Risk and Safety

in

Engineering

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Contents of Today's Lecture

- **Probabilistic Modelling of Loads**
- **Probabilistic Modelling of Resistances**
- **Probabilistic Modelling of Model Uncertainties**
- **The Joint Committee on Structural Safety Probabilistic Model Code**

Probabilistic Modelling of Loads

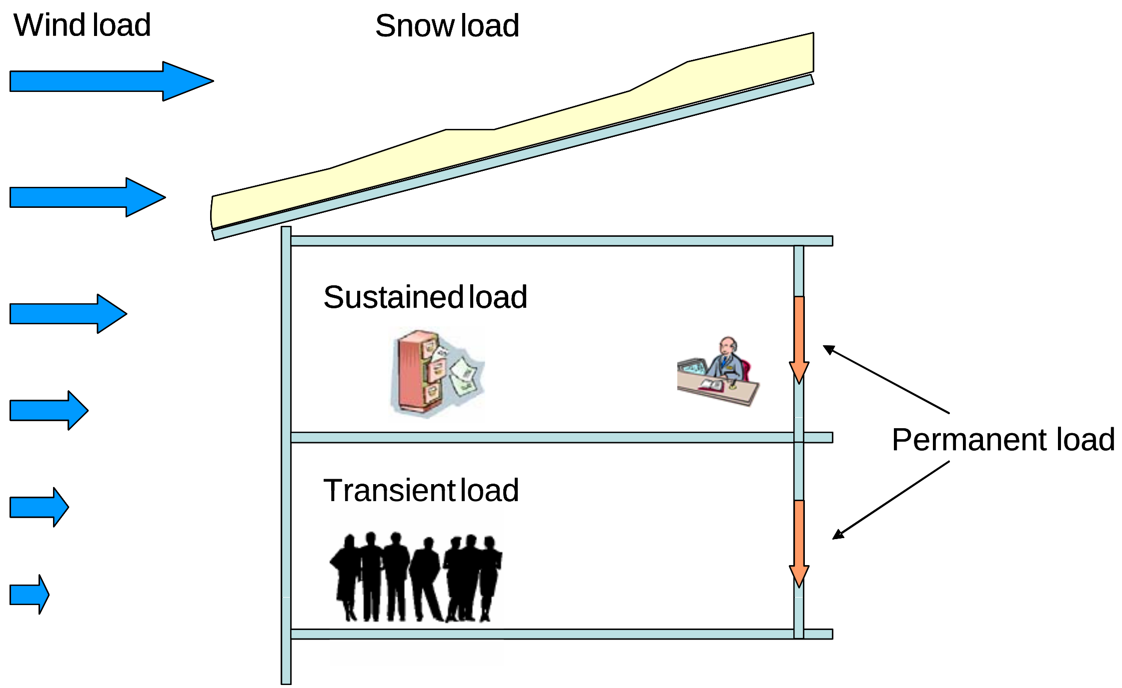
- **Loads on Structures**

Loads are uncertain due to:

- Random variations in space and time

- Model uncertainties

- Statistical uncertainties



Probabilistic Modelling of Loads

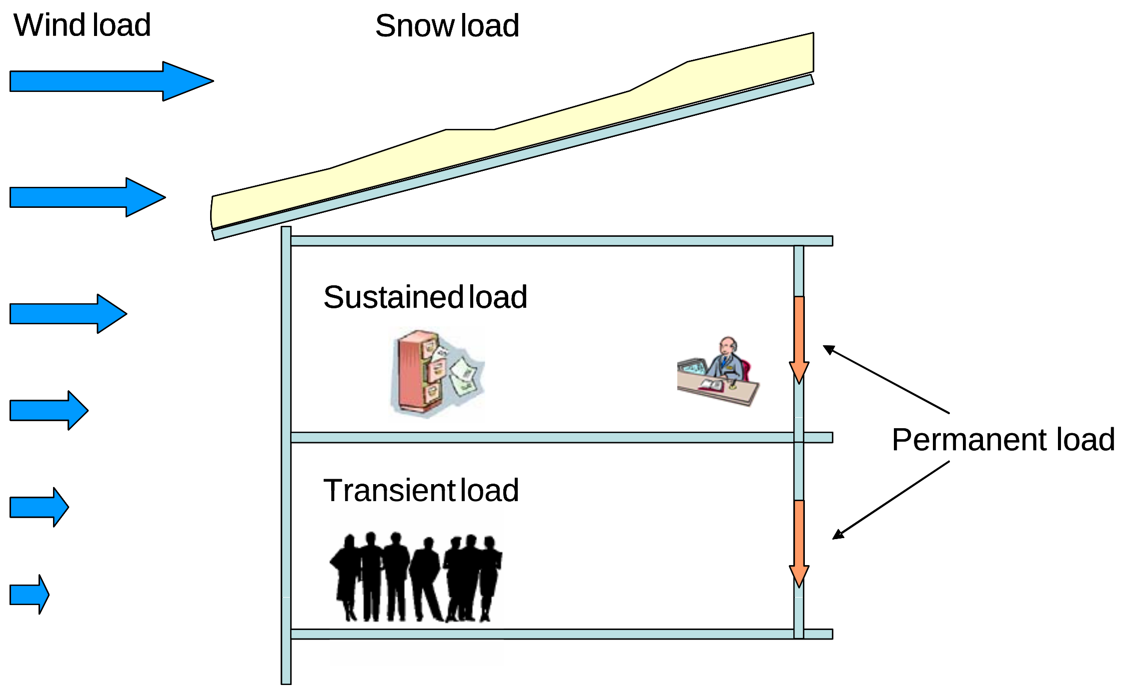
- **Loads on Structures**

It is often useful to characterize loads as:

- Permanent or variable

- Fixed or free

- Static or dynamic



Probabilistic Modelling of Loads

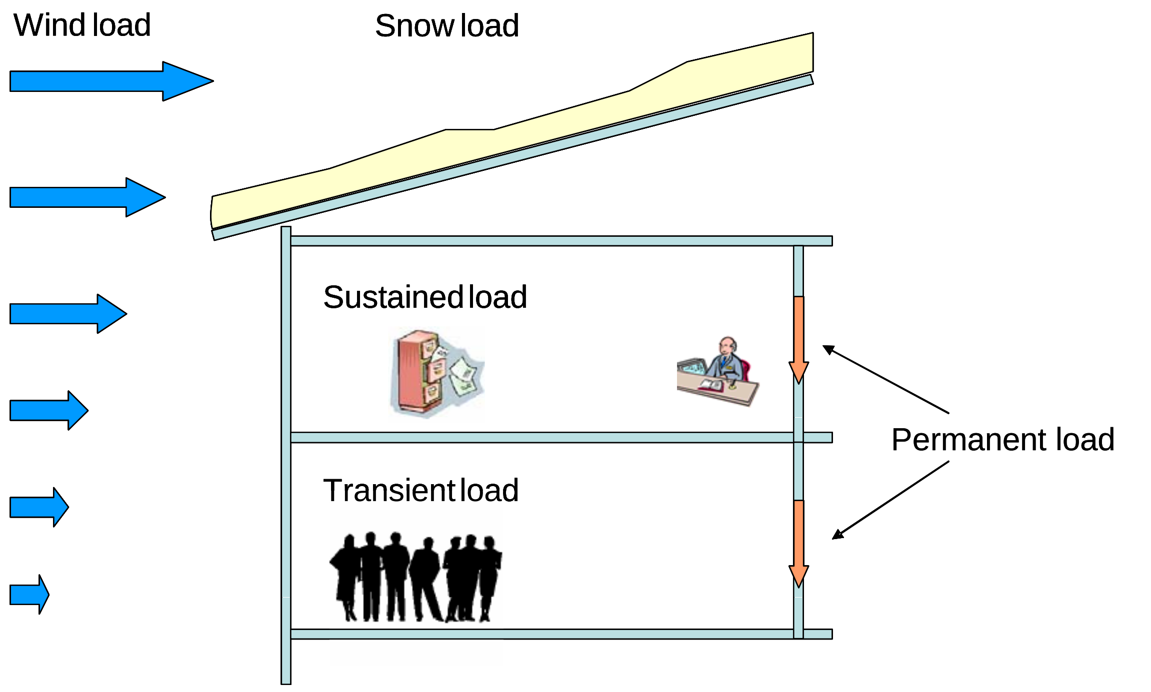
- **Loads on Structures**

The probabilistic modelling includes the following steps:

- specifying the definition of the random variables used to represent the uncertainties in the loading

- selecting a suitable distribution type to represent the random variable

- assigning the distribution parameters of the selected distribution.



Probabilistic Modelling of Loads

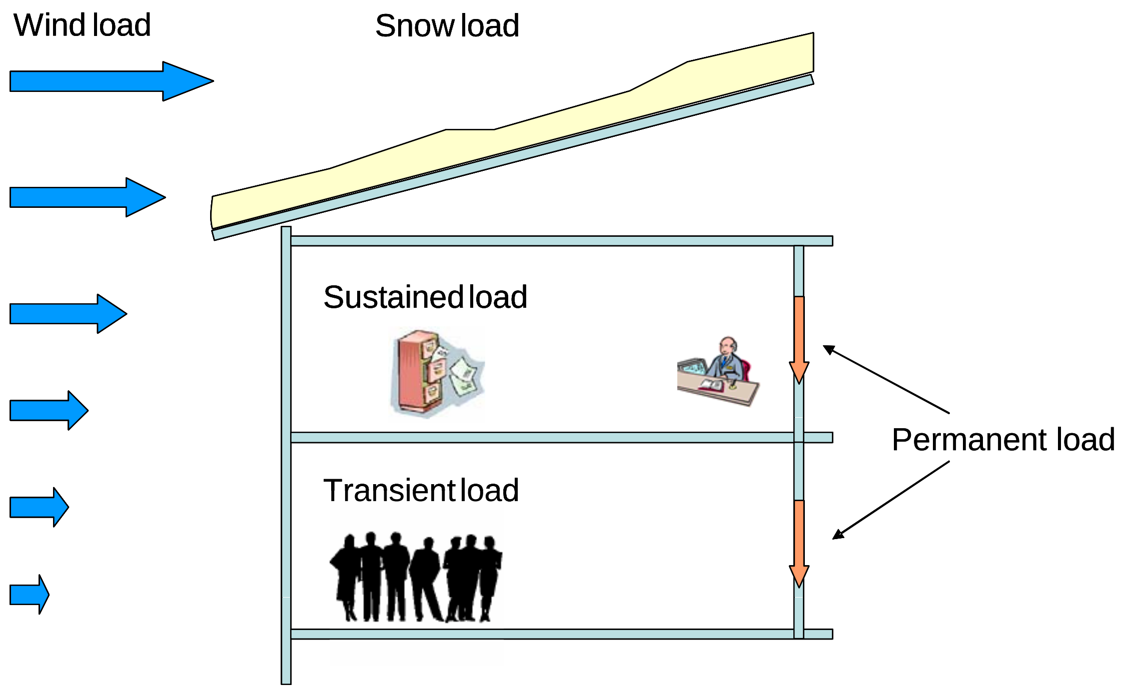
- ## Loads on Structures

Permanent loads:

$$G = \int_V \gamma \, dV$$

Density

Material	**COV**
Construction Steel	0.01
Concrete	0.04
Timber - sawn beam or strut - laminated beam, planed	 0.12 0.10

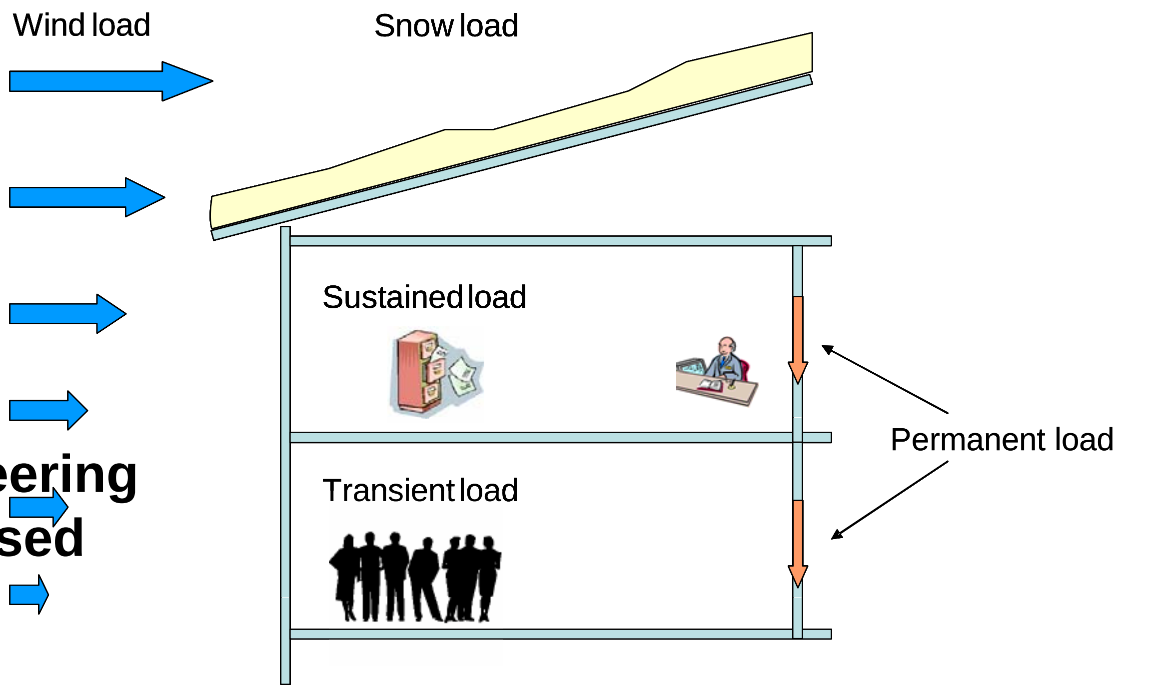


Probabilistic Modelling of Loads

- **Loads on Structures**

Permanent loads:

$$G = \int_V \gamma \, dV$$



The mean value as assessed by engineering assessments has been found to be biased about 5% to the low side

Log-Normal and Normal distributions are good candidates to represent the uncertainty

Probabilistic Modelling of Loads

- **Loads on Structures**

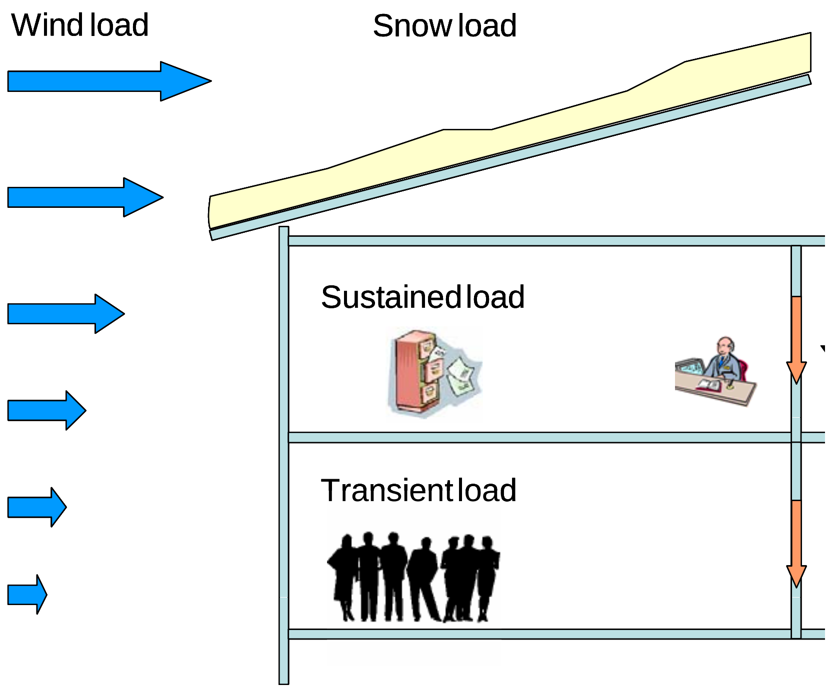
Live floor loads:

$$W(x,y) = m + V + U(x,y)$$

m **is the overall mean value for a given use category**

V **is a zero mean random variable**

U(x,y) **is a zero mean random field**



Probabilistic Modelling of Loads

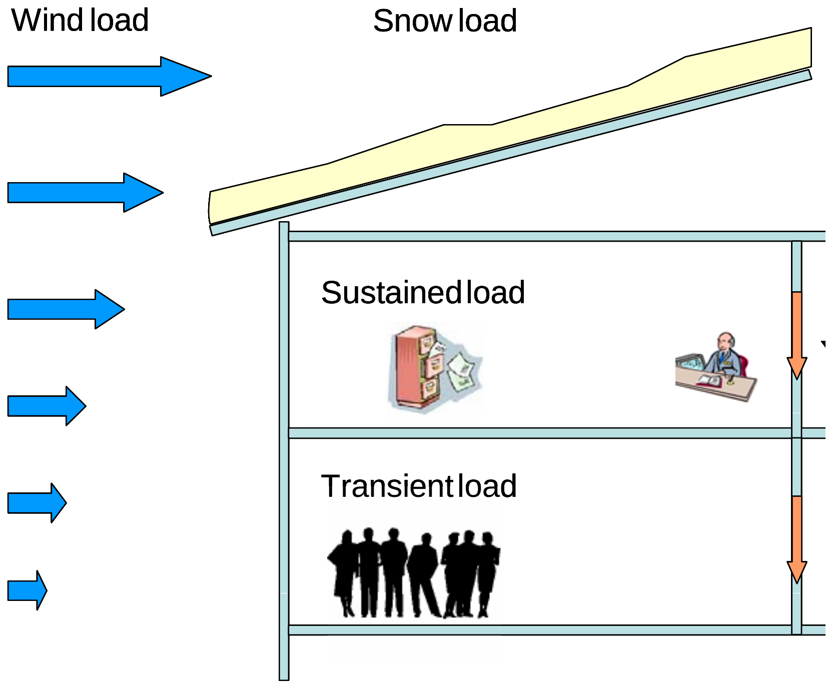
- **Loads on Structures**

Live floor loads:

$$W(x,y) = m + V + U(x,y)$$

The random load effect in linear systems due to the spatially distributed load *W(x,y)* is represented by an equivalent uniformly distributed load Q_{equ}

$$S = \int_A W(x,y)i(x,y)dA = Q_{equ}\int_A i(x,y)dA$$



Probabilistic Modelling of Loads

- ## Loads on Structures

Live floor loads:

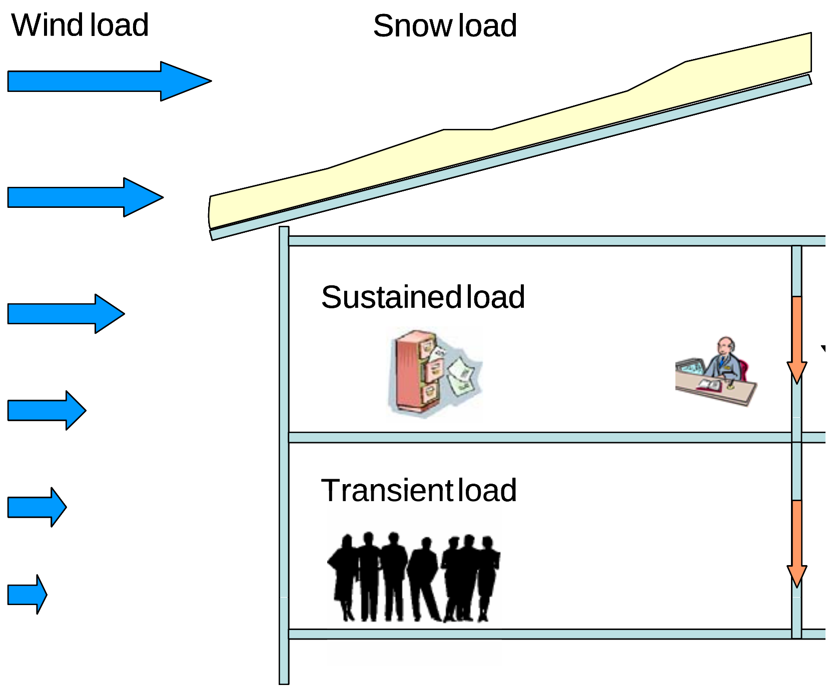
The mean value of the Q_{equ}

$$E\left[Q_{equ}\right] = m$$

The variance is

$$Var\left[Q_{equ}\right] = \frac{Var\left[\int_A W(x,y)i(x,y)dA\right]}{\left[\int_A i(x,y)dA\right]^2} =$$

$$\sigma_V^2 + \sigma_U^2 \frac{\int_{A_1}\int_{A_2} i(x_1,y_1)i(x_2,y_2)\rho_{U(x_1,y_1),U(x_2,y_2)}dA_1dA_2}{\left[\int_A i(x,y)dA\right]^2}$$



Probabilistic Modelling of Loads

- **Loads on Structures**

Live floor loads:

If the correlation radius ρ_0 is small then:

$$Var\left[Q_{equ}\right] =$$

$$\sigma_V^2 + \sigma_U^2 \frac{\int_A i(x,y)^2 \, dA}{\left[\int_A i(x,y) \, dA\right]^2} = \sigma_V^2 + \sigma_U^2 \kappa_{red}$$

$$\kappa_{red} = \frac{A_0}{A} \kappa(A)$$

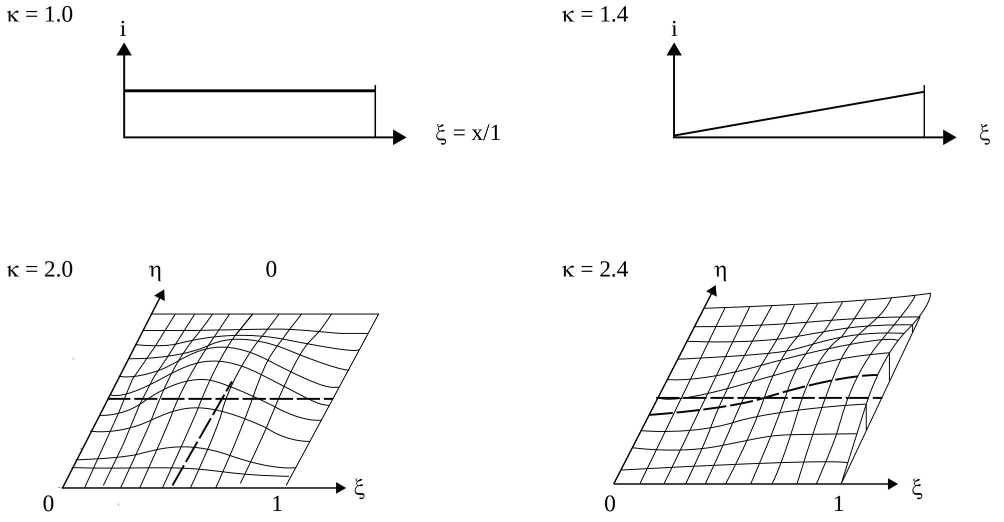
Probabilistic Modelling of Loads

- **Loads on Structures**

Live floor loads:

In principle the variance reduction factor may be determined from

$$\kappa_{red} = \frac{A_0}{A}\kappa(A)$$



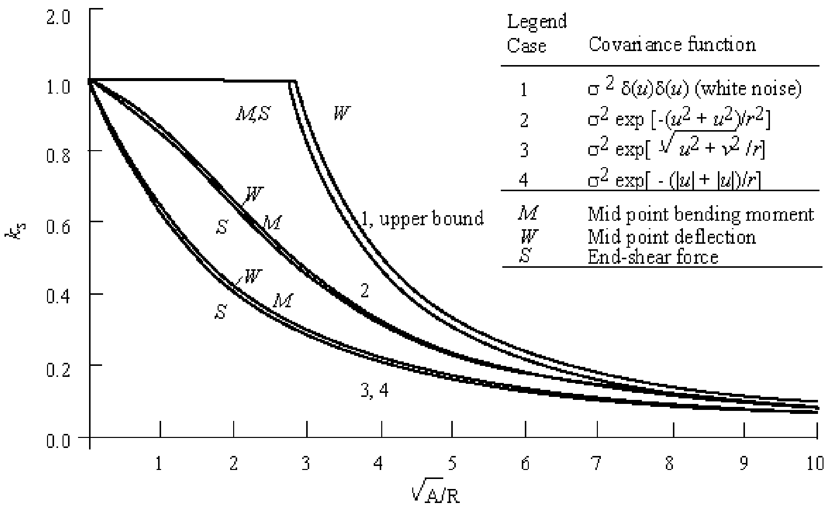
Probabilistic Modelling of Loads

- **Loads on Structures**

Live floor loads:

In principle the variance reduction factor may be determined from

$$\kappa_{red} = \frac{A_0}{A}\kappa(A)$$



But for most practical purposes it can be assumed to be equal to zero

$$E\left[Q_{equ}\right] = m_q$$

$$Var\left[Q_{equ}\right] = \sigma_V^2$$

Probabilistic Modelling of Loads

- **Loads on Structures**

Live floor loads (sustained loads):

The maximum EUDL load has been found to be Gamma distributed but is sometimes modelled by a Type I extreme value distribution.

Assuming that changes of the sustained load follow a Poisson process with rate λ the probability distribution function of the maximum load in a time reference *T* may be determined from

$$F_{Q,\max}(x) = \exp(-\lambda T(1 - F_Q(x)))$$

Probabilistic Modelling of Loads

- **Loads on Structures**

Live floor loads (transient loads):

The maximum EUDL load has been found to be Exponential distributed.

$$E\left[P_{equ}\right] = m_p \qquad Var\left[P_{equ}\right] = \sigma_V^2$$

Assuming that changes of the sustained load follow a Poisson process with rate ν the probability distribution function of the maximum load in a time reference *T* may be determined from

$$F_{p,\max} = \exp\left(-\nu T\left(1 - F_p\left(\mathrm{x}\right)\right)\right)$$

Probabilistic Modelling of Loads

- ## Loads on Structures

Live floor loads (sustained/transient loads):

		Sustained Load				Transient Load			
Category	A_0 [m²]	m_q [kN/m²]	σ_V [kN/m²]	σ_U [kN/m²]	$1/\lambda$ [y]	m_p [kN/m²]	σ_V [kN/m²]	$1/\nu$ [y]	d_p [d]
Office	20	0.5	0.3	0.6	5	0.2	0.4	0.3	1 - 3
Lobby	20	0.2	0.15	0.3	10	0.4	0.6	1.0	1 – 3
Residence	20	0.3	0.15	0.3	7	0.3	0.4	1.0	1 – 3
Hotel guest room	20	0.3	0.05	0.1	10	0.2	0.4	0.1	1 – 3
Patient room	20	0.4	0.3	0.6	5 – 10	0.2	0.4	1.0	1 – 3
Laboratory	20	0.7	0.4	0.8	5 – 10				
Libraries	20	1.7	0.5	1.0	>10				

Probabilistic Modelling of Loads

- **Loads on Structures**

The combined sustained and transient loads can be assessed as the maximum of

$$L_1 = L_{Q,\max} + L_p$$

$$L_2 = L_Q + L_{p,\max}$$

and modelled as a type I extreme value distribution

Probabilistic Modelling of Loads

- **Loads on Structures**

Wind loads

$$w = c_a c_g c_r \overline{Q}_{ref} = c_a c_e \overline{Q}_{ref}$$

Smaller rigid structures

$$w = c_d c_a c_e \overline{Q}_{ref}$$

Taller flexible structures

$$Q = \frac{1}{2}\rho U^2$$

Q_{ref}: 10 min mean U

c_r: roughness factor
c_g: gust factor
c_a: aero-dynamic shape factor
c_d: dynamic factor
c_e: exposure factor
ρ: 1.25 kg/m^3

Probabilistic Modelling of Loads

- ## Loads on Structures

Wind loads

c_r: roughness factor
c_g: gust factor
c_a: aero-dynamic shape factor
c_d: dynamic factor
c_e: exposure factor
ρ: 1.25 kg/m³

Variable	Type	V
Q_{ref}	Gumbel	0.20 - 0.30
c_r	Lognormal	0.10 - 0.20
c_a - coefficient pressure coefficient force	Lognormal Lognormal	0.10 - 0.30 0.10 - 0.15
c_g	Lognormal	0.10 - 0.15
c_d	Lognormal	0.10 - 0.20

Wind load can be assumed to be Log-Normal distributed

$$V_w^2 \cong V_{c_d}^2 + V_{c_a}^2 + V_{c_r}^2 + V_{\bar{Q}_{ref}}^2$$

Dynamic sensitive

$$V_w^2 \cong V_{c_a}^2 + V_{c_r}^2 + V_{\bar{Q}_{ref}}^2$$

Rigid

Probabilistic Modelling of Loads

- **Loads on Structures**

Snow loads

$$S_r = S_g \; r \; k^{\frac{h}{h_r}}$$

$$S_g = d \cdot \gamma\left(d\right)$$

S_r: Snow load on roof
S_g: Snow load on ground
r: ground to roof conversion factor
k: location factor (1.25 coastal, 1,5 inland)
h: altitude in meters
h_r: reference altitude (300 meters)
ρ: 1.25 kg/m^3

Typically the snow load is modelled by a Gamma or a Gumbel distribution

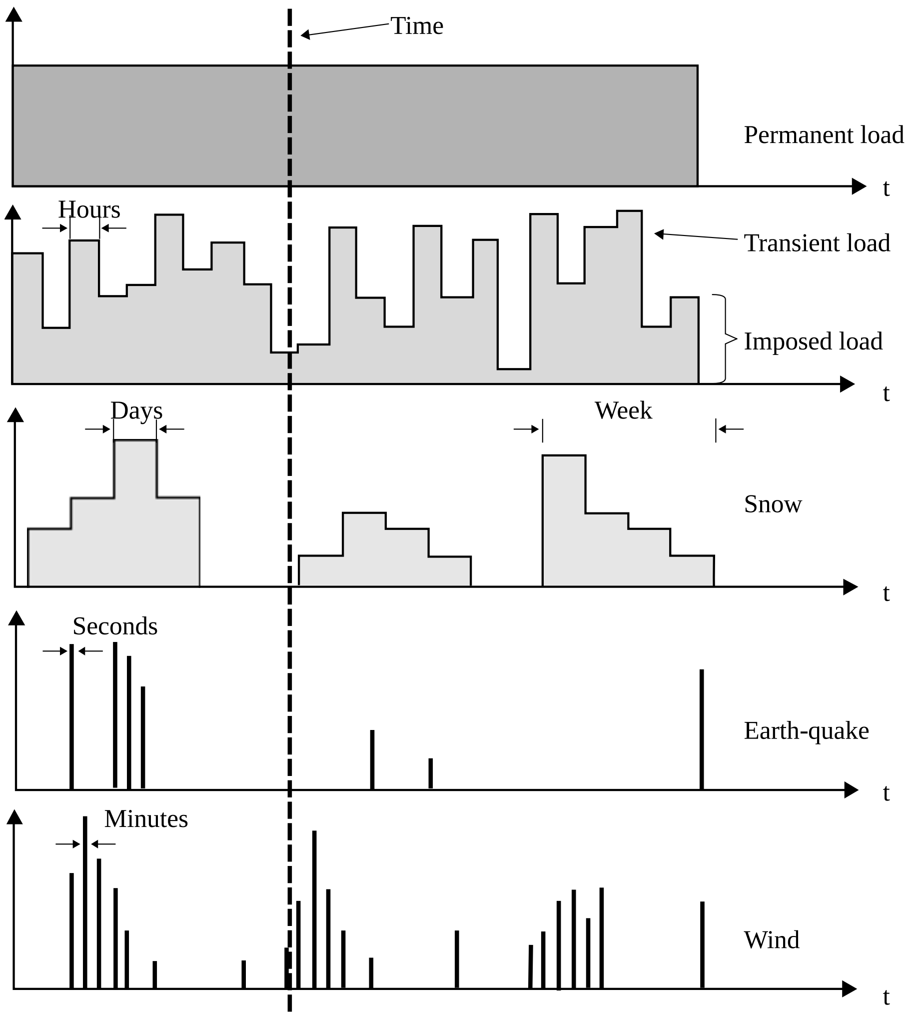
Probabilistic Modelling of Loads

- **Loads on Structures**

  Combination of loads

  We are interested in the maximum of a sum of load effects from different loads

$$X_{max}(T) = \max_{T}\left\{X_1(t) + X_2(t) + \ldots + X_n(t)\right\}$$



Probabilistic Modelling of Loads

- **Loads on Structures**

  Combination of loads

  Turkstra's load combination rule

  We take the max of the following combinations

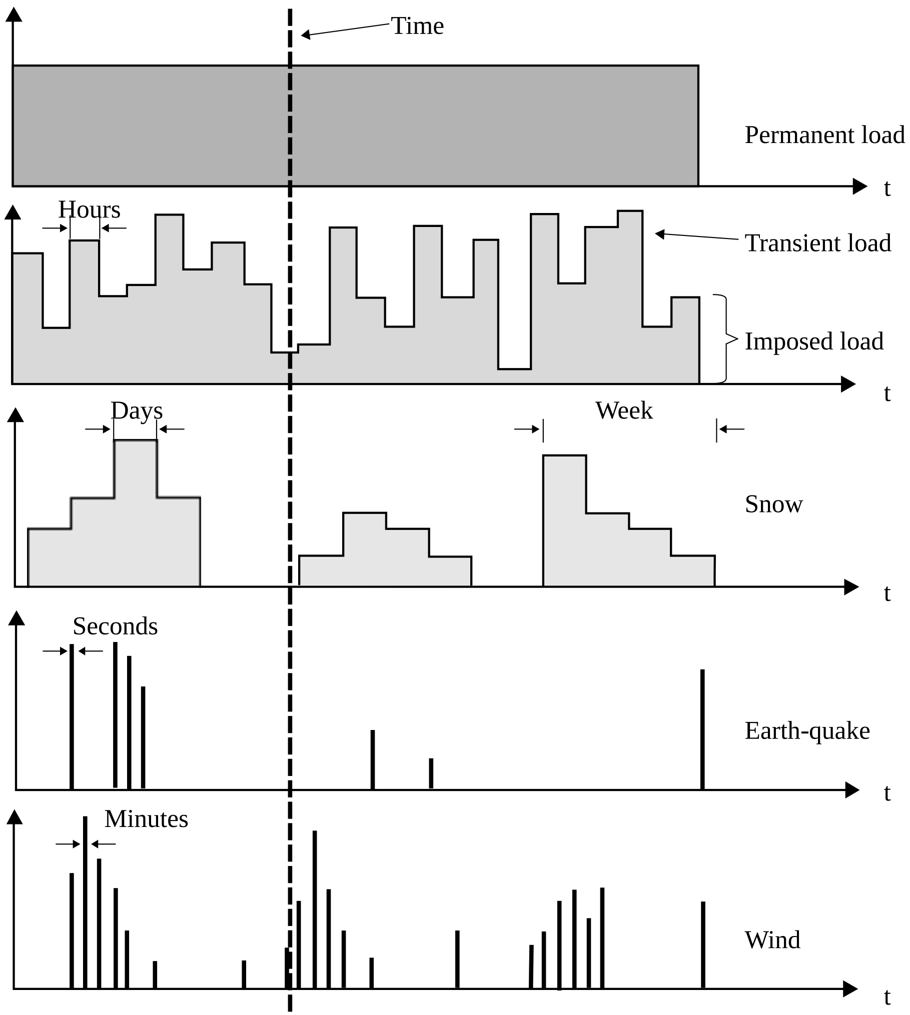
$$Z_1 = \max_T\{X_1(t)\} + X_2(t^*) + X_3(t^*) + \ldots + X_n(t^*)$$

$$Z_2 = X_1(t^*) + \max_T\{X_2(t)\} + X_3(t^*) + \ldots + X_n(t^*)$$

$$\vdots$$

$$Z_n = X_1(t^*) + X_2(t^*) + X_3(t^*) + \ldots + \max_T\{X_n(t)\}$$

$$X_{max}(T) \approx \max_i\{Z_i\}$$



Probabilistic Modelling of Loads

- **Loads on Structures**

Combination of loads

Ferry Borges-Castanheta's load combination rule

$$X_{max}(T) \approx \max_i \{Z_i\}$$

Load combination	Repetition numbers		
	Load 1	Load 2	Load 3
1	n_1	n_2 / n_1	n_3 / n_1
2	1	n_2	n_3 / n_2
3	1	1	n_3
4	n_1	1	n_3 / n_1

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

In structural engineering resistances include the following uncertainties

- Geometrical uncertainties
- Material characteristics
- Model uncertainties

} **Random variation in time and space**

The steps in the modelling process are:

- define the random variables used to represent the uncertainties in the resistances
- select a suitable distribution type to represent the random variable
- to assign the distribution parameters of the selected distribution.

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Concrete compressive strength

$$f_c = \alpha(t,\tau) f_{co}^{\lambda}$$

f_{co}:	**28 day compressive strength**
$\alpha(t,\tau)$:	**spatial stress and loading time function**
λ:	**conversion factor between in-situ concrete strength and cylinder compressive strength**

The concrete compressive strength can be assumed to be Log-Normal distributed with a coefficient of variation equal to 15%

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Reinforcement steel yield strength

$$f_s = X_1 + X_2 + X_3$$

X_1 normal distributed random variable representing the variation in the mean of different mills.

X_2 normal distributed zero mean random variable, which takes into account the variation between batches

X_3 normal distributed zero mean random variable, which takes into account the variation within a batch.

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Reinforcement steel yield strength

Variable	Type	$E[X]$	$\sigma_x[MPa]$	V_x
X_1	Normal	μ	19	-
X_2	Normal	0	22	-
X_3	Normal	0	8	-
A	-	A_{nom}	-	0.02

μ: nominal steel grade + two standard deviations of X_1

Yield stress depends on diameter of reinforcement bars

$$\mu(d) = \mu(0.87 + 0.13\exp(-0.08d))^{-1}$$

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Structural steel yield strength

Description	Variable	Type	$E[X]$	V_X
Yield stress	f_y	Lognormal	$f_{y\,sp}\alpha\, e^{-uV_{fy}} - C$	0.07
ultimate stress	f_u	Lognormal	$B\, E[f_u]$	0.04
modulus of elasticity	E	Lognormal	E_{sp}	0.03
Poisson's ratio	v	Lognormal	v_{sp}	0.03
ultimate strain	ε_u	Lognormal	$\varepsilon_{u\,sp}$	0.06

Distribution characteristics

	f_y	f_u	E	v	ε_u
f_y	1	0.75	0	0	-0.45
f_u		1	0	0	-0.60
E			1	0	0
v	Symmetry			1	0
ε_u					1

Dependencies

Probabilistic Modelling of Resistances

- **Model uncertainties**

Model uncertainties relate engineering model results with actual structural behaviour

$$X = \Xi \cdot X_{\text{mod}}$$

$$\xi = \frac{x_{\text{mod}}}{x_{\text{exp}}}$$

X:	**true value**
Ξ:	**model uncertainty**
X_{mod}:	**model value**
x_{exp}:	**experimentally obtained value**

Probabilistic Modelling of Resistances

- **Model uncertainties**

Model uncertainties may be introduced in different ways:

$$Y = \Xi f(\mathbf{X})$$

$$Y = \Xi + f(\mathbf{X})$$

$$Y = f(\Xi_1 X_1, \Xi_2 X_2, \ldots, \Xi_n X_n)$$

Y	structural performance
$f(.)$	model function
Ξ	random variable representing the model uncertainty
X_i	basic variables
$\mathbf{X}$	vector of basic random variables

Probabilistic Modelling of Resistances

- **The JCSS Probabilistic Model Code (PMC)**
  http://www.jcss.ethz.ch/publications/publications_pmc.html

  Part I : Basis of design
  Part II: Load models
  Part III: Resistance models
  Part IV: Examples

Probabilistic Modelling of Resistances

- ## The JCSS PMC – Load Models

2.00	GENERAL PRINCIPLES	05.2001
2.01	SELF WEIGHT	06.2001
2.02	LIVE LOAD	05.2001
2.06	LOAD IN CAR PARKS	05.2001
2.12	SNOW LOAD	05.2001
2.13	WIND LOAD	05.2001
2.15	WAVE LOAD	05.2006
2.17	EARTHQUAKE	09.2002
2.18	IMPACT LOAD	05.2001
2.20	FIRE	05.2001

Probabilistic Modelling of Resistances

- **The JCSS PMC – Resistance models**

3.00	GENERAL PRINCIPLES	03.2001
3.01	CONCRETE	05.2002
3.02	STRUCTURAL STEEL	03.2001
3.0*	REINFORCING STEEL	03.2001
3.04	PRESTRESSING STEEL	04.2005
3.05	TIMBER	05.2006
3.07	SOIL PROPERTIES	06.2002
3.09	MODELUNCERTAINTIES	03.2001
3.10	DIMENSIONS	03.2001
3.11	EXCENTRICITIES	03.2001