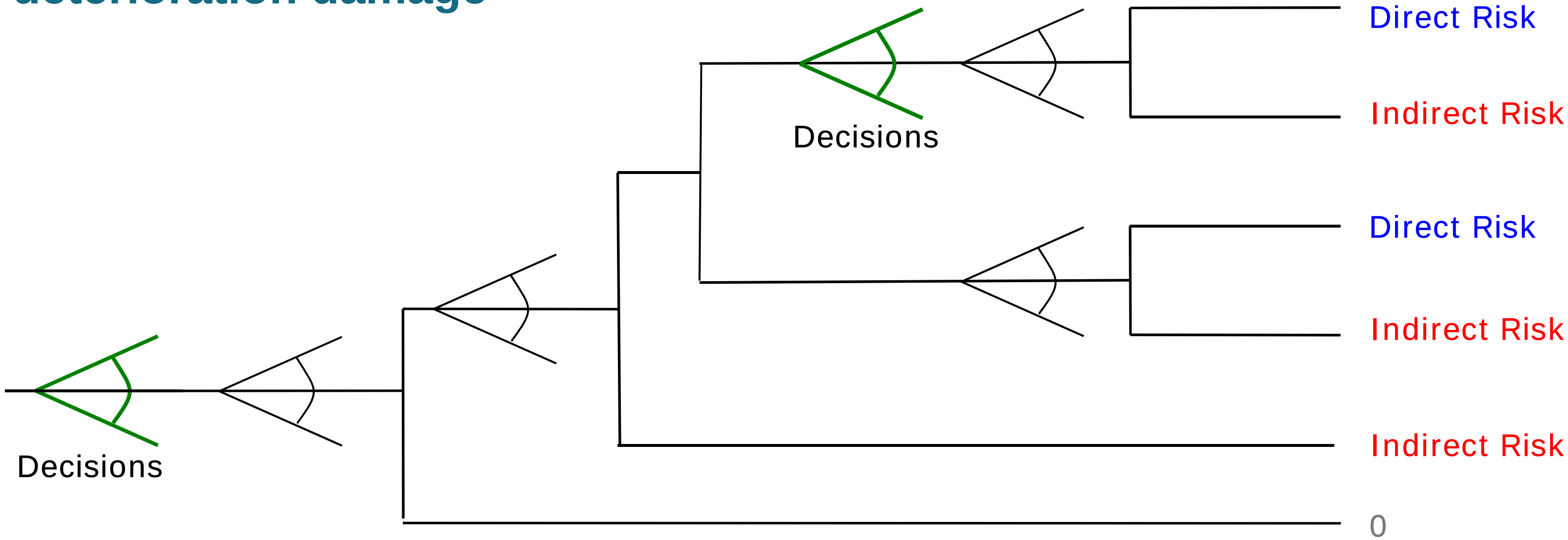
- **Robustness-based design**
  - Acceptable levels of direct risk are achieved by other design requirements
  - Here the goal is indirect risk-reduction
  - Choices are facilitated using the decision trees in this framework
  - The choices can be framed as an optimization problem



Assessing robustness – a risk based framework

- **Robustness-based design options:**

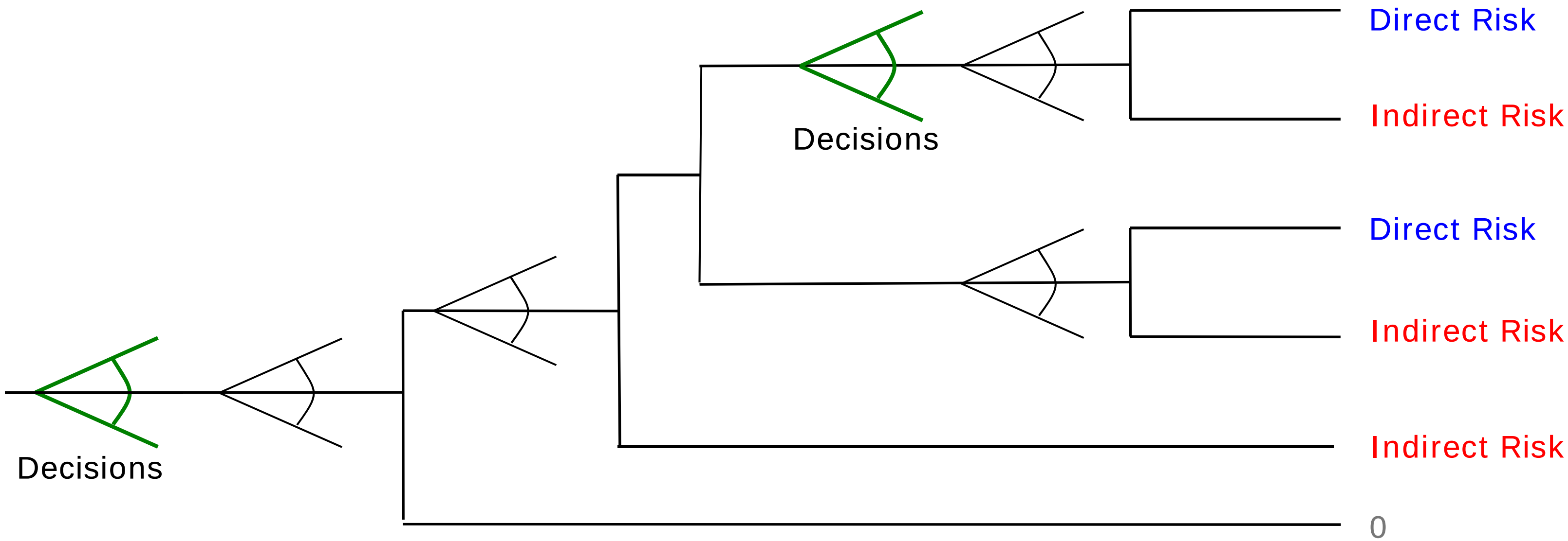
  - Change structural detailing to provide load transfer
  - Increase redundancy of elements
  - Reduce consequences of failure
  - Reduce exposures
  - Add inspection and maintenance to address deterioration damage



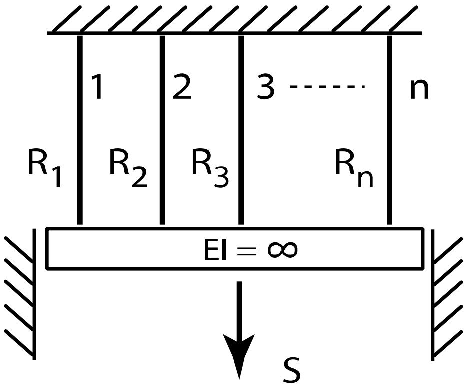
Assessing robustness – a risk based framework

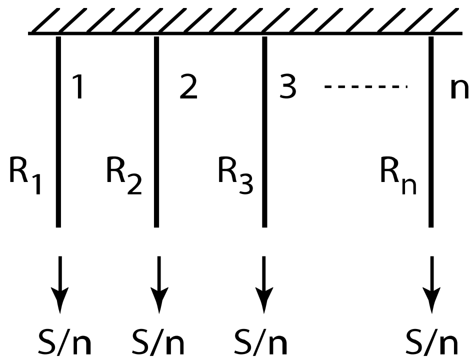
- **Robustness-based design calibration**

  - By benchmarking the robustness of a variety of structures, general patterns can be found
  - This should lead to simplified requirements that do not require complete risk assessments



Assessing robustness – a risk based framework



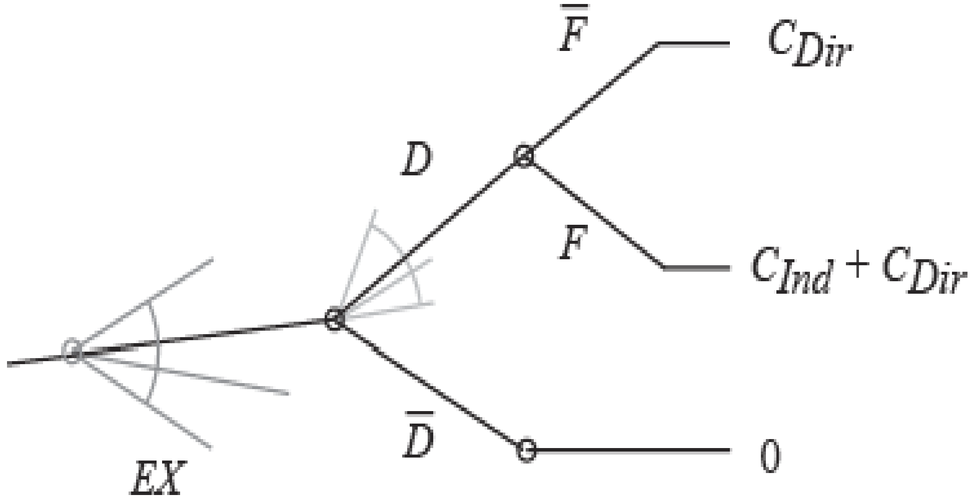


Example - Structural Systems

- Parallel system with *n* elements
- Subjected to different types of exposures
- Perfect ductile / brittle
- Load distribution after component failure
- Element damage / system failure
- The one element case represents series systems
- Consequences of system failure is set equal to 100 times the consequences of component failure

Assessing robustness – a risk based framework

A simplified event/decision tree is considered



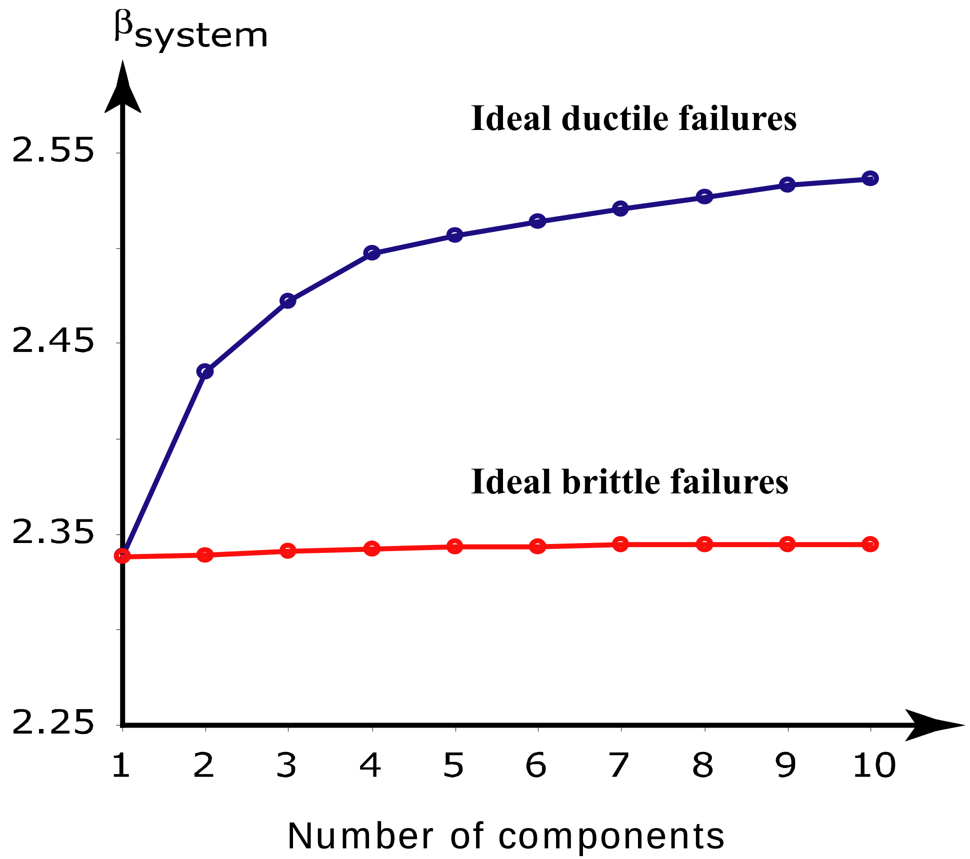
Assessing robustness – a risk based framework

Exposures

- Dead load and live load
  - Weibull distribution

- Applied load is the yearly maximum

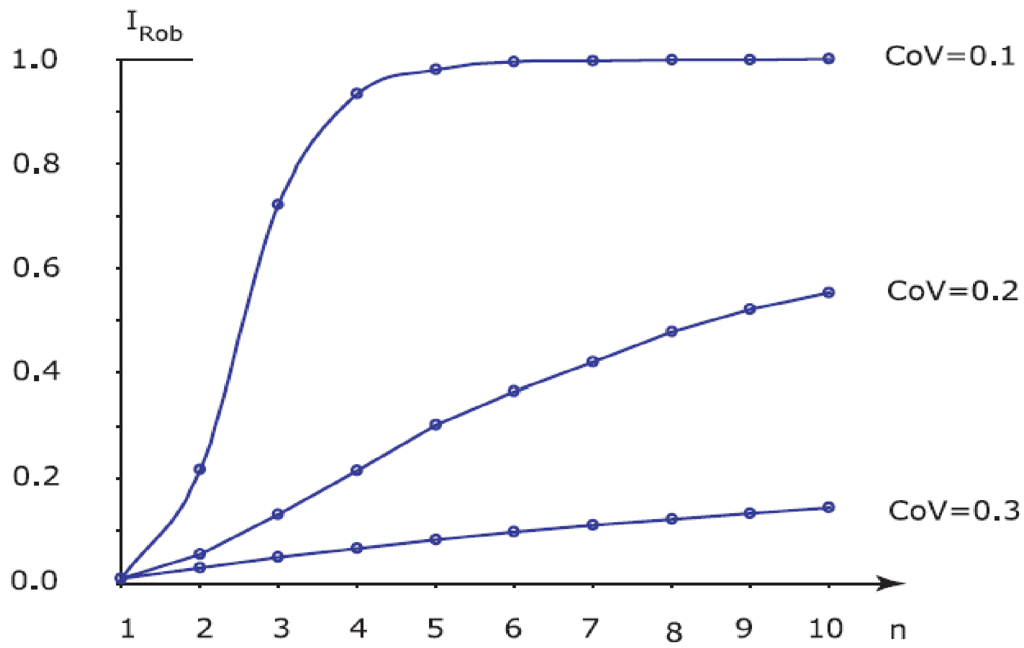
- Each component has the same probability of failure



Assessing robustness – a risk based framework

Number of components – ductile material

- The greater the number of components, the more robust
- One component – Small robustness
- One component – Series system



Assessing robustness – a risk based framework

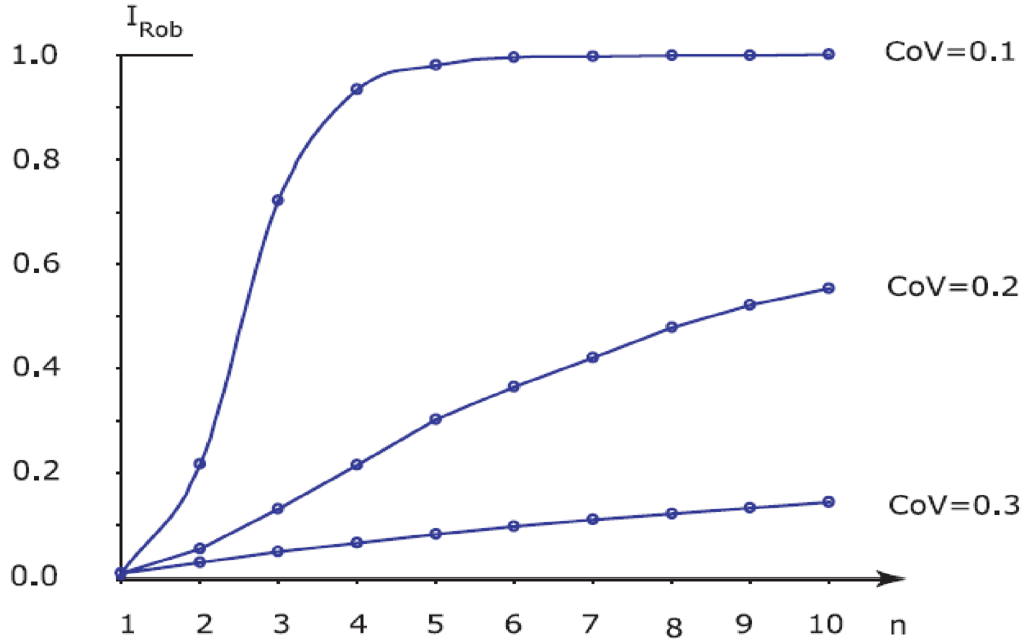
Load variability – ductile material

- Higher CoV leads to less robustness

- Higher Cov increases the probability that the system fails if one component is damaged

- Here uncorrelated resistance is assumed

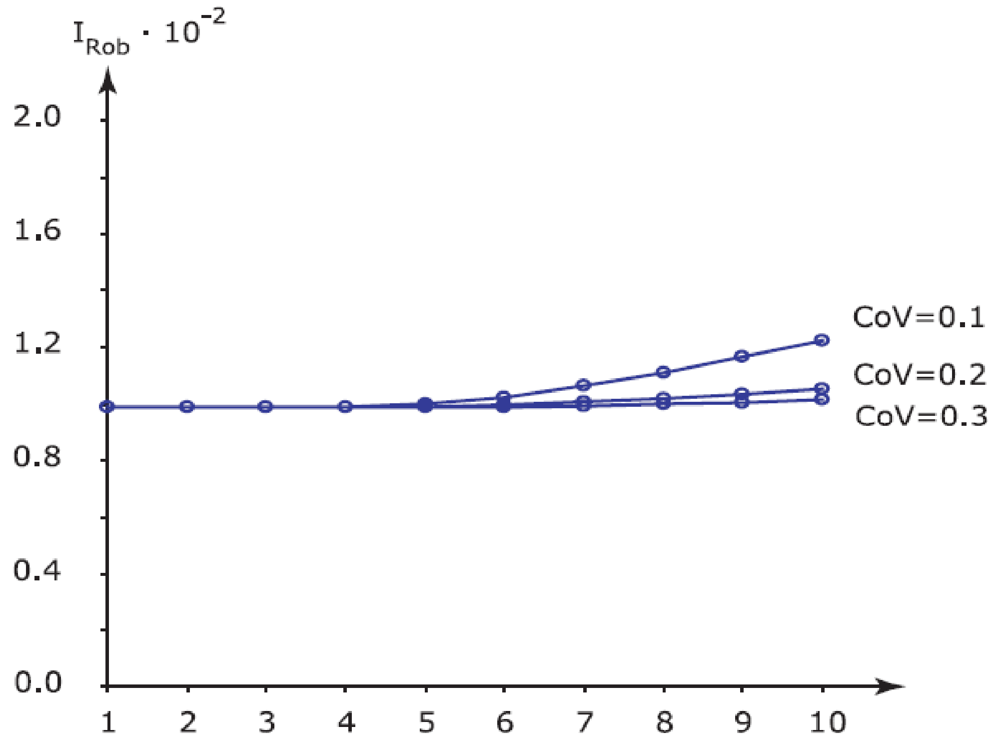
– Correlation has the same effect as reducing the number of components



Assessing robustness – a risk based framework

Load variability – brittle material

- No residual carrying capacity
- Cascading system failure
- The robustness is close to zero
- Indirect risks are dominating
- Probabilities for damage states are low – or failure consequences high



Assessing robustness – a risk based framework

Failure Consequences

- The higher the indirect consequences, the lower the robustness

- Increase the robustness with

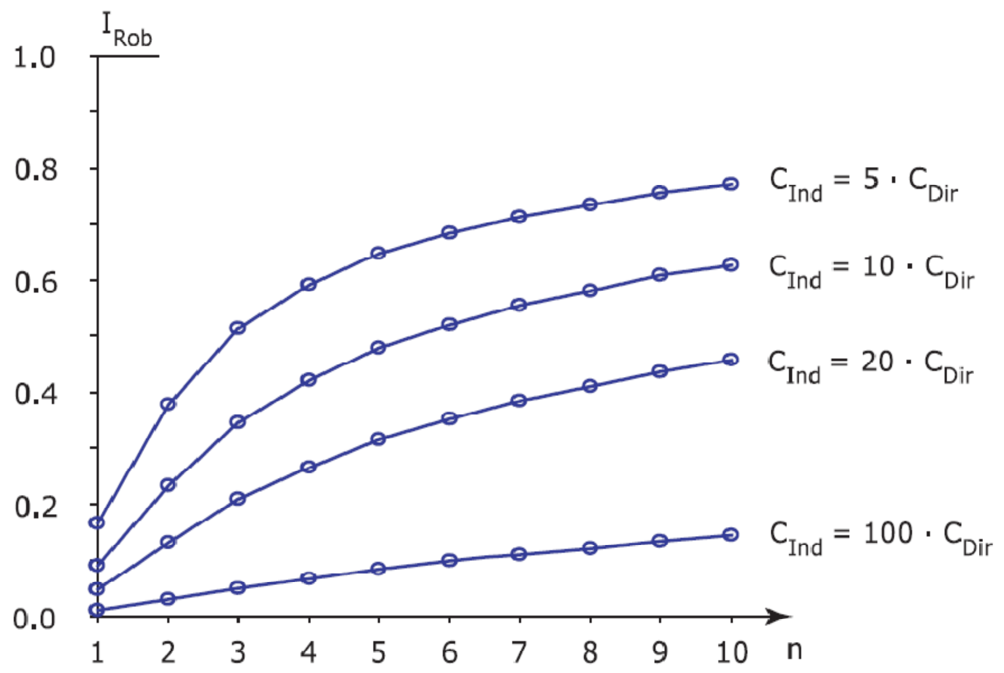
    - effective egress routes

    - decisions in rescue action

    - effective warning systems

- Effect of increasing the damage consequences

-The robustness is related to reliability



Assessing robustness – a risk based framework

Load redistribution

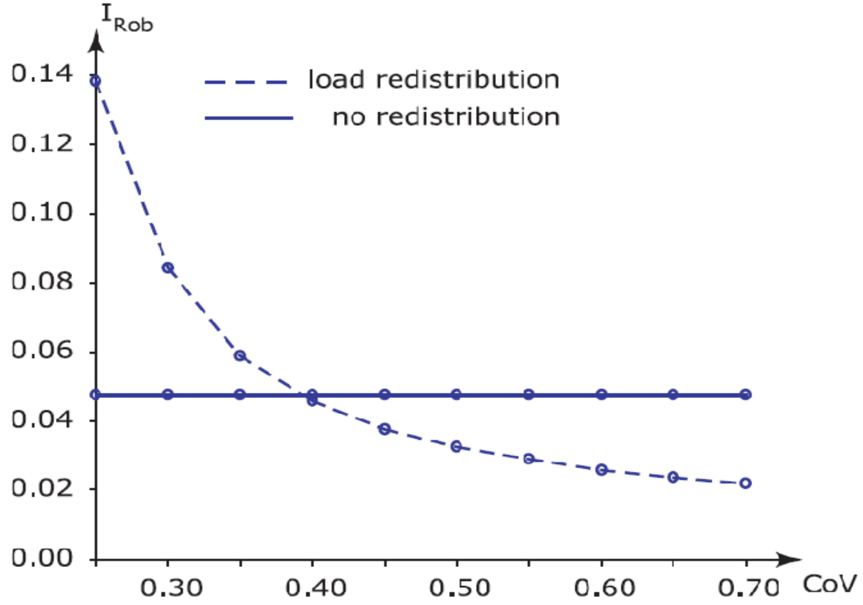
- How is the load carried by the structure? Tie together or accept local failure?

- Load redistribution might increase system failure probability

- Indirect consequences occur in the case of local failure

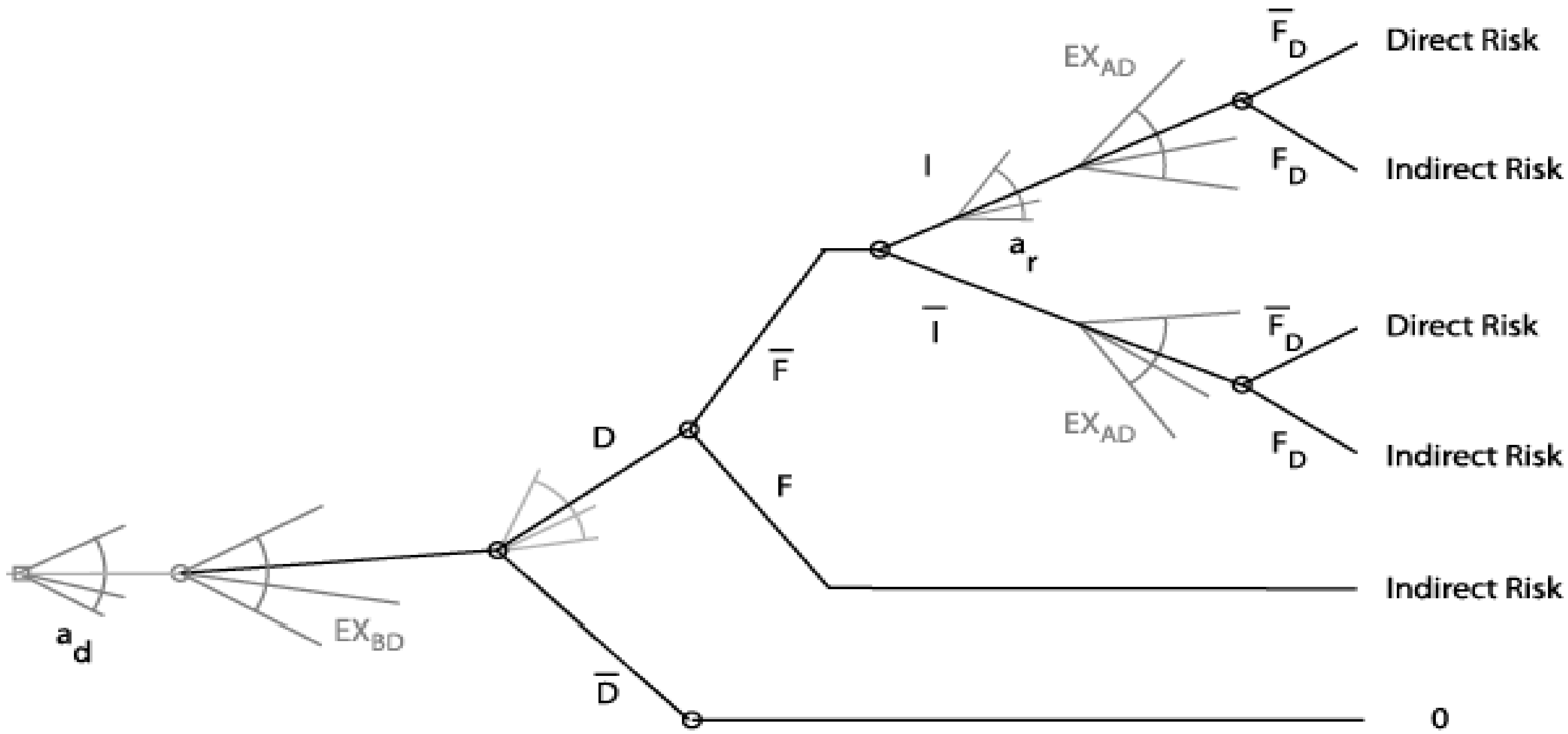
- In some cases it is better to tie the structure together – but not in all cases.

- This robustness assessment can help to identify the proper strategy



Assessing robustness – a risk based framework

Extraordinary loads / repair actions



Assessing robustness – a risk based framework

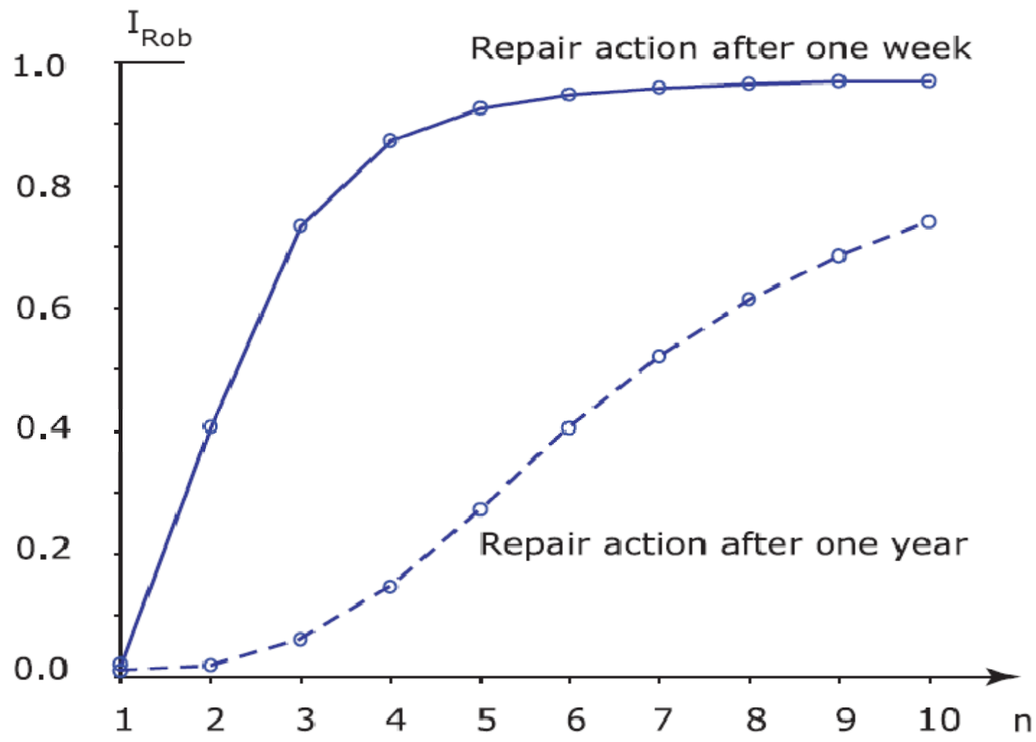
Extraordinary loads / repair actions

- Random load in time + accidental loss of one component

- The structure is more robust when damage can be detected

- The robustness is also affected by actions such as monitoring and repair

- Imperfect damage detection or partial repairs can easily be included



Assessing robustness – a risk based framework

Conditional robustness

- Loss of one component is assumed
- Provides information about structural performance
- Other damage states can be investigated
- Useful if the triggering event or the probability is unknown
- Different strategies can be investigated to identify highest conditional robustness