Risk and Safety

in

Engineering

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Decision Analysis in Engineering

- **Introduction to Decision Theory**
  - **The problem**
  - **The decision tree**
  - **Prior decision analysis**
  - **Posterior decision analysis**
  - **Pre-posterior decision analysis**

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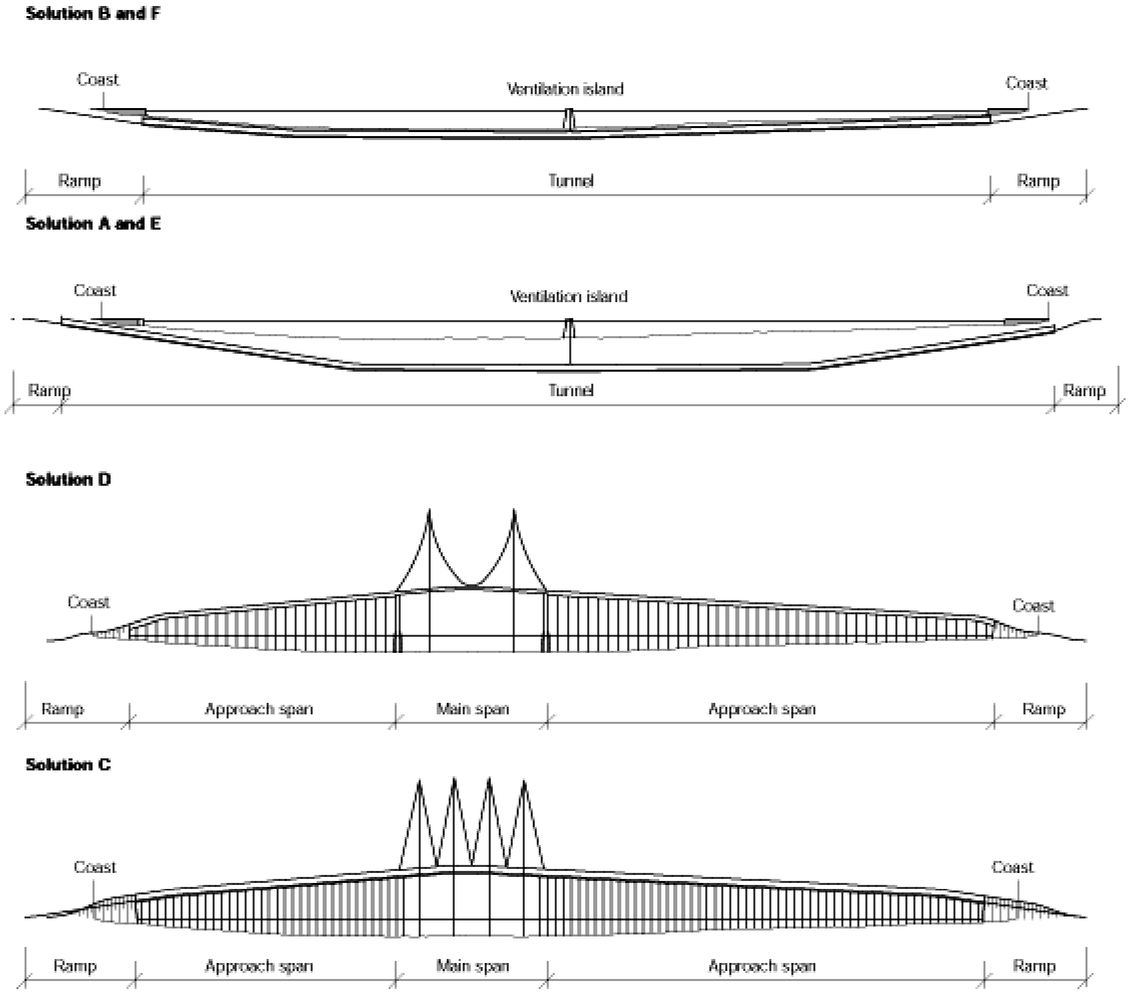
- The basic engineering problem

Several solutions may be identified

The available information is uncertain

A decision must be made !



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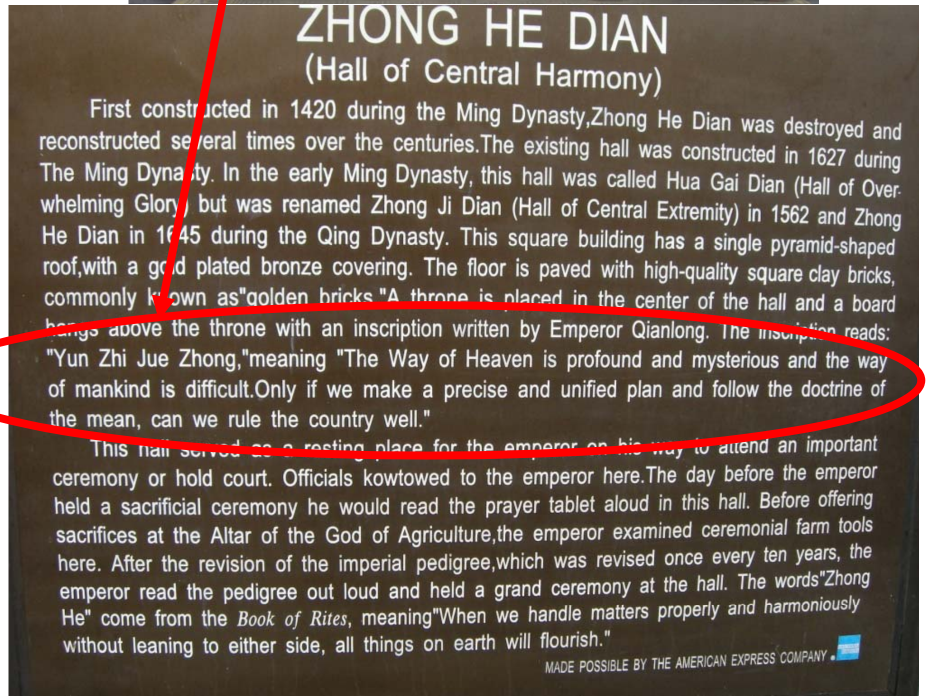
Emperor Qianlong

Qing dynasty

Reign : 18 October 1735 – 8 February 1796

The Chinese knew long ago what the principles of decision making were!



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Approach

- **Formulation of the decision problem**
  - Identification of the decision maker and the preferences of the decision maker
  - Mapping of the decision process
  - Identification of the possible decision alternatives
  - Identification of the contributing uncertainties

- **Identification of potential consequences and their utility (cost/benefit)**

- **Assessment of the probabilities of the consequences**

- **Comparison of the different decision alternatives based on their expected utilities**

- **Final decision making and reporting of the assumptions underlying the selected alternative**

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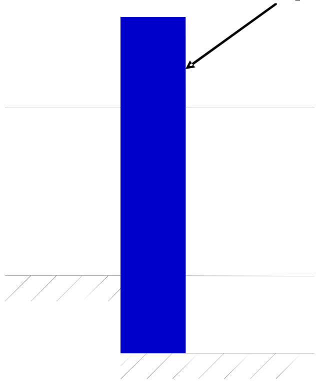
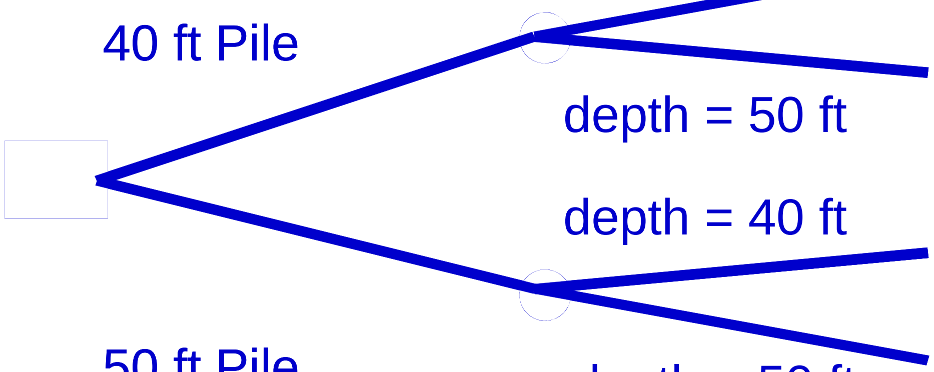
- **The decision tree**



Action alternatives **Outcome** **Consequence** **Utility(consequence)**



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Assignment of utility

- The assignment of utility must reflect the preferences of the decision maker
- Utility functions may be defined as linear functions in monetary unity
- It is important to include all monetary consequences in the utility function

$$u(a_i) = \sum_{j=1}^{n} p_j \cdot u(K_j)$$

$u(a_i)\ldots$ Utility (cost/benefit) associated with action a_i

$p_j \cdot u(K_j)\ldots$ Expected utility associated with consequence K_j

$p_j \ldots$ Probability of the occurence of the consequence K_j

$u(K_j)\ldots$ Utility associated with the consequence K_j

$K_j \ldots$ A potential consequence associated with the action a_i

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The different types of decision analysis

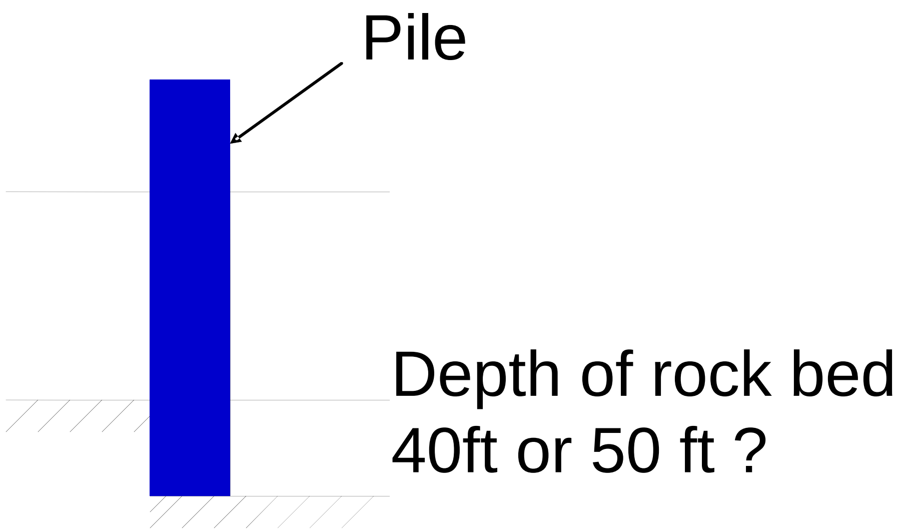
- Prior
- Posterior
- Pre-posterior

Illustrated on an example :

Question : What pile length should be applied ?

Alternatives :
a_0 : Choose a 40 ft pile
a_1 : Choose a 50 ft pile

States of nature (depth to rock bed)
θ_0 : Rock bed at 40 ft
θ_1 : Rock bed at 50 ft



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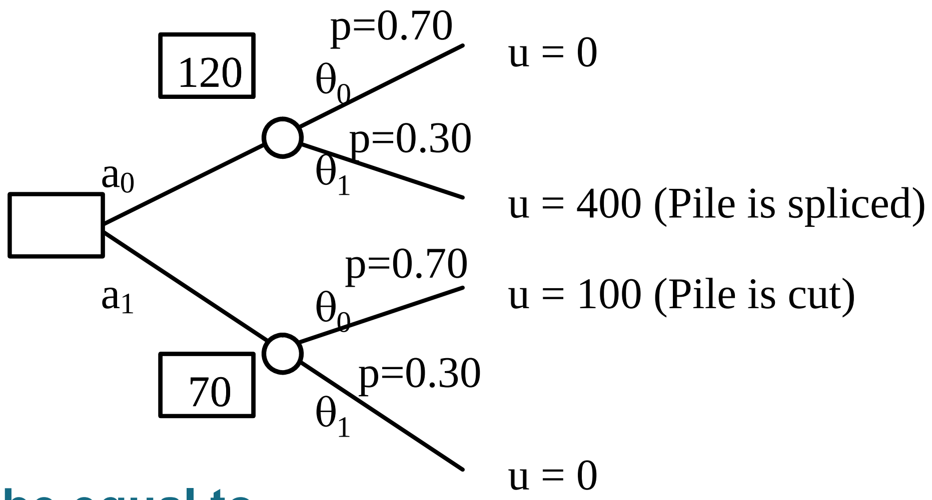
Prior Analysis



P'[θ$_0$] = 0.70
P'[θ$_1$] = 0.30

The expected utility is calculated to be equal to

$$
\begin{aligned}
E'[u] &= \min\{u[a_0], u[a_1]\} \\
&= \min\{P'[\theta_0]\times u[\theta_0|a_0] + P'[\theta_1]\times u[\theta_1|a_0], \\
&\qquad P'[\theta_0]\times u[\theta_0|a_1] + P'[\theta_1]\times u[\theta_1|a_1]\} \\
&= \min\{0.7\times 0 + 0.3\times 400, 0.7\times 100 + 0.3\times 0\} \\
&= \min\{120, 70\} = 70 \quad \Rightarrow \text{Decision for } a_1 \text{ (50ft Pile)}
\end{aligned}
$$

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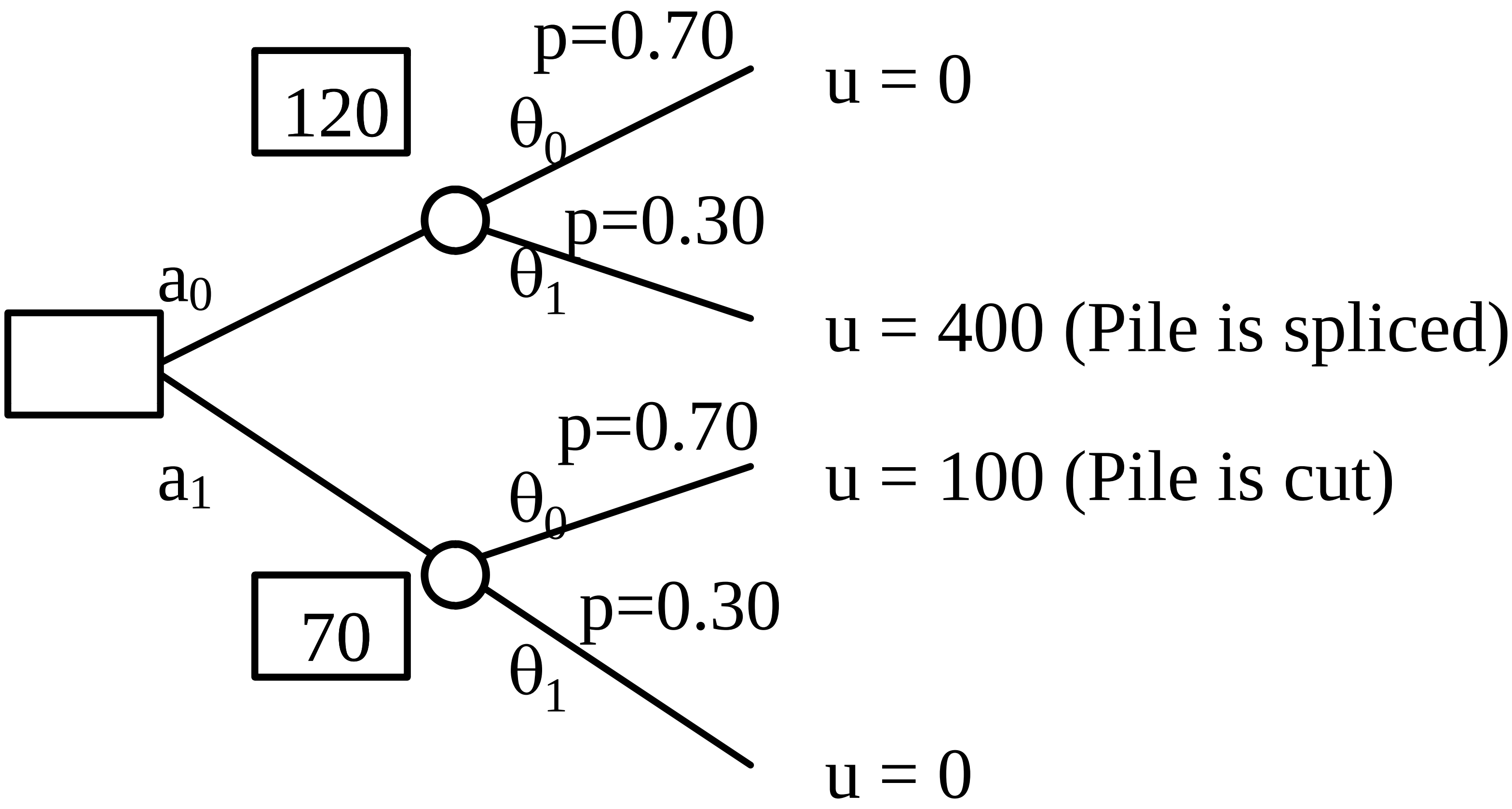

$\Rightarrow$ **Choice of pile a_1 (50ft Pile)**

Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

$$\begin{pmatrix} \text{Posterior probability of } \theta_i \\ \text{with given sample outcome} \end{pmatrix} = \begin{pmatrix} \text{Normalising} \\ \text{constant} \end{pmatrix} \times \begin{pmatrix} \text{Sample likelihood} \\ \text{given } \theta \end{pmatrix} \times \begin{pmatrix} \text{prior probability} \\ \text{of } \theta \end{pmatrix}$$

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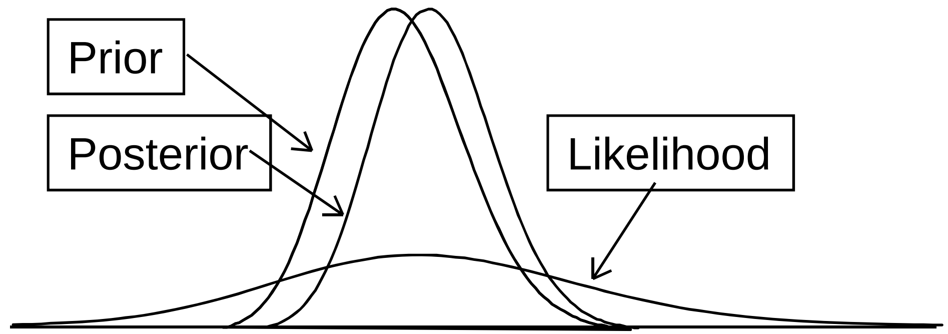
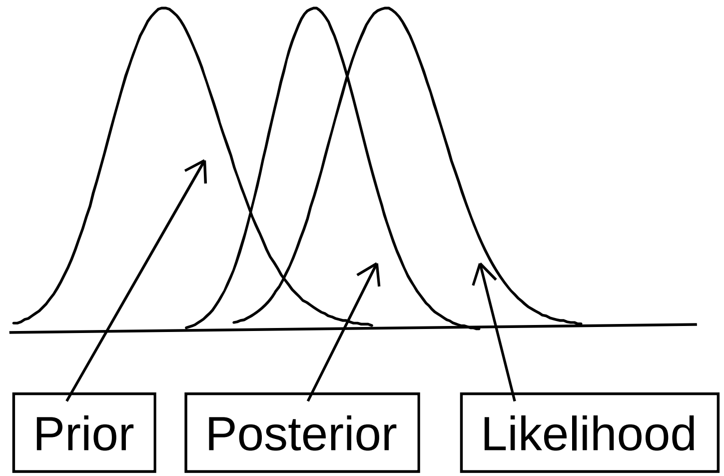
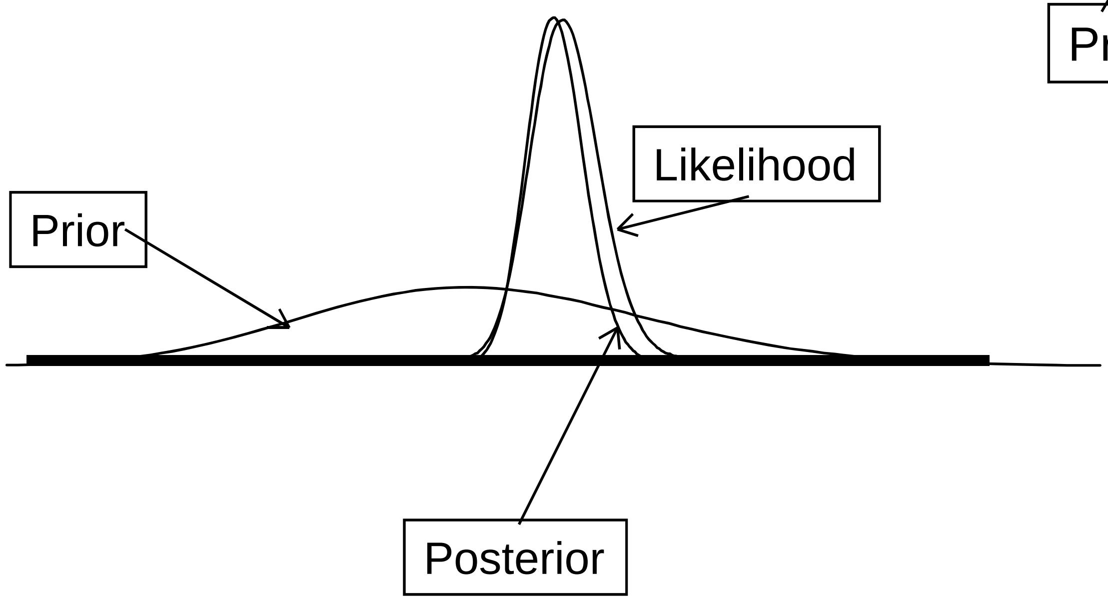
Posterior Analysis







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Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'[\theta_i]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

Ultrasonic tests to determine the depth to bed rock

True state / Test result	θ_0 40 ft – depth	θ_1 50 ft – depth
z_0 - 40 ft indicated	0.6	0.1
z_1 - 50 ft indicated	0.1	0.7
z_2 - 45 ft indicated	0.3	0.2

Likelihoods of the different indications/test results given the various possible states of nature – ultrasonic test methods $P\left[z_k \middle| \theta_j\right]$

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Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

It is assumed that a test gives a 45 ft indication

$$P''\left[\theta_0\right] = P\left[\theta_0 \middle| z_2\right] \propto P\left[z_2 \middle| \theta_0\right] P\left[\theta_0\right] = 0.3\ x\ 0.7\ = 0.21$$

$$P''\left[\theta_1\right] = P\left[\theta_1 \middle| z_2\right] \propto P\left[z_2 \middle| \theta_1\right] P\left[\theta_1\right] = 0.2\ x\ 0.3 = 0.06$$

$$P''\left[\theta_0 \middle| z_2\right] = \frac{0.21}{0.21 + 0.06} \ = \ 0.78$$

$$P''\left[\theta_1 \middle| z_2\right] = \frac{0.06}{0.21 + 0.06} \ = \ 0.22$$

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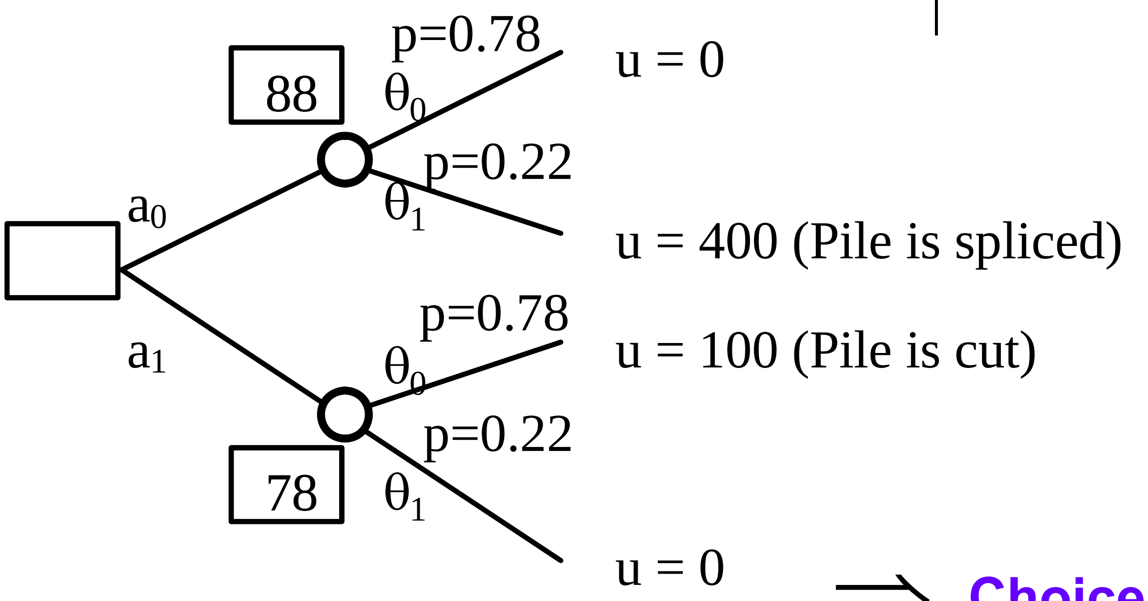
Posterior Analysis

Test result indicates 45ft to rock bed



$\Rightarrow$ **Choice of alternative a_1 (50ft Pile)**

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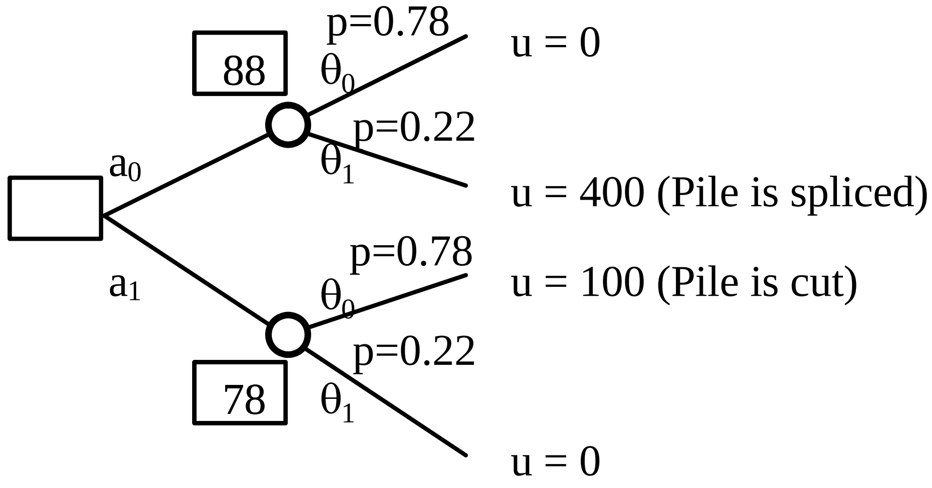
Posterior Analysis



$$E''\left[u \middle| z_2\right] = \min_j \{E''\left[u(a_j) \middle| z_2\right]\}$$

$$= \min\{P''[\theta_0] \times 0 + P''[\theta_1] \times 400,\ P''[\theta_0] \times 100 + P''[\theta_1] \times 0\}$$

$$= \min\{0.78 \times 0 + 0.22 \times 400,\ 0.78 \times 100 + 0.22 \times 0\}$$

$$= \min\{88\ ,\ 78\} = 78$$

$\Longrightarrow$ **Choice of alternative a_1 (50ft Pile)**

Pre-posterior Analysis

$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''[u|z_i] = \sum_{i=1}^{n} P'[z_i] \times \min_{j=1,m}\{E''[u(a_j)|z_i]\}$$

$$P'[z_i] = P[z_i|\theta_0] \times P'[\theta_0] + P[z_i|\theta_1] \times P'[\theta_1]$$

$$P'[z_0] = P[z_0|\theta_0] \times P'[\theta_0] + P[z_0|\theta_1] \times P'[\theta_1] = 0.6 \times 0.7 + 0.1 \times 0.3 = 0.45$$

$$P'[z_1] = P[z_1|\theta_0] \times P'[\theta_0] + P[z_1|\theta_1] \times P'[\theta_1] = 0.1 \times 0.7 + 0.7 \times 0.3 = 0.28$$

$$P'[z_2] = P[z_2|\theta_0] \times P'[\theta_0] + P[z_2|\theta_1] \times P'[\theta_1] = 0.3 \times 0.7 + 0.2 \times 0.3 = 0.27$$

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Pre-posterior Analysis

$$E''\left[u \middle| z_0\right] = \min_j \{E''\left[u(a_j) \middle| z_0\right]\}$$

$$\overbrace{\underbrace{}_{\textbf{do nothing}} \quad \underbrace{}_{\textbf{splicing}}}^{\mathbf{a_0}} \qquad \overbrace{\underbrace{}_{\textbf{cutting}} \quad \underbrace{}_{\textbf{do nothing}}}^{\mathbf{a_1}}$$

$$= \min\{P''\left[\theta_0 \middle| z_0\right]\times 0 + P''\left[\theta_1 \middle| z_0\right]\times 400,\ P''\left[\theta_0 \middle| z_0\right]\times 100 + P''\left[\theta_1 \middle| z_0\right]\times 0\}$$

$$= \min\{0.93\times 0 + 0.07\times 400,\ 0.93\times 100 + 0.07\times 0\}$$

$$= 0.07\times 400 + 0.93\times 0 = 28$$

Pre-posterior Analysis

$$E''\left[u\left|z_1\right.\right] = \min_j \{E''\left[u(a_j)\left|z_1\right.\right]\}$$

a₀: **do nothing**, **splicing** — **a₁**: **cutting**, **do nothing**

$$= \min\{P''\left[\theta_0\left|z_1\right.\right]\times 0 + P''\left[\theta_1\left|z_1\right.\right]\times 400,\ P''\left[\theta_0\left|z_1\right.\right]\times 100 + P''\left[\theta_1\left|z_1\right.\right]\times 0\}$$

$$= \min\{0.25\times 0 + 0.75\times 400,\ 0.25\times 100 + 0.75\times 0\}$$

$$= 0.25\times 100 + 0.75\times 0 = 25$$

Pre-posterior Analysis

The minimum expected costs based on pre-posterior decision analysis – not including costs of experiments

$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''[u|z_i] = 28 \times 0.45 + 25 \times 0.28 + 78 \times 0.27 = 40.66$$

Allowable costs for the experiment

$$E'[u] - E[u] = 70.00 - 40.66 = 29.34$$

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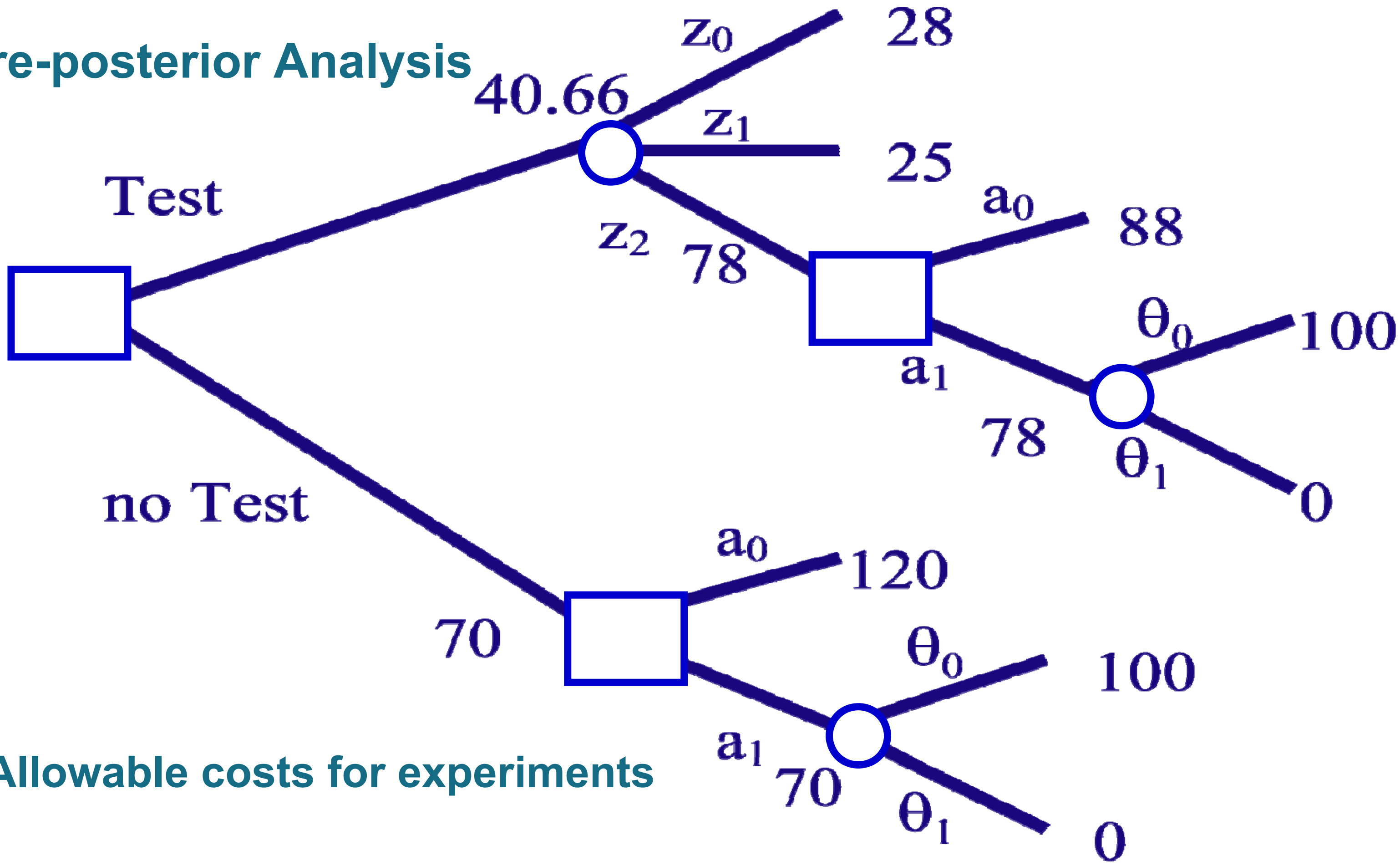
Pre-posterior Analysis



Allowable costs for experiments

$$E'[u] - E[u] = 70.00 - 40.66 = 29.34$$