

Risk and Safety in Engineering

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Contents of Today's Lecture

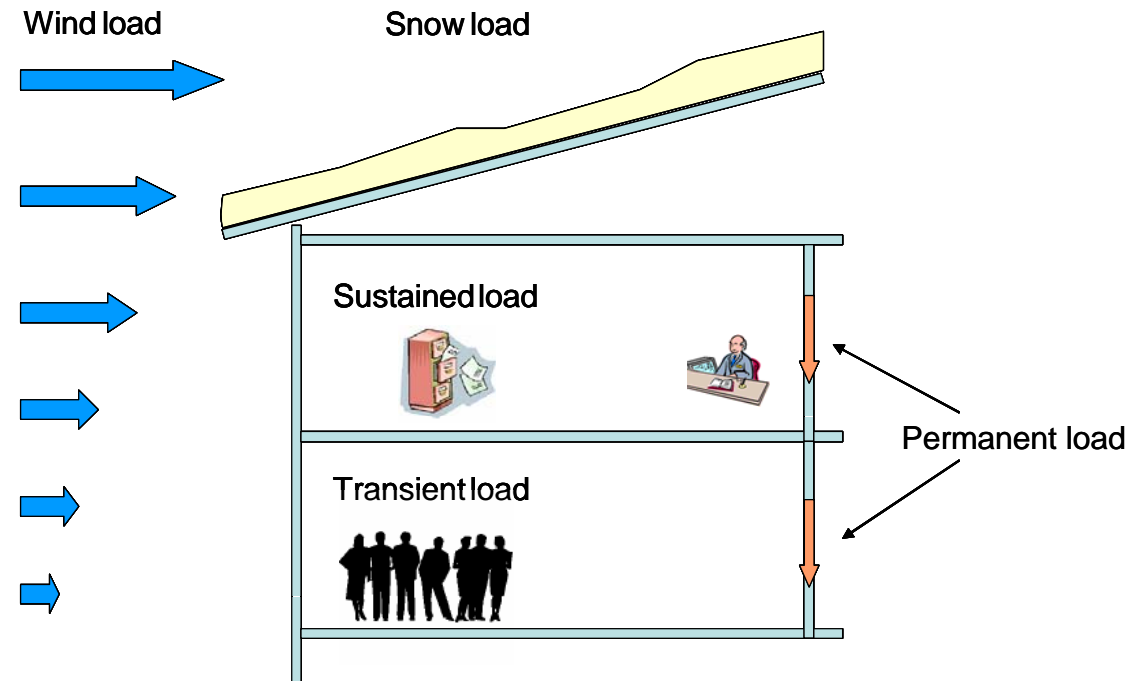
- Probabilistic Modelling of Loads
- Probabilistic Modelling of Resistances
- Probabilistic Modelling of Model Uncertainties
- The Joint Committee on Structural Safety Probabilistic Model Code

Probabilistic Modelling of Loads

• Loads on Structures

Loads are uncertain due to:

- Random variations in space and time
- Model uncertainties
- Statistical uncertainties

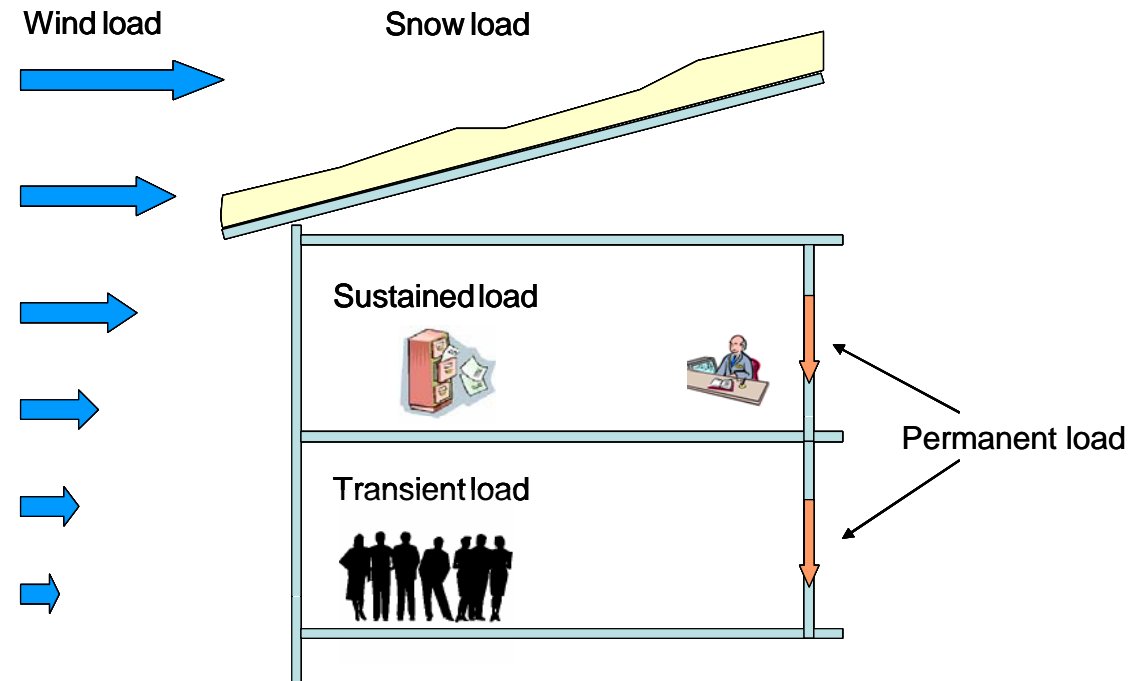


Probabilistic Modelling of Loads

• Loads on Structures

It is often useful to characterize loads as:

- Permanent or variable
- Fixed or free
- Static or dynamic



Probabilistic Modelling of Loads

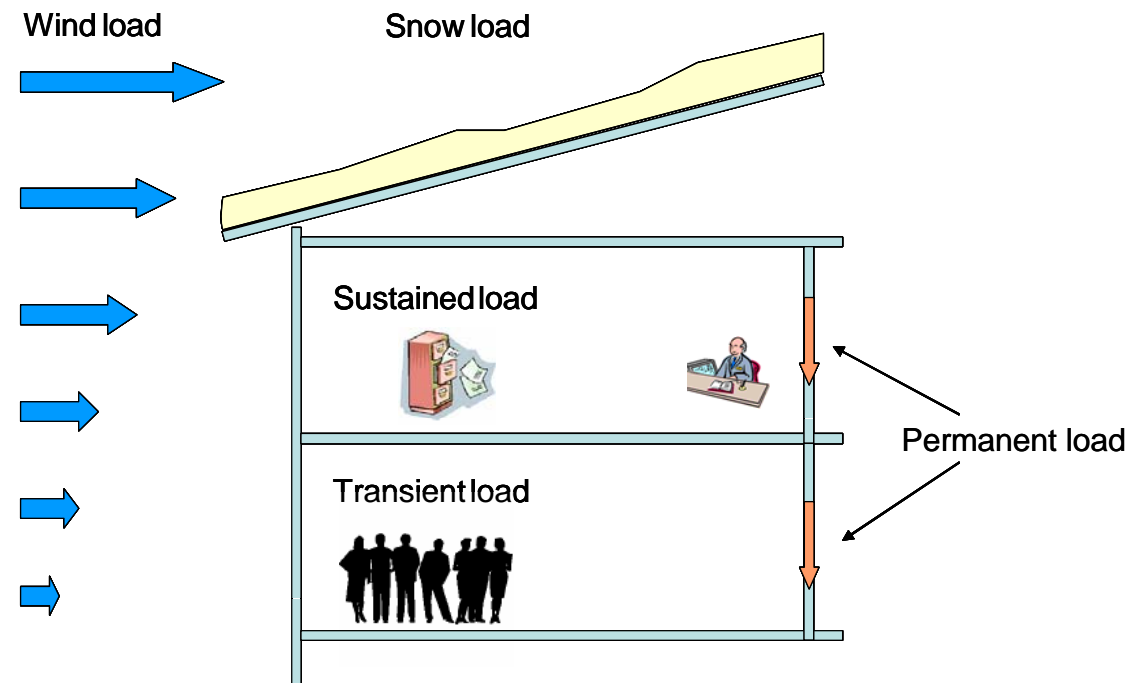
• Loads on Structures

The probabilistic modelling includes the following steps:

- specifying the definition of the random variables used to represent the uncertainties in the loading

- selecting a suitable distribution type to represent the random variable

- assigning the distribution parameters of the selected distribution.



Probabilistic Modelling of Loads

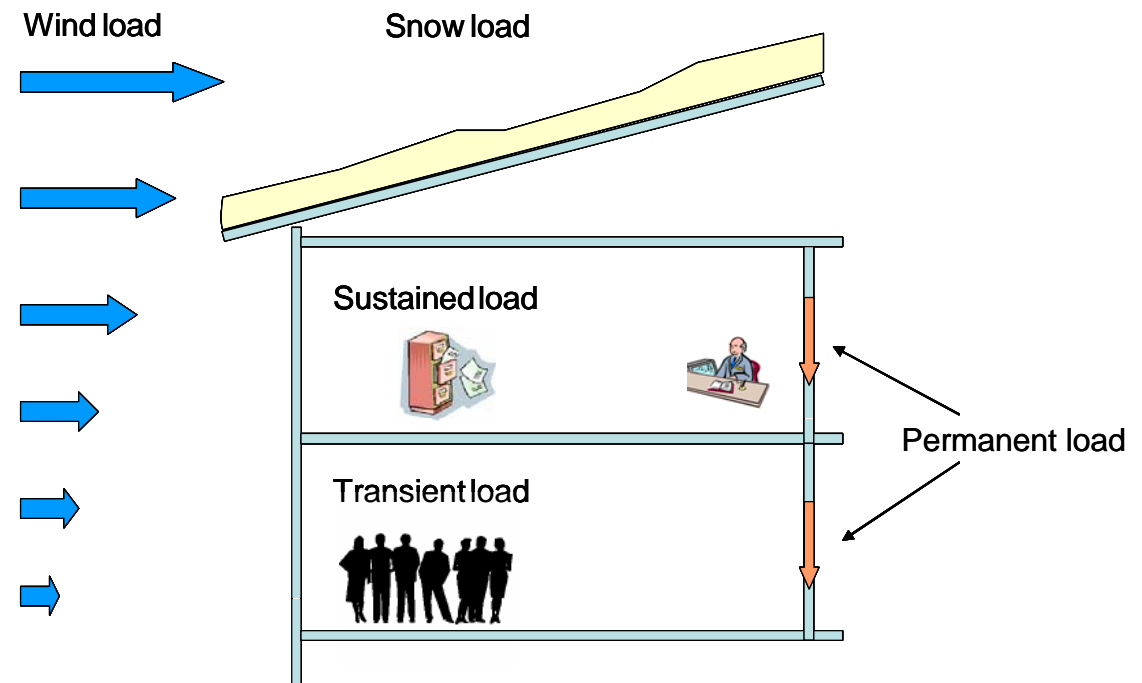
- Loads on Structures

Permanent loads:

$$G = \int_V \gamma dV$$

Density

Material	COV
Construction Steel	0.01
Concrete	0.04
Timber	
- sawn beam or strut	0.12
- laminated beam, planed	0.10



Probabilistic Modelling of Loads

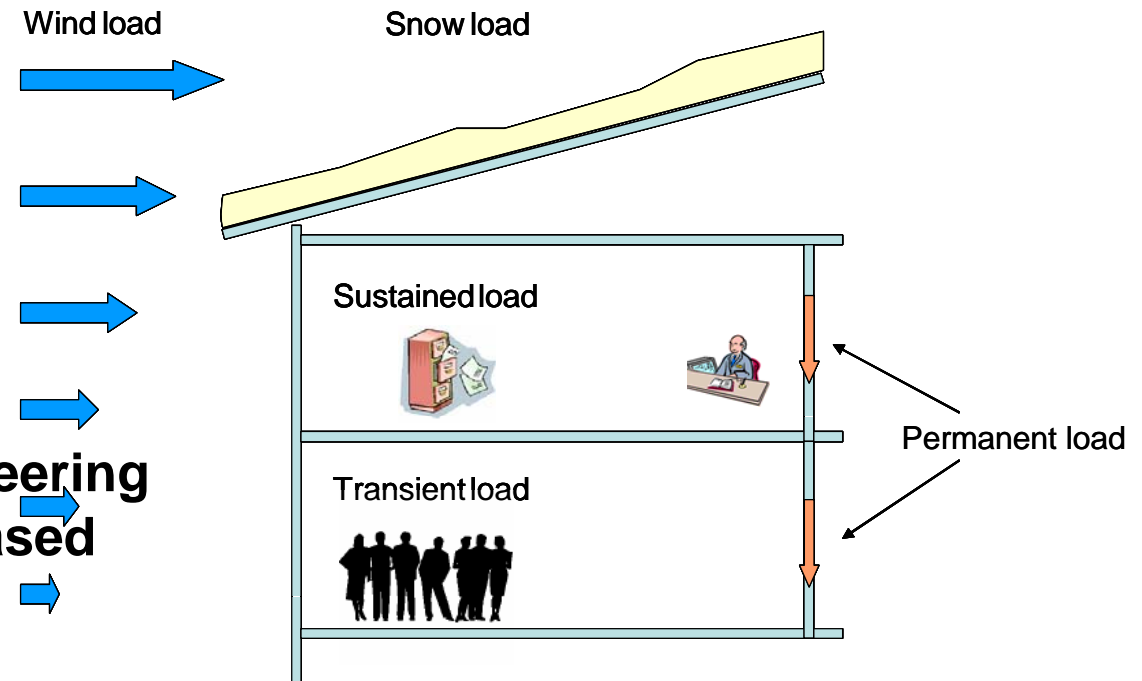
- Loads on Structures

Permanent loads:

$$G = \int_V \gamma dV$$

The mean value as assessed by engineering assessments has been found to be biased about 5% to the low side

Log-Normal and Normal distributions are good candidates to represent the uncertainty



Probabilistic Modelling of Loads

- **Loads on Structures**

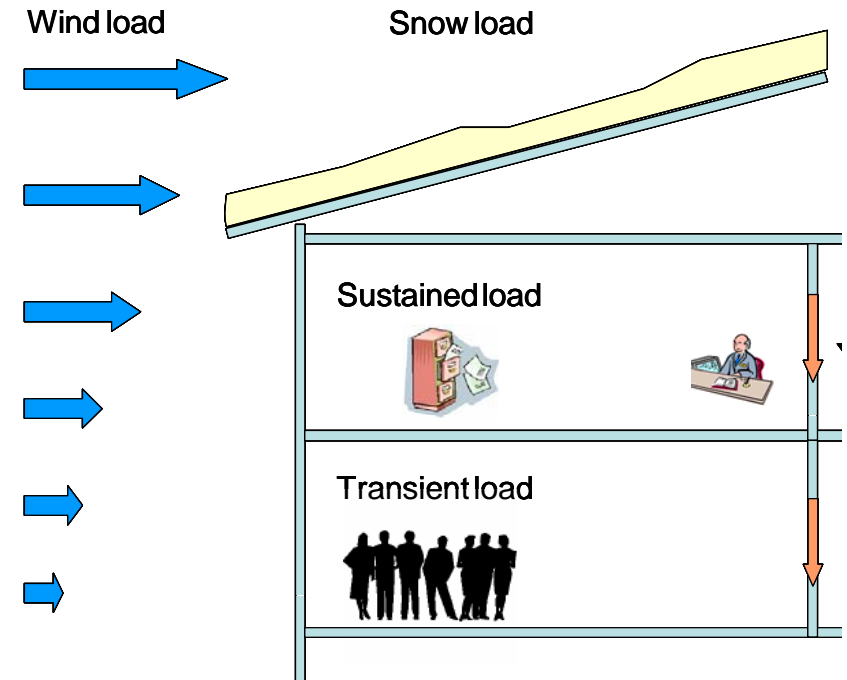
Live floor loads:

$$W(x, y) = m + V + U(x, y)$$

m is the overall mean value for a given use category

V is a zero mean random variable

$U(x,y)$ is a zero mean random field



Probabilistic Modelling of Loads

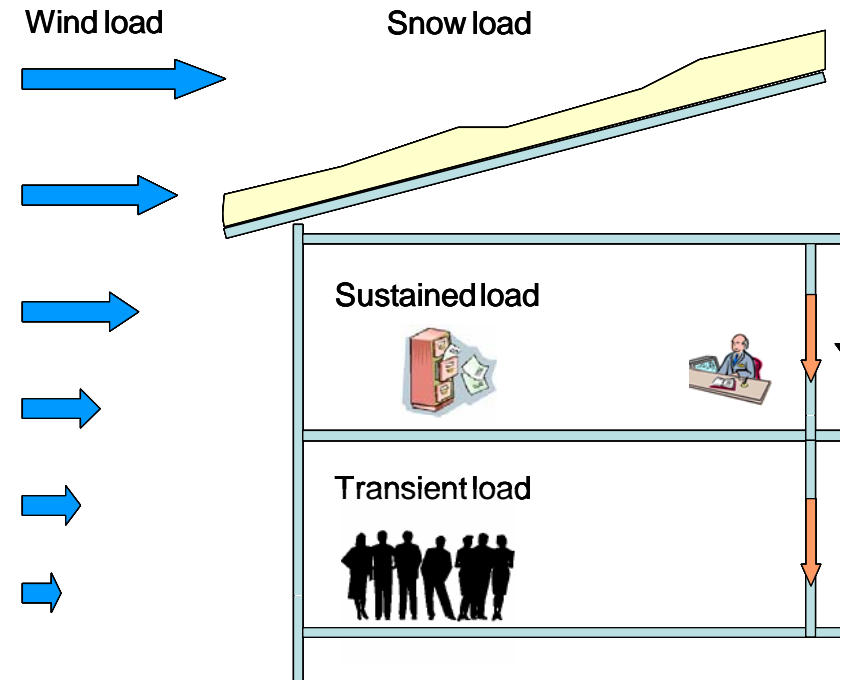
- Loads on Structures

Live floor loads:

$$W(x, y) = m + V + U(x, y)$$

The random load effect in linear systems due to the spatially distributed load $W(x, y)$ is represented by an equivalent uniformly distributed load Q_{equ}

$$S = \int_A W(x, y) i(x, y) dA = Q_{equ} \int_A i(x, y) dA$$



Probabilistic Modelling of Loads

- **Loads on Structures**

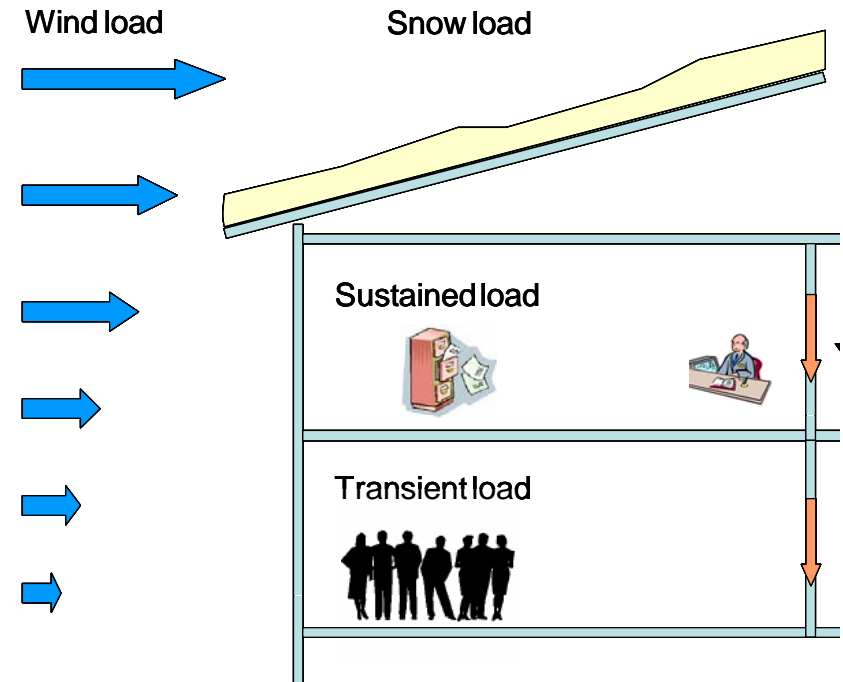
Live floor loads:

The mean value of the Q_{equ}

$$E[Q_{equ}] = m$$

The variance is

$$Var[Q_{equ}] = \frac{Var\left[\int_A W(x, y)i(x, y)dA\right]}{\left[\int_A i(x, y)dA\right]^2} = \frac{\sigma_V^2 + \sigma_U^2 \int_{A_1} \int_{A_2} i(x_1, y_1)i(x_2, y_2)\rho_{U(x_1, y_1), U(x_2, y_2)} dA_1 dA_2}{\left[\int_A i(x, y)dA\right]^2}$$



Probabilistic Modelling of Loads

- Loads on Structures

Live floor loads:

If the correlation radius ρ_0 is small then:

$$\text{Var}[Q_{equ}] = \sigma_V^2 + \sigma_U^2 \frac{\int i(x, y)^2 dA}{\left[\int_A i(x, y) dA \right]^2} = \sigma_V^2 + \sigma_U^2 \kappa_{red} \quad \kappa_{red} = \frac{A_0}{A} \kappa(A)$$

Probabilistic Modelling of Loads

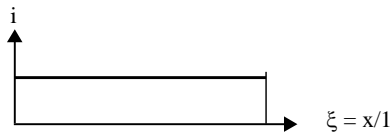
- Loads on Structures

Live floor loads:

In principle the variance reduction factor may be determined from

$$\kappa_{red} = \frac{A_0}{A} \kappa(A)$$

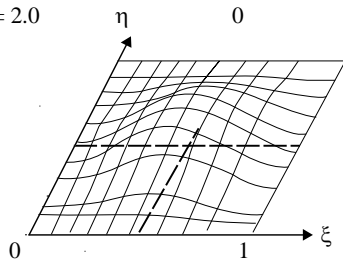
$\kappa = 1.0$



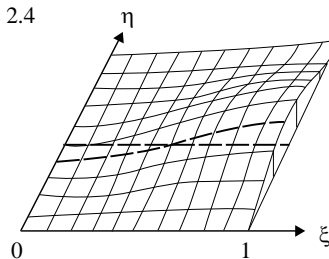
$\kappa = 1.4$



$\kappa = 2.0$



$\kappa = 2.4$



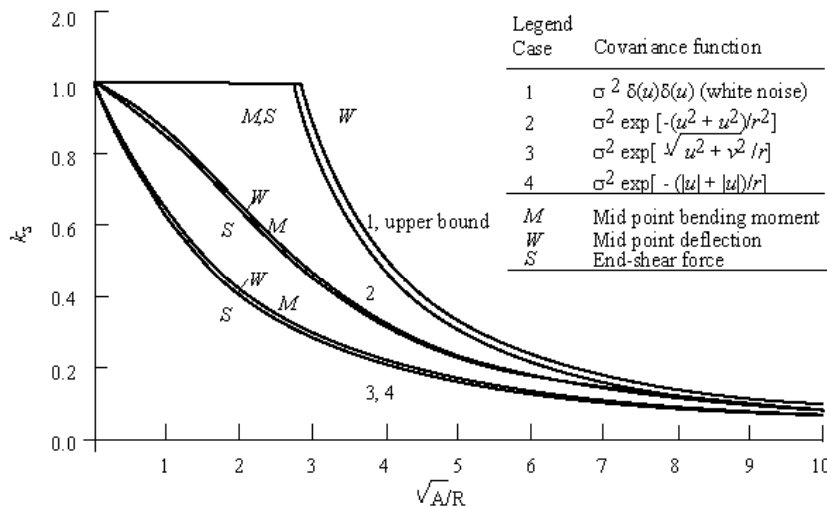
Probabilistic Modelling of Loads

- Loads on Structures

Live floor loads:

In principle the variance reduction factor may be determined from

$$K_{red} = \frac{A_0}{A} K(A)$$



But for most practical purposes it can be assumed to be equal to zero

$$E[Q_{equ}] = m_q$$

$$Var[Q_{equ}] = \sigma_V^2$$

Probabilistic Modelling of Loads

- **Loads on Structures**

Live floor loads (sustained loads):

The maximum EUDL load has been found to be Gamma distributed but is sometimes modelled by a Type I extreme value distribution.

Assuming that changes of the sustained load follow a Poisson process with rate λ the probability distribution function of the maximum load in a time reference T may be determined from

$$F_{Q,\max}(x) = \exp(-\lambda T(1 - F_Q(x)))$$

Probabilistic Modelling of Loads

- **Loads on Structures**

Live floor loads (transient loads):

The maximum EUDL load has been found to be Exponential distributed.

$$E[P_{equ}] = m_p \quad Var[P_{equ}] = \sigma_V^2$$

Assuming that changes of the sustained load follow a Poisson process with rate ν the probability distribution function of the maximum load in a time reference T may be determined from

$$F_{p,\max} = \exp\left(-\nu T \left(1 - F_p(x)\right)\right)$$

Probabilistic Modelling of Loads

- Loads on Structures

Live floor loads (sustained/transient loads):

		Sustained Load				Transient Load			
Category	A_0 [m ²]	m_q [kN/m ²]	σ_V [kN/m ²]	σ_U [kN/m ²]	$1/\lambda$ [y]	m_p [kN/m ²]	σ_V [kN/m ²]	$1/\nu$ [y]	d_p [d]
Office	20	0.5	0.3	0.6	5	0.2	0.4	0.3	1 - 3
Lobby	20	0.2	0.15	0.3	10	0.4	0.6	1.0	1 - 3
Residence	20	0.3	0.15	0.3	7	0.3	0.4	1.0	1 - 3
Hotel guest room	20	0.3	0.05	0.1	10	0.2	0.4	0.1	1 - 3
Patient room	20	0.4	0.3	0.6	5 - 10	0.2	0.4	1.0	1 - 3
Laboratory	20	0.7	0.4	0.8	5 - 10				
Libraries	20	1.7	0.5	1.0	>10				

Probabilistic Modelling of Loads

- **Loads on Structures**

The combined sustained and transient loads can be assessed as the maximum of

$$L_1 = L_{Q,\max} + L_p$$

$$L_2 = L_Q + L_{p,\max}$$

and modelled as a type I extreme value distribution

Probabilistic Modelling of Loads

- Loads on Structures

Wind loads

$$w = c_a c_g c_r \bar{Q}_{ref} = c_a c_e \bar{Q}_{ref}$$

Smaller rigid structures

$$w = c_d c_a c_e \bar{Q}_{ref}$$

Taller flexible structures

$$Q = \frac{1}{2} \rho U^2$$

c_r : roughness factor

c_g : gust factor

c_a : aero-dynamic shape factor

c_d : dynamic factor

c_e : exposure factor

ρ : 1.25 kg/m³

Q_{ref} : 10 min mean U

Probabilistic Modelling of Loads

- Loads on Structures

Wind loads

c_r : roughness factor

c_g : gust factor

c_a : aero-dynamic shape factor

c_d : dynamic factor

c_e : exposure factor

ρ : 1.25 kg/m³

Variable	Type	V
Q_{ref}	Gumbel	0.20 - 0.30
c_r	Lognormal	0.10 - 0.20
c_a - coefficient pressure coefficient force	Lognormal	0.10 - 0.30
	Lognormal	0.10 - 0.15
c_g	Lognormal	0.10 - 0.15
c_d	Lognormal	0.10 - 0.20

Wind load can be assumed to be Log-Normal distributed

$$V_w^2 \cong V_{c_d}^2 + V_{c_a}^2 + V_{c_r}^2 + V_{Q_{ref}}^2$$

Dynamic sensitive

$$V_w^2 \cong V_{c_a}^2 + V_{c_r}^2 + V_{Q_{ref}}^2$$

Rigid

Probabilistic Modelling of Loads

- Loads on Structures

Snow loads

$$S_r = S_g \cdot r \cdot k^{\frac{h}{h_r}}$$

$$S_g = d \cdot \gamma(d)$$

S_r : Snow load on roof

S_g : Snow load on ground

r : ground to roof conversion factor

k : location factor (1.25 coastal, 1.5 inland)

h : altitude in meters

h_r : reference altitude (300 meters)

ρ : 1.25 kg/m³

Typically the snow load is modelled by a Gamma or a Gumbel distribution

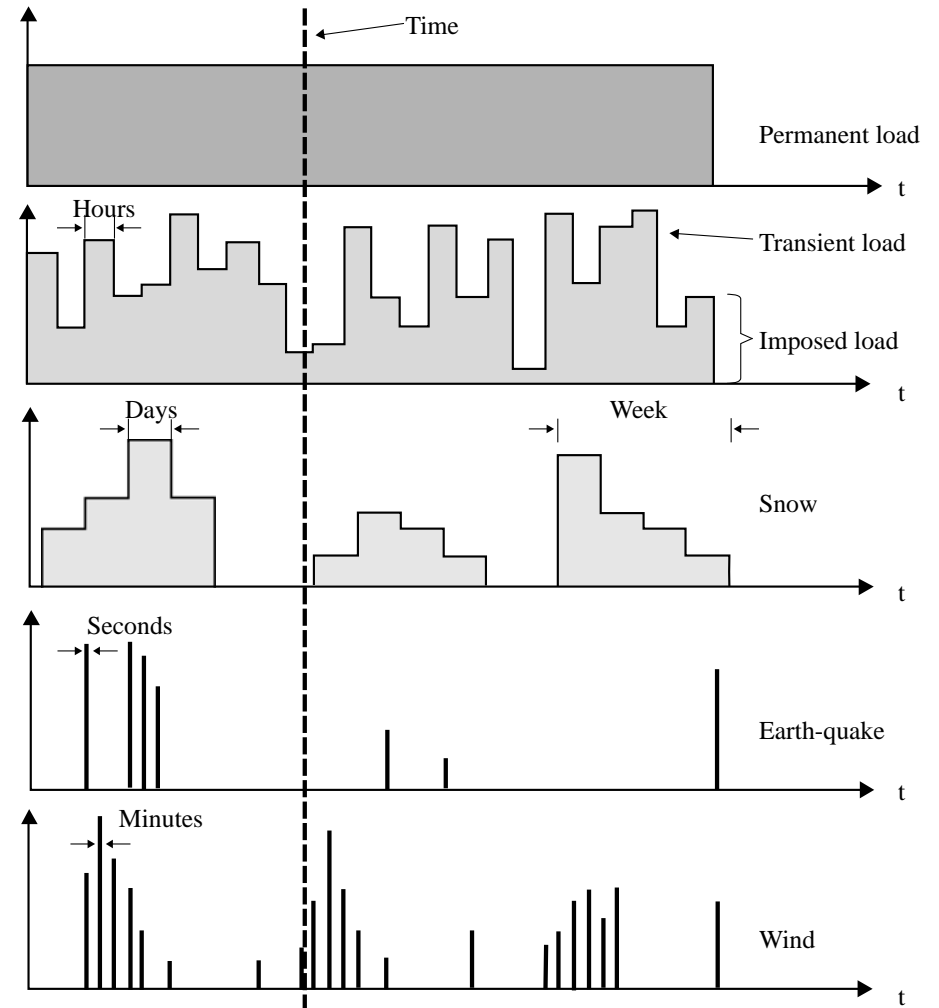
Probabilistic Modelling of Loads

- **Loads on Structures**

Combination of loads

We are interested in the maximum of a sum of load effects from different loads

$$X_{max}(T) = \max_T \{X_1(t) + X_2(t) + \dots + X_n(t)\}$$



Probabilistic Modelling of Loads

- **Loads on Structures**

Combination of loads

Turkstra's load combination rule

We take the max of the following combinations

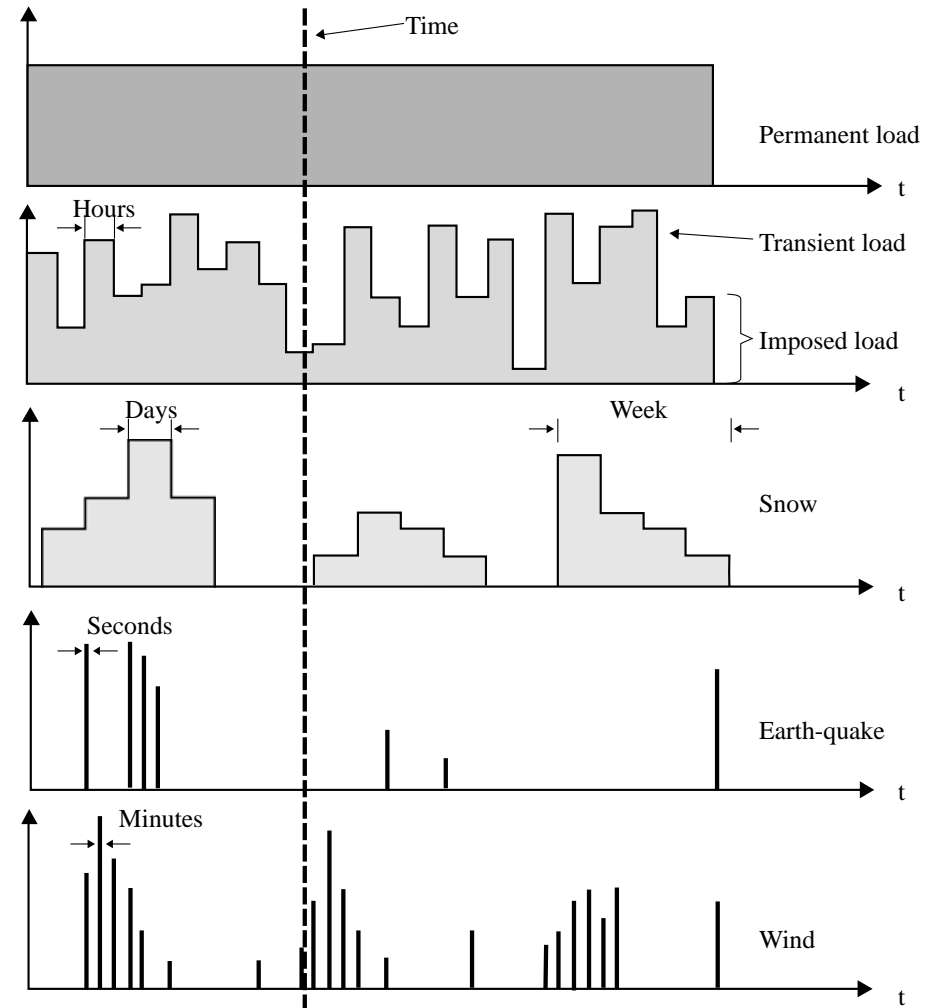
$$Z_1 = \max_T \{X_1(t)\} + X_2(t^*) + X_3(t^*) + \dots + X_n(t^*)$$

$$Z_2 = X_1(t^*) + \max_T \{X_2(t)\} + X_3(t^*) + \dots + X_n(t^*)$$

⋮

$$Z_n = X_1(t^*) + X_2(t^*) + X_3(t^*) + \dots + \max_T \{X_n(t)\}$$

$$X_{max}(T) \approx \max_i \{Z_i\}$$



Probabilistic Modelling of Loads

- Loads on Structures

Combination of loads

Ferry Borges-Castanheta's load combination rule

$$X_{max}(T) \approx \max_i \{Z_i\}$$

Load combination	Repetition numbers		
	Load 1	Load 2	Load 3
1	n_1	n_2 / n_1	n_3 / n_1
2	1	n_2	n_3 / n_2
3	1	1	n_3
4	n_1	1	n_3 / n_1

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

In structural engineering resistances include the following uncertainties

- Geometrical uncertainties
- Material characteristics
- Model uncertainties



Random variation in time and space

The steps in the modelling process are:

- define the random variables used to represent the uncertainties in the resistances
- select a suitable distribution type to represent the random variable
- to assign the distribution parameters of the selected distribution.

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Concrete compressive strength

$$f_c = \alpha(t, \tau) f_{co}^\lambda$$

f_{co} : 28 day compressive strength
 $\alpha(t, \tau)$: spatial stress and loading time function
 λ : conversion factor between in-situ concrete strength and cylinder compressive strength

The concrete compressive strength can be assumed to be Log-Normal distributed with a coefficient of variation equal to 15%

Probabilistic Modelling of Resistances

- **Uncertainties of resistances**

Reinforcement steel yield strength

$$f_s = X_1 + X_2 + X_3$$

X_1 normal distributed random variable representing the variation in the mean of different mills.

X_2 normal distributed zero mean random variable, which takes into account the variation between batches

X_3 normal distributed zero mean random variable, which takes into account the variation within a batch.

Probabilistic Modelling of Resistances

- Uncertainties of resistances

Reinforcement steel yield strength

Variable	Type	$E[X]$	$\sigma_x [MPa]$	V_x
X_1	Normal	μ	19	-
X_2	Normal	0	22	-
X_3	Normal	0	8	-
A	-	A_{nom}	-	0.02

μ : nominal steel grade + two standard deviations of X_1

Yield stress depends on diameter of reinforcement bars

$$\mu(d) = \mu(0.87 + 0.13 \exp(-0.08d))^{-1}$$

Probabilistic Modelling of Resistances

- Uncertainties of resistances

Structural steel yield strength

Description	Variable	Type	$E[X]$	V_X
Yield stress	f_y	Lognormal	$f_{y\ sp} \alpha e^{-uV_{f_y}} - C$	0.07
ultimate stress	f_u	Lognormal	$B E[f_u]$	0.04
modulus of elasticity	E	Lognormal	E_{sp}	0.03
Poisson's ratio	ν	Lognormal	ν_{sp}	0.03
ultimate strain	ε_u	Lognormal	$\varepsilon_{u\ sp}$	0.06

Distribution characteristics

	f_y	f_u	E	ν	ε_u
f_y	1	0.75	0	0	-0.45
f_u		1	0	0	-0.60
E			1	0	0
ν	Symmetry			1	0
ε_u					1

Dependencies

Probabilistic Modelling of Resistances

- **Model uncertainties**

Model uncertainties relate engineering model results with actual structural behaviour

$$X = \varepsilon \cdot X_{\text{mod}}$$

X : true value
 ε : model uncertainty
 X_{mod} : model value

$$\xi = \frac{x_{\text{mod}}}{x_{\text{exp}}}$$

x_{exp} : experimentally obtained value

Probabilistic Modelling of Resistances

- **Model uncertainties**

Model uncertainties may be introduced in different ways:

$$Y = \mathcal{E} f(\mathbf{X})$$

$$Y = \mathcal{E} + f(\mathbf{X})$$

$$Y = f(\mathcal{E}_1 X_1, \mathcal{E}_2 X_2, \dots, \mathcal{E}_n X_n)$$

Y structural performance

$f(\cdot)$ model function

\mathcal{E} random variable representing the model uncertainty

X_i basic variables

\mathbf{X} vector of basic random variables

Probabilistic Modelling of Resistances

- The JCSS Probabilistic Model Code (PMC)
http://www.jcss.ethz.ch/publications/publications_pmc.html

Part I : Basis of design

Part II: Load models

Part III: Resistance models

Part IV: Examples

Probabilistic Modelling of Resistances

- The JCSS PMC – Load Models

2.00	<u>GENERAL PRINCIPLES</u>	05.2001
2.01	<u>SELF WEIGHT</u>	06.2001
2.02	<u>LIVE LOAD</u>	05.2001
2.06	<u>LOAD IN CAR PARKS</u>	05.2001
2.12	<u>SNOW LOAD</u>	05.2001
2.13	<u>WIND LOAD</u>	05.2001
2.15	<u>WAVE LOAD</u>	05.2006
2.17	<u>EARTHQUAKE</u>	09.2002
2.18	<u>IMPACT LOAD</u>	05.2001
2.20	<u>FIRE</u>	05.2001

Probabilistic Modelling of Resistances

- The JCSS PMC – Resistance models

3.00	<u>GENERAL PRINCIPLES</u>	03.2001
3.01	<u>CONCRETE</u>	05.2002
3.02	<u>STRUCTURAL STEEL</u>	03.2001
3.0*	<u>REINFORCING STEEL</u>	03.2001
3.04	<u>PRESTRESSING STEEL</u>	04.2005
3.05	<u>TIMBER</u>	05.2006
3.07	<u>SOIL PROPERTIES</u>	06.2002
3.09	<u>MODELUNCERTAINTIES</u>	03.2001
3.10	<u>DIMENSIONS</u>	03.2001
3.11	<u>EXCENTRICITIES</u>	03.2001