

# **Risk and Safety** **in** **Engineering**

**Prof. Dr. Michael Havbro Faber**  
**Swiss Federal Institute of Technology**  
**ETH Zurich, Switzerland**

# Decision Analysis in Engineering

- **Introduction to Decision Theory**
  - **The problem**
  - **The decision tree**
  - **Prior decision analysis**
  - **Posterior decision analysis**
  - **Pre-posterior decision analysis**

# Decision Analysis in Engineering

- The basic engineering problem

Several solutions may be identified

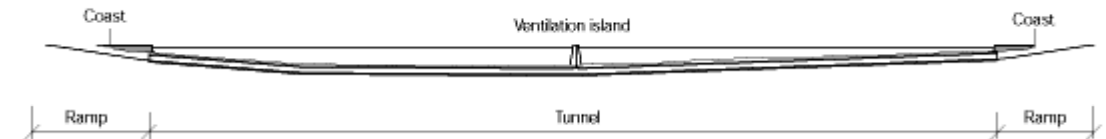


The available information is uncertain

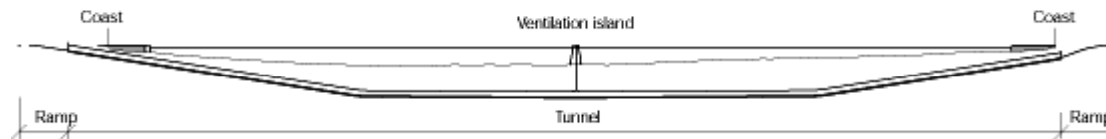
A decision must be made !



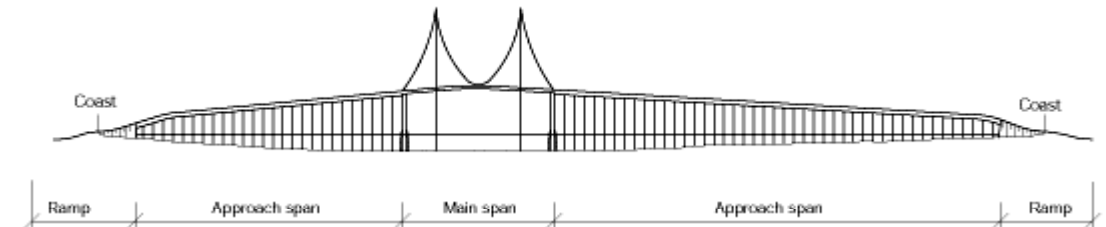
**Solution B and F**



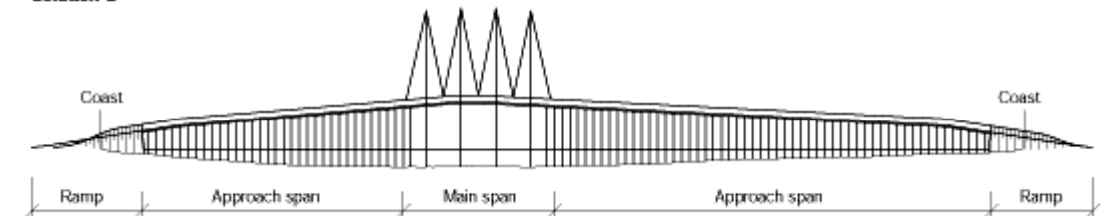
**Solution A and E**



**Solution D**



**Solution C**



# Decision Analysis in Engineering

## Emperor Qianlong

Qing dynasty

Reign : 18 October 1735 – 8 February 1796

The Chinese knew long ago what the principles of decision making were!



### ZHONG HE DIAN (Hall of Central Harmony)

First constructed in 1420 during the Ming Dynasty, Zhong He Dian was destroyed and reconstructed several times over the centuries. The existing hall was constructed in 1627 during the Ming Dynasty. In the early Ming Dynasty, this hall was called Hua Gai Dian (Hall of Overwhelming Glory) but was renamed Zhong Ji Dian (Hall of Central Extremity) in 1562 and Zhong He Dian in 1645 during the Qing Dynasty. This square building has a single pyramid-shaped roof, with a gold-plated bronze covering. The floor is paved with high-quality square clay bricks, commonly known as "golden bricks." A throne is placed in the center of the hall and a board hangs above the throne with an inscription written by Emperor Qianlong. The inscription reads: "Yun Zhi Jue Zhong," meaning "The Way of Heaven is profound and mysterious and the way of mankind is difficult. Only if we make a precise and unified plan and follow the doctrine of the mean, can we rule the country well."

This hall served as a resting place for the emperor on his way to attend an important ceremony or hold court. Officials kowtowed to the emperor here. The day before the emperor held a sacrificial ceremony he would read the prayer tablet aloud in this hall. Before offering sacrifices at the Altar of the God of Agriculture, the emperor examined ceremonial farm tools here. After the revision of the imperial pedigree, which was revised once every ten years, the emperor read the pedigree out loud and held a grand ceremony at the hall. The words "Zhong He" come from the *Book of Rites*, meaning "When we handle matters properly and harmoniously without leaning to either side, all things on earth will flourish."

MADE POSSIBLE BY THE AMERICAN EXPRESS COMPANY

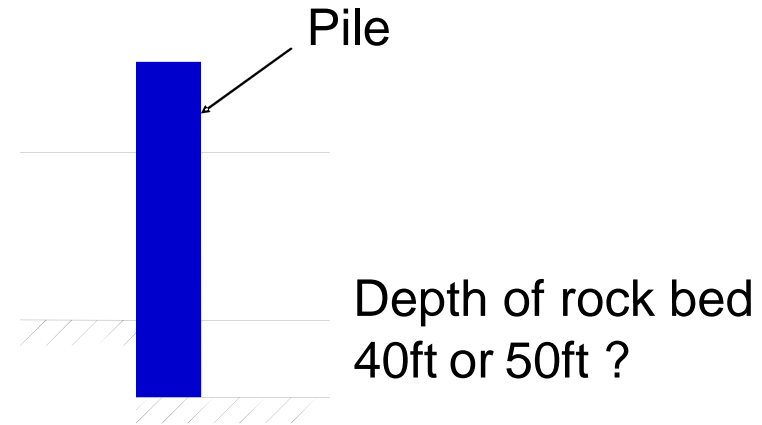
# Decision Analysis in Engineering

## Approach

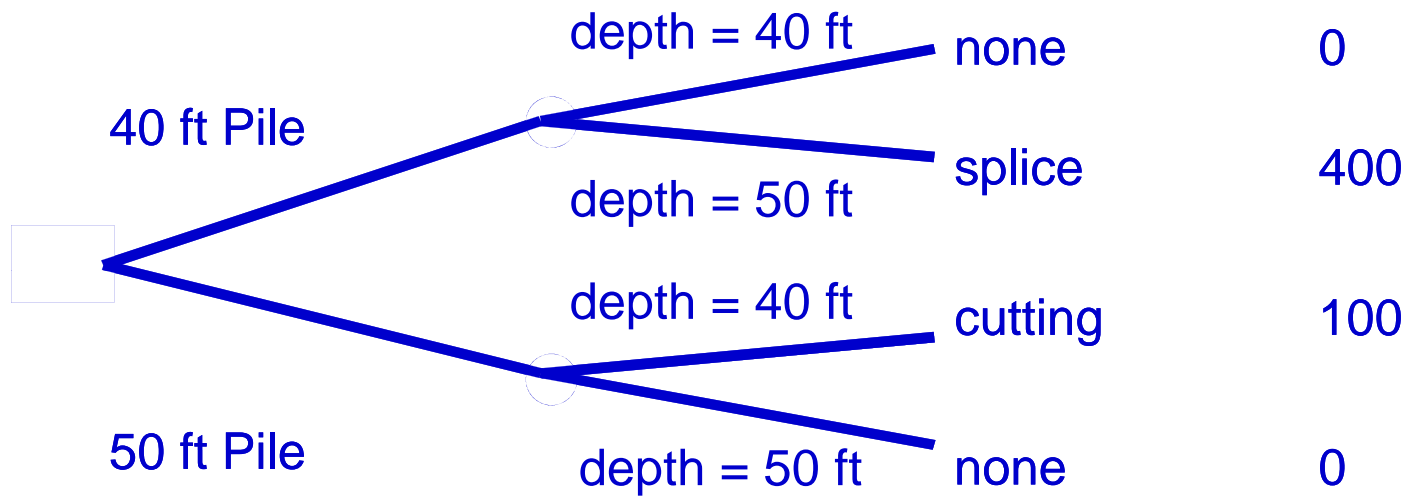
- **Formulation of the decision problem**
  - Identification of the decision maker and the preferences of the decision maker
  - Mapping of the decision process
  - Identification of the possible decision alternatives
  - Identification of the contributing uncertainties
- **Identification of potential consequences and their utility (cost/benefit)**
- **Assessment of the probabilities of the consequences**
- **Comparison of the different decision alternatives based on their expected utilities**
- **Final decision making and reporting of the assumptions underlying the selected alternative**

# Decision Analysis in Engineering

- The decision tree



**Action alternatives    Outcome    Consequence    Utility(consequence)**



# Decision Analysis in Engineering

## Assignment of utility

- The assignment of utility must reflect the preferences of the decision maker
- Utility functions may be defined as linear functions in monetary unity
- It is important to include all monetary consequences in the utility function

$$u(a_i) = \sum_{j=1}^n p_j \cdot u(K_j)$$

$u(a_i)$ ... Utility (cost/benefit) associated with action  $a_i$

$p_j \cdot u(K_j)$ ... Expected utility associated with consequence  $K_j$

$p_j$ ... Probability of the occurrence of the consequence  $K_j$

$u(K_j)$ ... Utility associated with the consequence  $K_j$

$K_j$ ... A potential consequence associated with the action  $a_i$

# Decision Analysis in Engineering

## The different types of decision analysis

- Prior
- Posterior
- Pre-posterior

Illustrated on an example :

**Question :** What pile length should be applied ?

### Alternatives :

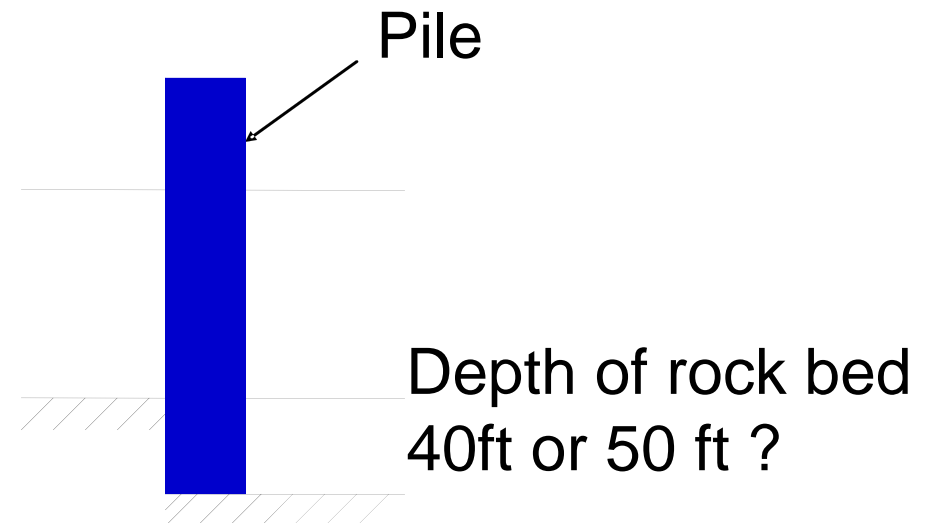
$a_0$  : Choose a 40 ft pile

$a_1$  : Choose a 50 ft pile

### States of nature (depth to rock bed)

$\theta_0$  : Rock bed at 40 ft

$\theta_1$  : Rock bed at 50 ft



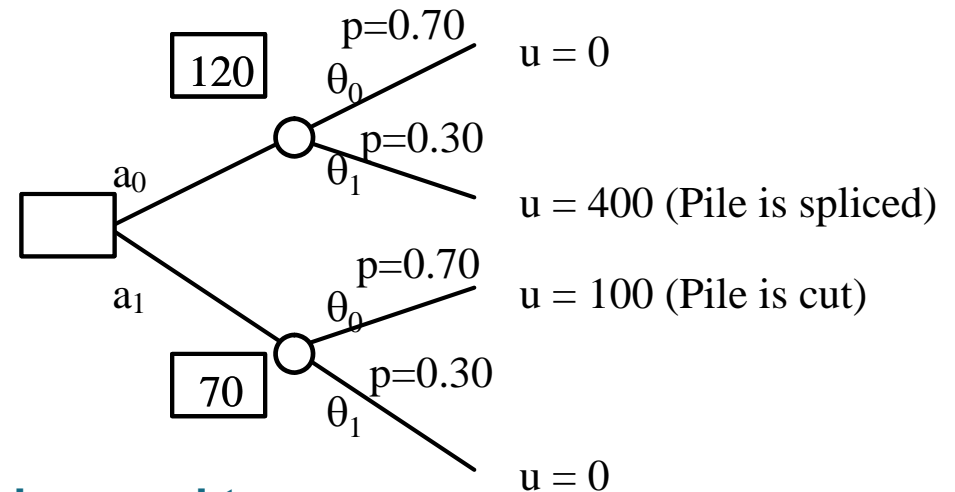


# Decision Analysis in Engineering

## Prior Analysis

$$P'[\theta_0] = 0.70$$

$$P'[\theta_1] = 0.30$$



The expected utility is calculated to be equal to

$$E'[u] = \min \{u[a_0], u[a_1]\}$$

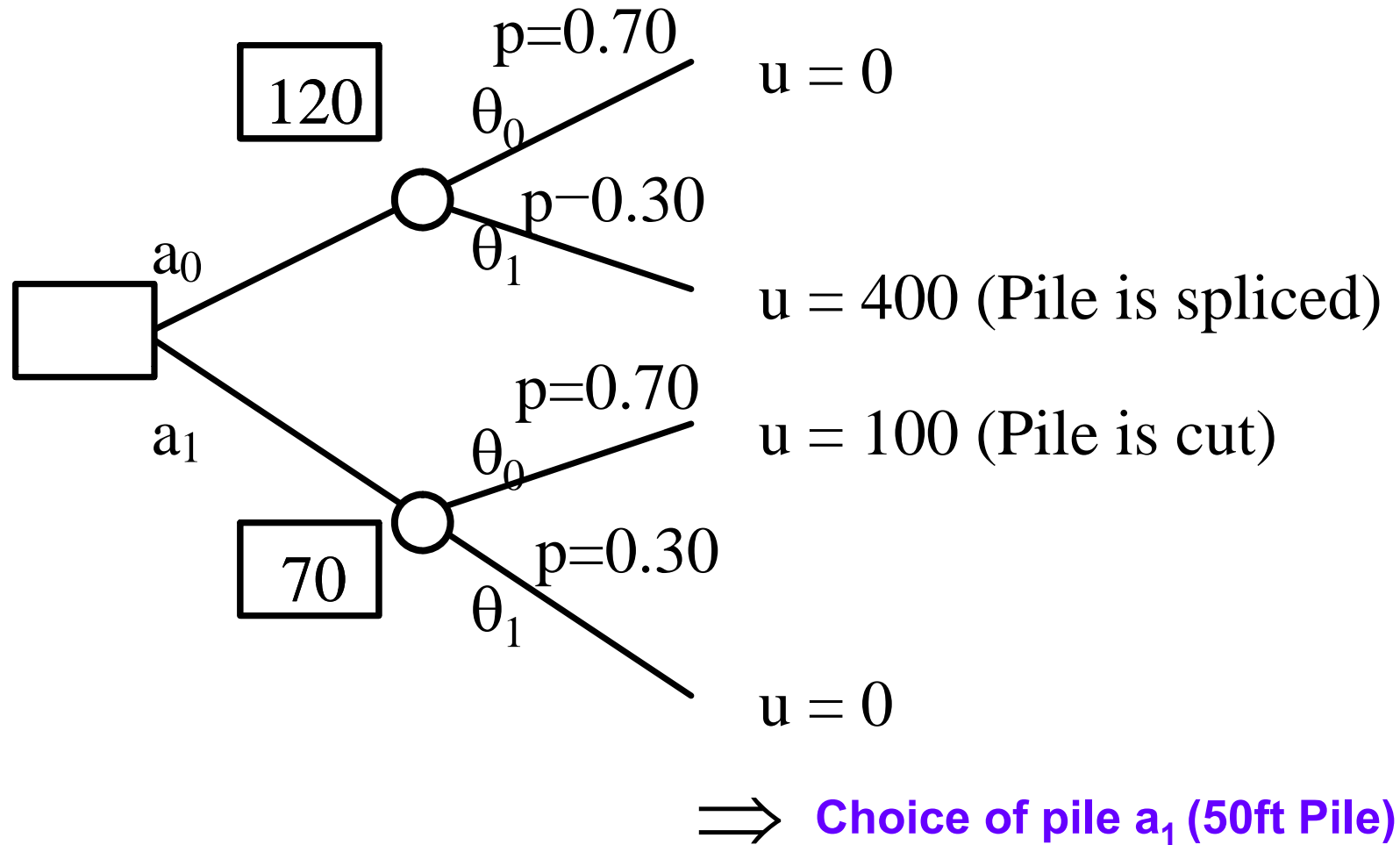
$$= \min \{P'[\theta_0] \times u[\theta_0|a_0] + P'[\theta_1] \times u[\theta_1|a_0],$$

$$P'[\theta_0] \times u[\theta_0|a_1] + P'[\theta_1] \times u[\theta_1|a_1]\}$$

$$= \min \{0.7 \times 0 + 0.3 \times 400, 0.7 \times 100 + 0.3 \times 0\}$$

$$= \min \{120, 70\} = 70 \Rightarrow \text{Decision for } a_1 \text{ (50ft Pile)}$$

# Decision Analysis in Engineering



# Decision Analysis in Engineering

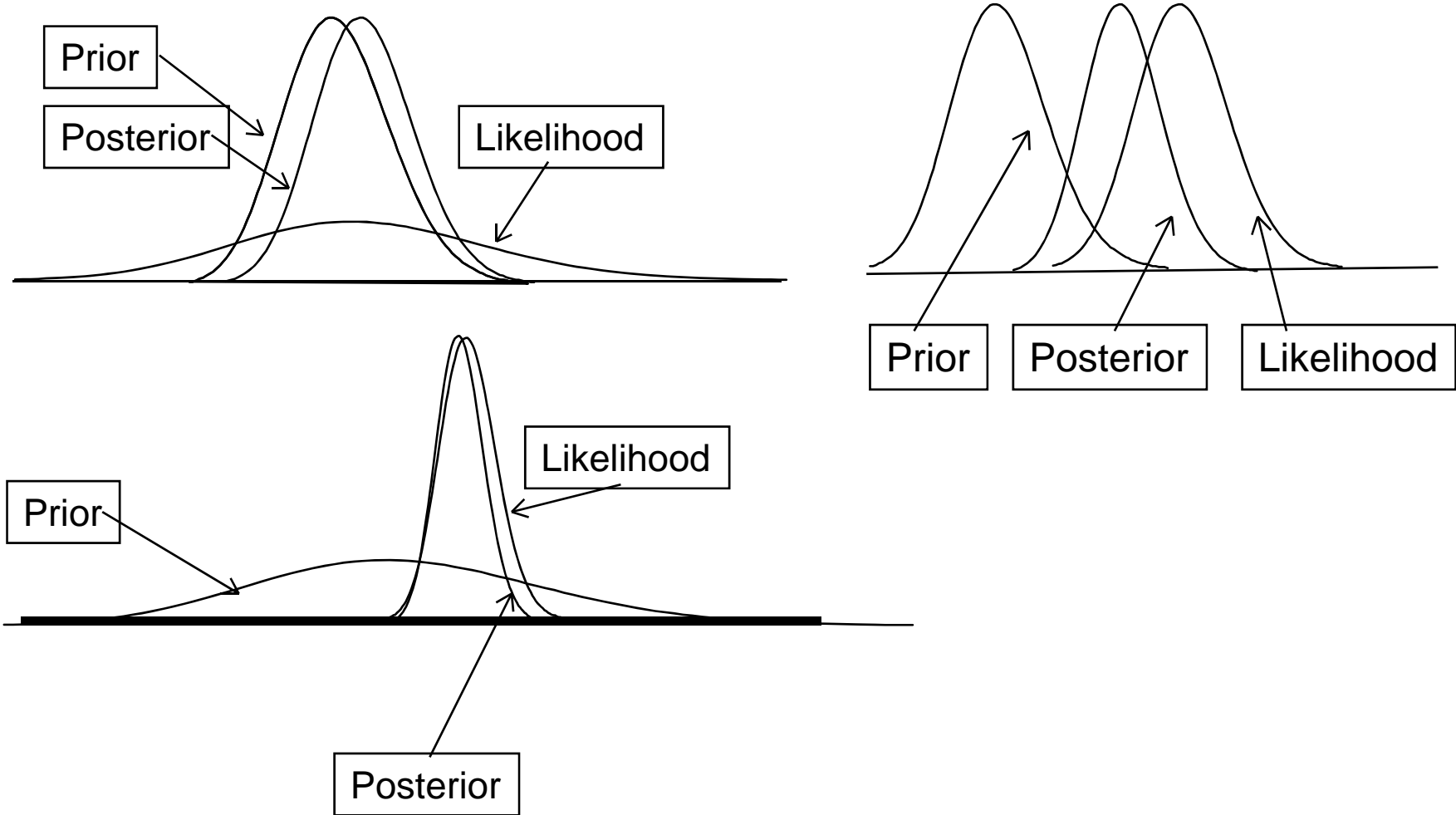
## Posterior Analysis

$$P''(\theta_i) = \frac{P[z_k | \theta_i] P'[\theta_i]}{\sum_j P[z_k | \theta_j] P'[\theta_j]}$$

$$\left( \begin{array}{l} \text{Posterior probability of } \theta_i \\ \text{with given sample outcome} \end{array} \right) = \left( \begin{array}{l} \text{Normalising} \\ \text{constant} \end{array} \right) \times \left( \begin{array}{l} \text{Sample likelihood} \\ \text{given } \theta \end{array} \right) \times \left( \begin{array}{l} \text{prior probability} \\ \text{of } \theta \end{array} \right)$$

# Decision Analysis in Engineering

## Posterior Analysis



# Decision Analysis in Engineering

## Posterior Analysis

$$P''(\theta_i) = \frac{P[z_k | \theta_i] P'[\theta_i]}{\sum_j P[z_k | \theta_j] P'[\theta_j]}$$

Ultrasonic tests to determine the depth to bed rock

True state \ Test result	$\theta_0$ 40 ft – depth	$\theta_1$ 50 ft – depth
$z_0$ - 40 ft indicated	0.6	0.1
$z_1$ - 50 ft indicated	0.1	0.7
$z_2$ - 45 ft indicated	0.3	0.2

Likelihoods of the different indications/test results given the various possible states of nature – ultrasonic test methods  $P[z_k | \theta_j]$

# Decision Analysis in Engineering

## Posterior Analysis

It is assumed that a test gives a 45 ft indication

$$P''(\theta_i) = \frac{P[z_k | \theta_i] P'[\theta_i]}{\sum_j P[z_k | \theta_j] P'[\theta_j]}$$

$$P''[\theta_0] = P[\theta_0 | z_2] \propto P[z_2 | \theta_0] P[\theta_0] = 0.3 \times 0.7 = 0.21$$

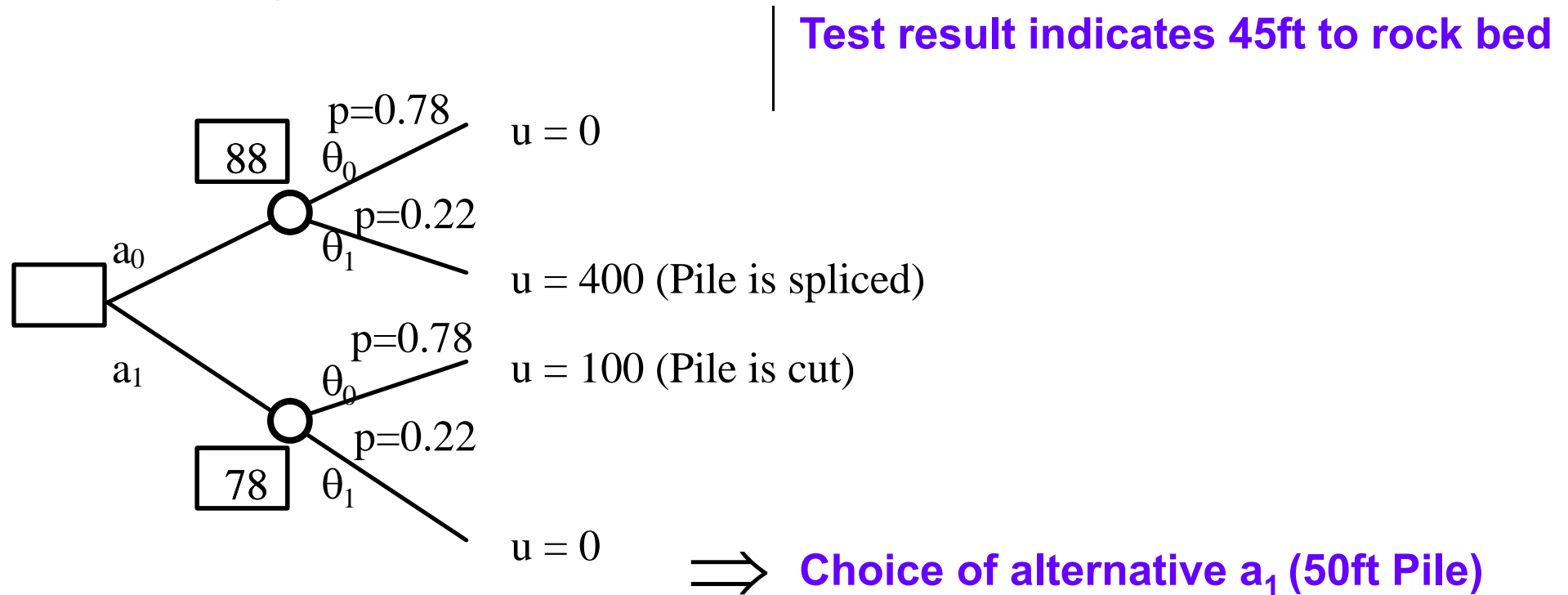
$$P''[\theta_1] = P[\theta_1 | z_2] \propto P[z_2 | \theta_1] P[\theta_1] = 0.2 \times 0.3 = 0.06$$

$$P''[\theta_0 | z_2] = \frac{0.21}{0.21 + 0.06} = 0.78$$

$$P''[\theta_1 | z_2] = \frac{0.06}{0.21 + 0.06} = 0.22$$

# Decision Analysis in Engineering

## Posterior Analysis



# Decision Analysis in Engineering

## Posterior Analysis

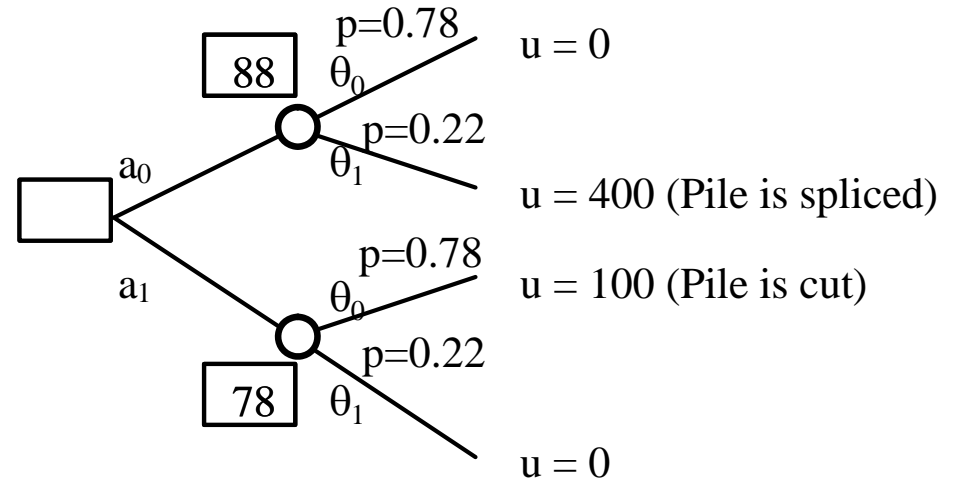
$$E[u | z_2] = \min_j \{ E[u(a_j) | z_2] \}$$

$$= \min \{ P[\theta_0] \times 0 + P[\theta_1] \times 400, P[\theta_0] \times 100 + P[\theta_1] \times 0 \}$$

$$= \min \{ 0.78 \times 0 + 0.22 \times 400, 0.78 \times 100 + 0.22 \times 0 \}$$

$$= \min \{ 88, 78 \} = 78$$

⇒ **Choice of alternative a<sub>1</sub> (50ft Pile)**





# Decision Analysis in Engineering

## Pre-posterior Analysis

$$E[u] = \sum_{i=1}^n P'[z_i] \times E''[u|z_i] = \sum_{i=1}^n P'[z_i] \times \min_{j=1,m} \{E''[u(a_j)|z_i]\}$$

$$P'[z_i] = P[z_i|\theta_0] \times P'[\theta_0] + P[z_i|\theta_1] \times P'[\theta_1]$$

$$P'[z_0] = P[z_0|\theta_0] \times P'[\theta_0] + P[z_0|\theta_1] \times P'[\theta_1] = 0.6 \times 0.7 + 0.1 \times 0.3 = 0.45$$

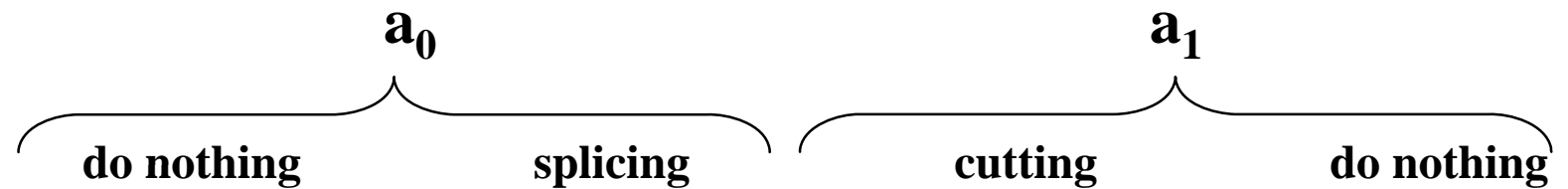
$$P'[z_1] = P[z_1|\theta_0] \times P'[\theta_0] + P[z_1|\theta_1] \times P'[\theta_1] = 0.1 \times 0.7 + 0.7 \times 0.3 = 0.28$$

$$P'[z_2] = P[z_2|\theta_0] \times P'[\theta_0] + P[z_2|\theta_1] \times P'[\theta_1] = 0.3 \times 0.7 + 0.2 \times 0.3 = 0.27$$

# Decision Analysis in Engineering

## Pre-posterior Analysis

$$E^*[u|z_0] = \min_j \{E^*[u(a_j)|z_0]\}$$



$$\begin{aligned} &= \min\{P^*[\theta_0|z_0] \times 0 + P^*[\theta_1|z_0] \times 400, P^*[\theta_0|z_0] \times 100 + P^*[\theta_1|z_0] \times 0\} \\ &= \min\{0.93 \times 0 + 0.07 \times 400, 0.93 \times 100 + 0.07 \times 0\} \\ &= 0.07 \times 400 + 0.93 \times 0 = 28 \end{aligned}$$

# Decision Analysis in Engineering

## Pre-posterior Analysis

$$\begin{aligned} E[u|z_1] &= \min_j \{ E[u(a_j)|z_1] \} \\ &\quad \underbrace{\qquad \qquad \qquad \mathbf{a}_0 \qquad \qquad \qquad}_{\text{do nothing} \quad \text{splicing}} \quad \underbrace{\qquad \qquad \qquad \mathbf{a}_1 \qquad \qquad \qquad}_{\text{cutting} \quad \text{do nothing}} \\ &= \min \{ P[\theta_0|z_1] \times 0 + P[\theta_1|z_1] \times 400, P[\theta_0|z_1] \times 100 + P[\theta_1|z_1] \times 0 \} \\ &= \min \{ 0.25 \times 0 + 0.75 \times 400, 0.25 \times 100 + 0.75 \times 0 \} \\ &= 0.25 \times 100 + 0.75 \times 0 = 25 \end{aligned}$$

# Decision Analysis in Engineering

## Pre-posterior Analysis

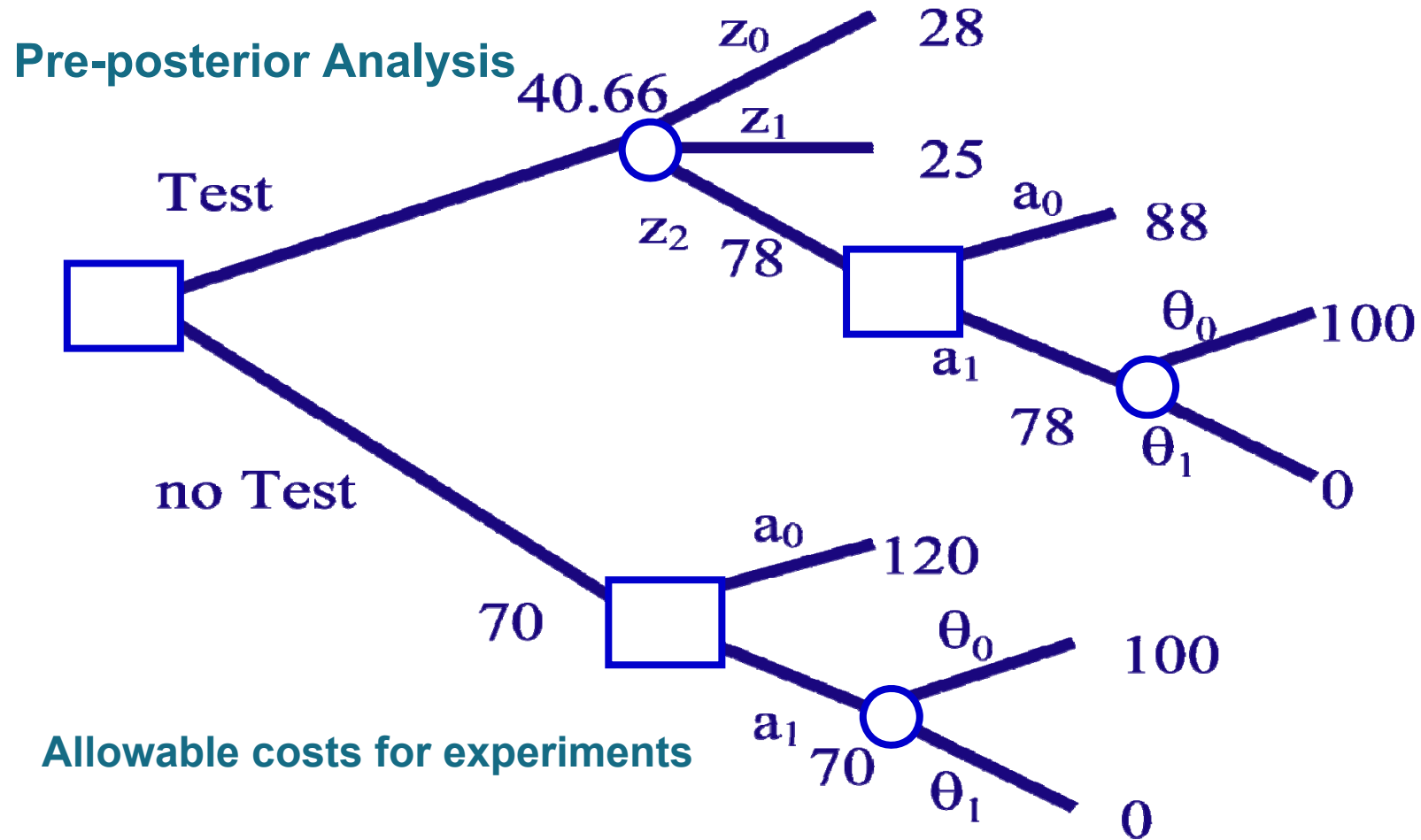
The minimum expected costs based on pre-posterior decision analysis  
– not including costs of experiments

$$E[u] = \sum_{i=1}^n P'[z_i] \times E''[u|z_i] = 28 \times 0.45 + 25 \times 0.28 + 78 \times 0.27 = 40.66$$

Allowable costs for the experiment

$$E'[u] - E[u] = 70.00 - 40.66 = 29.34$$

# Decision Analysis in Engineering



$$E'[u] - E[u] = 70.00 - 40.66 = 29.34$$