Risk and Safety

in

Engineering

Prof. Dr. Michael Havbro Faber

Swiss Federal Institute of Technology

ETH Zurich, Switzerland

Exercise 1

Consider a steel rod. The rod will fail if the applied load on the rod exceeds the resistance of the rod.

The resistance *R* of the rod is a Normal distributed random variable with parameters:

$$\mu_R = 350\ MPa \text{ and } \sigma_R = 35\ MPa$$

The applied load *S* on the rod is deterministic and equal to 200 MPa

➡ **We need to estimate the probability of failure of the rod**

Exercise 1

1. Safety margin $M = R - S$

2. In "Symbolic Expressions" write: $\mathrm{FLIM(1)}\left\{\mathrm{M}\right\} = R - S$

3. In "Stochastic Model" provide the name, distribution and distribution parameters of *R* and *S*

4. In "Job → Settings → Computation Options" choose FORM for the method of integration

5. Save the file with name "Exercise_1"

6. Run the reliability analysis

Exercise 2

Consider Exercise 1 again:

➡ **Carry out a parametric study for the loading stress *S***

- values ranging between 100 to 300 MPa with an interval of 10 MPa

- **Go to "Model → Parameter Study" and input the above information under lower and upper bound**
- **Proceed as before and save the file as "Exercise 2"**
- **Run the reliability analysis**

Exercise 3

Consider the same steel rod

The load *S* on the rod is now also Normal distributed with parameters:

$$\mu_S = 200\ MPa \text{ and } \sigma_S = 40\ MPa$$

➡ **a. Estimate again the probability of failure of the rod using FORM**

➡ **b. Use Monte Carlo Simulation to do the above estimation**

Exercise 3a

1. **Safety margin** $M = R - S$

2. **In "Symbolic Expressions" write:** $\mathrm{FLIM(1)}\{\mathrm{M}\} = R - S$

3. **In "Stochastic Model" provide the name, distribution and distribution parameters of *R* and *S* (both Normal distributed)**

$$\mu_R = 350\ MPa \text{ and } \sigma_R = 35\ MPa$$

$$\mu_S = 200\ MPa \text{ and } \sigma_S = 40\ MPa$$

Exercise 3a

1. Safety margin $M = R - S$

2. In "Symbolic Expressions" write: $\mathrm{FLIM(1)}\left\{\mathrm{M}\right\} = R - S$

3. In "Stochastic Model" provide the name, distribution and distribution parameters of *R* and *S* (both Normal distributed)

   $\mu_R = 350\ MPa$ and $\sigma_R = 35\ MPa$ $\mu_S = 200\ MPa$ and $\sigma_S = 40\ MPa$

4. In "Job → Settings → Computation Options" choose FORM for the method of integration

5. Save the file with name "Exercise_3a"

6. Run the reliability analysis

Exercise 3b

1. Safety margin $M = R - S$

2. In "Symbolic Expressions" write: $\mathrm{FLIM(1)}\{\mathrm{M}\} = R - S$

3. In "Stochastic Model" provide the name, distribution and distribution parameters of *R* and *S* (both Normal distributed)

   $\mu_R = 350\ MPa$ and $\sigma_R = 35\ MPa$ $\mu_S = 200\ MPa$ and $\sigma_S = 40\ MPa$

4. In "Job → Settings → Computation Options" choose Crude Monte Carlo Sampling for the method of integration and provide the Number of Samples

5. Save the file with name "Exercise_3b"

6. Run the reliability analysis

Exercise 4

Consider again a steel rod

The resistance *R* of the rod is given by: $R = A \cdot f_y$

Where *A* is the cross-sectional area and f_y *is the steel yield stress*

- ***A* is constant :** $A = 10\ mm^2$
- **f_y is Normal distributed with parameters:** $\mu_{f_y} = 350\ MPa$ and $\sigma_{f_y} = 35\ MPa$

The load *S* is Normal distributed with parameters: $\mu_S = 1500\ N$ and $\sigma_S = 300\ N$

Estimate the probability of failure of the rod using FORM carrying a parametric analysis for the cross sectional area of the rod (from 10 mm^2 to 20 mm^2, with step 2 mm^2)

Exercise 4

1. Safety margin $M = R - S$

2. In "Symbolic Expressions" write: $\text{DEFFUNC(1)=}A*fy$

   $\text{FLIM(1)}\{\text{M}\} = \text{FUNC(1)} - S$

3. In "Stochastic Model" provide the name, distribution and distribution parameters of all the variables.

4. In "Job → Settings → Computation Options" choose FORM for the method of integration.

5. Save the file with name "Exercise_4"

6. Run the reliability analysis

Exercise 5

Consider Exercise 4 but:

the steel yield stress f_y

- **is Normal distributed with parameters:** μ_{f_y} and $\sigma_{\mu_{f_y}} = 35\ MPa$

  where μ_{f_y} is Normal distributed: $\mu_{\mu_{f_y}} = 350\ MPa$ and $\sigma_{\mu_{f_y}} = 10\ MPa$

Remember that:

The resistance *R* of the rod is given by: $R = A \cdot f_y$

Where *A* is the cross-sectional area and *f_y is the steel yield stress*

_A_ is constant : $A = 10\ mm^2$

The load *S* is normal distributed with parameters: $\mu_S = 1500\ N$ and $\sigma_S = 300\ N$

Estimate the probability of failure of the rod using FORM

Exercise 5

1. Safety margin $M = A * f_y - S$

2. In "Symbolic Expressions" write: $\text{FLIM(1)}\{\text{M}\} = A * f_y - S$

   RF07(1)=mfy

3. In "Stochastic Model" provide the name, distribution and distribution parameters of all the variables. Note how f_y is defined

4. In "Job → Settings → Computation Options" choose FORM for the method of integration

5. Save the file with name "Exercise_5"

6. Run the reliability analysis

Exercise 6

Corrosion of reinforcement in reinforced concrete structures

The time to corrosion initiation is given by:

$$T_{I_{\mathrm{mod}}} = \frac{d^2}{4D}\left(erf^{-1}\left(1-\frac{C_{CR}}{C_S}\right)\right)^{-2}$$

$$erf(y) = \frac{2}{\sqrt{\pi}}\int_0^y e^{-t^2}\,dt$$

$$erf^{-1}(y) = 1 - erf(y) = \frac{2}{\sqrt{\pi}}\int_y^\infty e^{-t^2}\,dt$$

where:

d : the concrete cover depth to the reinforcement

D : the chloride diffusion coefficient

C_{CR} : the critical chloride concentration which when exceeded at the level of the reinforcement will lead to the initiation of corrosion

C_S : the chloride concentration at the surface of the concrete

$erf^{-1}(.)$:inverse error function expressed as: $erf^{-1}(x) = \dfrac{\Phi^{-1}\left(\dfrac{x}{2}+\dfrac{1}{2}\right)}{\sqrt{2}}$

Exercise 6

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$$erf^{-1}(y) = 1 - erf(y) = \frac{2}{\sqrt{\pi}}\int_y^\infty e^{-t^2}\,dt$$

Model uncertainty Ξ : $T_I = \Xi T_{I_{\text{mod}}}$

Statistical characteristics:

Description	Representation	Units	Distribution	μ	σ
Cover thickness	d	mm	Log-normal	55	11
Diffusion coefficient	D	mm^2 / yr	Log-normal	40	10
Surface concentration	C_S	Water % of concrete	Log-normal	0.4	0.08
Critical concentration	C_{CR}	Water % of concrete	Log-normal	0.15	0.05
Propagation time	T_P	years	Log-normal	7.5	1.88
Model uncertainty	Ξ	-	Log-normal	1	0.05

Exercise 6

Corrosion of reinforcement in reinforced concrete structures

a. **Estimate the probability of corrosion initiation after 50 years**

b. **Estimate the profile of corrosion initiation from year 1 to year 100**

Exercise 6

a. Estimate the probability of corrosion initiation after *t= 50 years*

1. ***Formulate safety margin M_CI:*** **M_CI=Xi*Ti_mod-t**

Exercise 6

a. Estimate the probability of corrosion initiation after *t= 50 years*

1. ***Formulate safety margin M_CI*** **M_CI=Xi*Ti_mod-t**

2. ***Write the expression in Comrel and declare the distributions and their statistics***

$$erf^{-1}(x) = \frac{\Phi^{-1}\left(\frac{x}{2}+\frac{1}{2}\right)}{\sqrt{2}}$$

$$T_{I_{\text{mod}}} = \frac{d^2}{4D}\left(erf^{-1}\left(\left(1-\frac{C_{CR}}{C_S}\right)\right)\right)^{-2}$$

DEFFUNC(1)(X)=IF(X<0,0,ICPHI(X/2+0.5)/SQRT(2))

Exercise 6

a. Estimate the probability of corrosion initiation after *t= 50 years*

1. ***Formulate safety margin M_CI***

2. ***Write the expression in Comrel and declare the distributions and their statistics***

DEFFUNC(1)(X)=IF(X<0,0,ICPHI(X/2+0.5)/SQRT(2))

FLIM(1){CI}=XI*((FUNC(1)(1-Ccr/Cs))^(-2)*d^2/(4*D))-t

3. Save as Exercise_6a

4. Run the reliability analysis

$$erf^{-1}(x) = \frac{\Phi^{-1}\left(\frac{x}{2}+\frac{1}{2}\right)}{\sqrt{2}}$$

M_CI=Xi*Ti_mod-t

$$T_{I_{\text{mod}}} = \frac{d^2}{4D}\left(erf^{-1}\left(1-\frac{C_{CR}}{C_S}\right)\right)^{-2}$$

Exercise 6

b. Estimate the profile of corrosion initiation from year 1 to year 100

1. ***Keep Exercise_6a***
2. ***In "Model → Parameter Study" provide the lower bound and upper bound equal to 1 and 100 years for t respectively***
3. ***Save as Exercise_6b***
4. ***Run the reliability analysis***