Risk and Safety

in

Civil, Environmental and Geomatic

Engineering

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Decision Analysis in Engineering

- **Introduction to Decision Theory**

    - **The problem**

    - **The decision tree**

    - **Prior decision analysis**

    - **Posterior decision analysis**

    - **Pre-posterior decision analysis**

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- **The basic engineering problem**

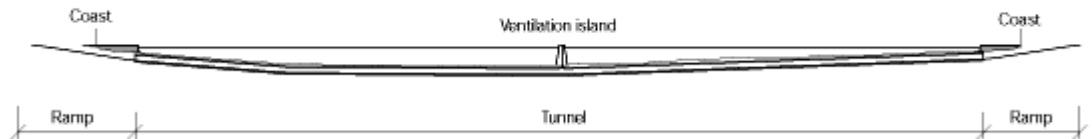Several solutions may be identified

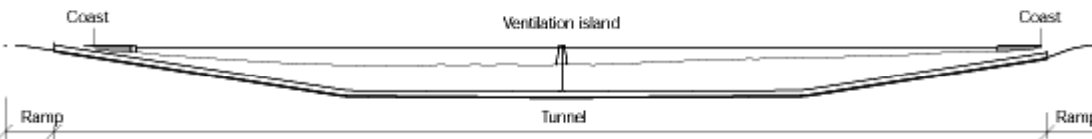The available information is uncertain
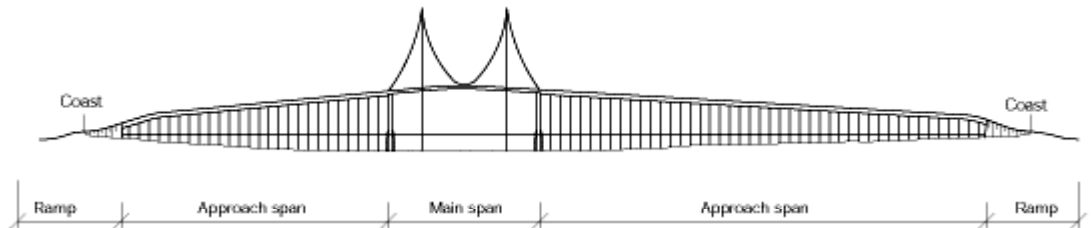
A decision must be made !
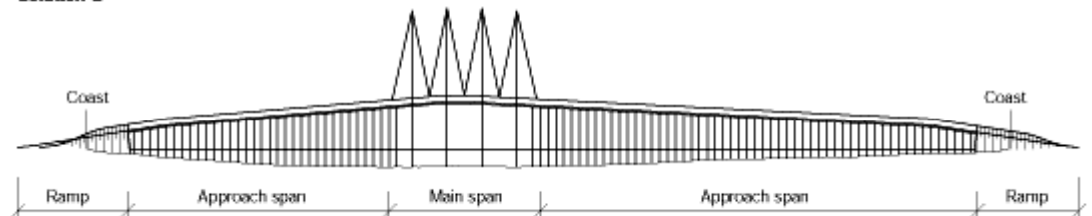

Solution B and F

Solution A and E

Solution D

Solution C

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Emperor Qianlong

Qing dynasty

Reign : 18 October 1735 – 8 February 1796

The Chinese knew long ago what the principles of decision making were!

ZHONG HE DIAN
(Hall of Central Harmony)

First constructed in 1420 during the Ming Dynasty, Zhong He Dian was destroyed and reconstructed several times over the centuries. The existing hall was constructed in 1627 during The Ming Dynasty. In the early Ming Dynasty, this hall was called Hua Gai Dian (Hall of Overwhelming Glory) but was renamed Zhong Ji Dian (Hall of Central Extremity) in 1562 and Zhong He Dian in 1645 during the Qing Dynasty. This square building has a single pyramid-shaped roof, with a gold plated bronze covering. The floor is paved with high-quality square clay bricks, commonly known as "golden bricks." A throne is placed in the center of the hall and a board hangs above the throne with an inscription written by Emperor Qianlong. The inscription reads: "Yun Zhi Jue Zhong," meaning "The Way of Heaven is profound and mysterious and the way of mankind is difficult. Only if we make a precise and unified plan and follow the doctrine of the mean, can we rule the country well."

This hall served as a resting place for the emperor on his way to attend an important ceremony or hold court. Officials kowtowed to the emperor here. The day before the emperor held a sacrificial ceremony he would read the prayer tablet aloud in this hall. Before offering sacrifices at the Altar of the God of Agriculture, the emperor examined ceremonial farm tools here. After the revision of the imperial pedigree, which was revised once every ten years, the emperor read the pedigree out loud and held a grand ceremony at the hall. The words "Zhong He" come from the *Book of Rites*, meaning "When we handle matters properly and harmoniously without leaning to either side, all things on earth will flourish."
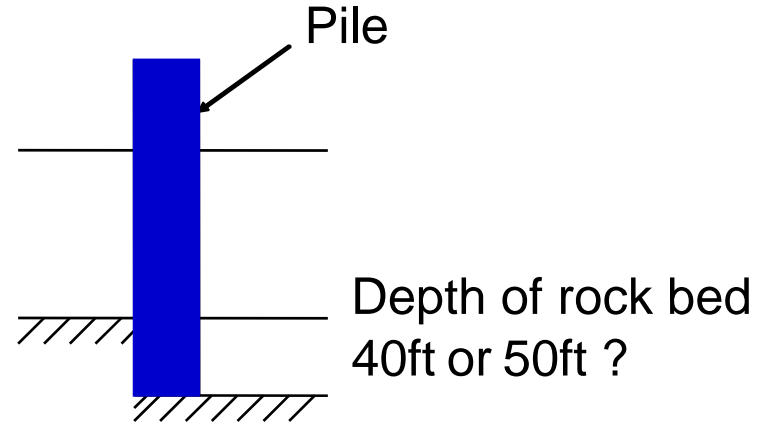
MADE POSSIBLE BY THE AMERICAN EXPRESS COMPANY.

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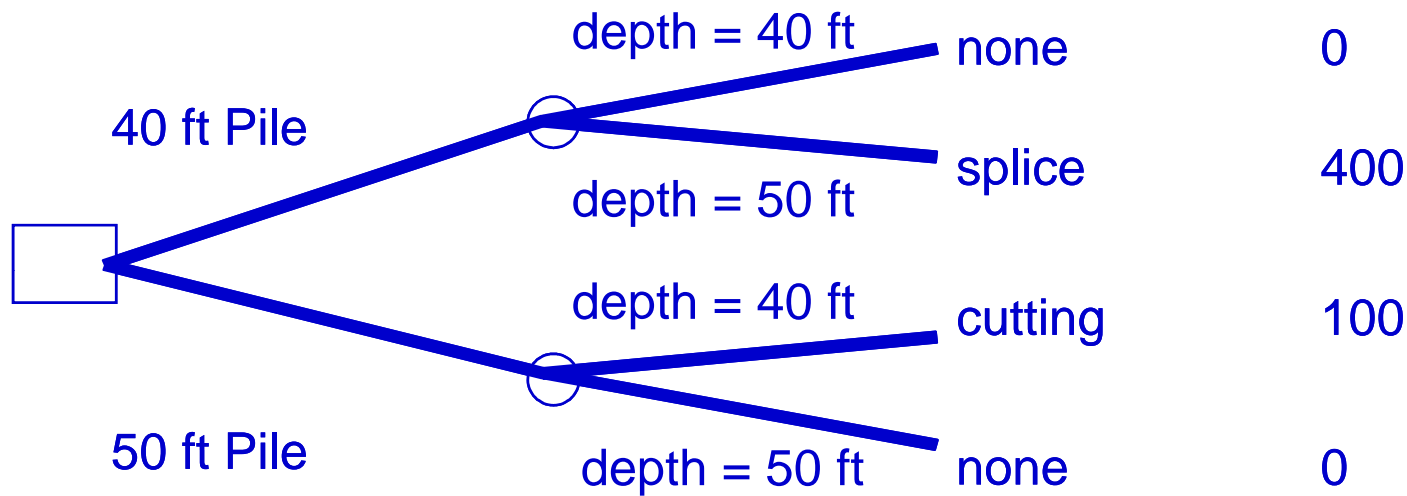
Approach

- **Formulation of the decision problem**
    - The decision maker and the preferences of the decision maker must be identified
    - Mapping of the decision process
    - All the possible decision alternatives must be identified
    - Identification of the contributing uncertainties

- **Identification of potential consequences and their utility (cost/benefit)**

- **Assessment of the probabilities of the consequences**

- **Comparison of the different decision alternatives based on their expected utilities**

- **Final decision making and reporting of the assumptions underlying the selected alternative**

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- **The decision tree**

Pile

Depth of rock bed
40ft or 50ft ?

Action alternatives	Outcome	Consequence	Utility(consequence)

40 ft Pile

depth = 40 ft — none — 0

depth = 50 ft — splice — 400

50 ft Pile

depth = 40 ft — cutting — 100

depth = 50 ft — none — 0

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Assignment of utility

- The assignment of utility must reflect the preferences of the decision maker

- Utility functions may be defined as linear functions in monetary unity

- It is important to include all monetary consequences in the utility function

$$u(a_i) = \sum_{j=1}^{n} p_j \cdot u(K_j)$$

$u(a_i)\ldots$ Utility (cost/benefit) associated with action a_i

$p_j \cdot u(K_j)\ldots$ Expected utility associated with consequence K_j

$p_j\ldots$ Probability of the occurence of the consequence K_j

$u(K_j)\ldots$ Utility associated with the consequence K_j

$K_j\ldots$ A potential consequence associated with the action a_i

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The different types of decision analysis

- **Prior**
- **Posterior**
- **Pre-posterior**

Illustrated on an example :

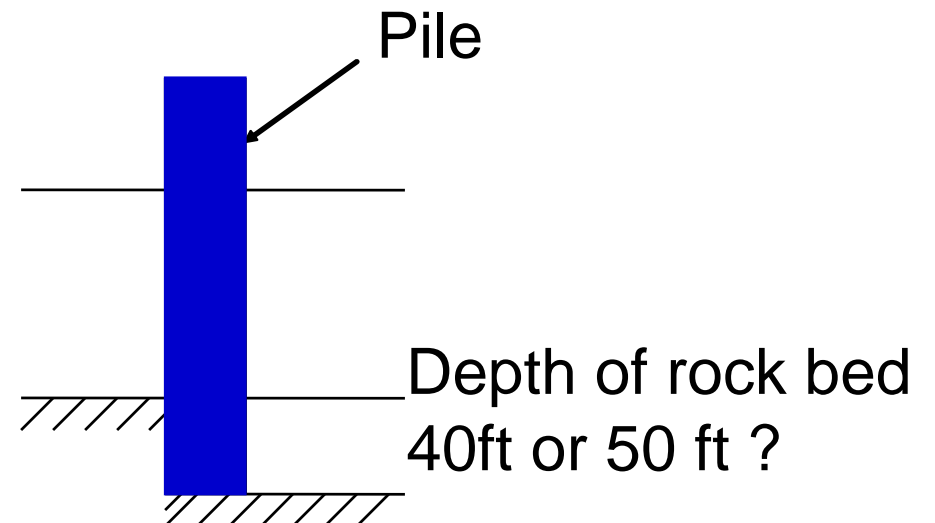**Question :** What pile length should be applied ?

Alternatives :
a_0 : Choose a 40 ft pile
a_1 : Choose a 50 ft pile

States of nature (depth to rock bed)
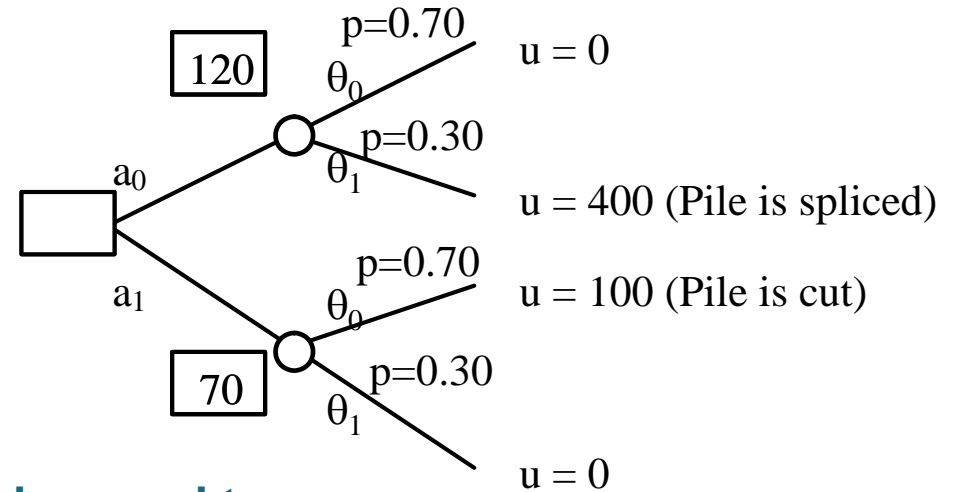0 : Rock bed in 40 ft
1 : Rock bed at 50 ft

Pile

Depth of rock bed
40ft or 50 ft ?

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Prior Analysis

P'[θ_0] = 0.70
P'[θ_1] = 0.30

The expected utility is calculated to be equal to
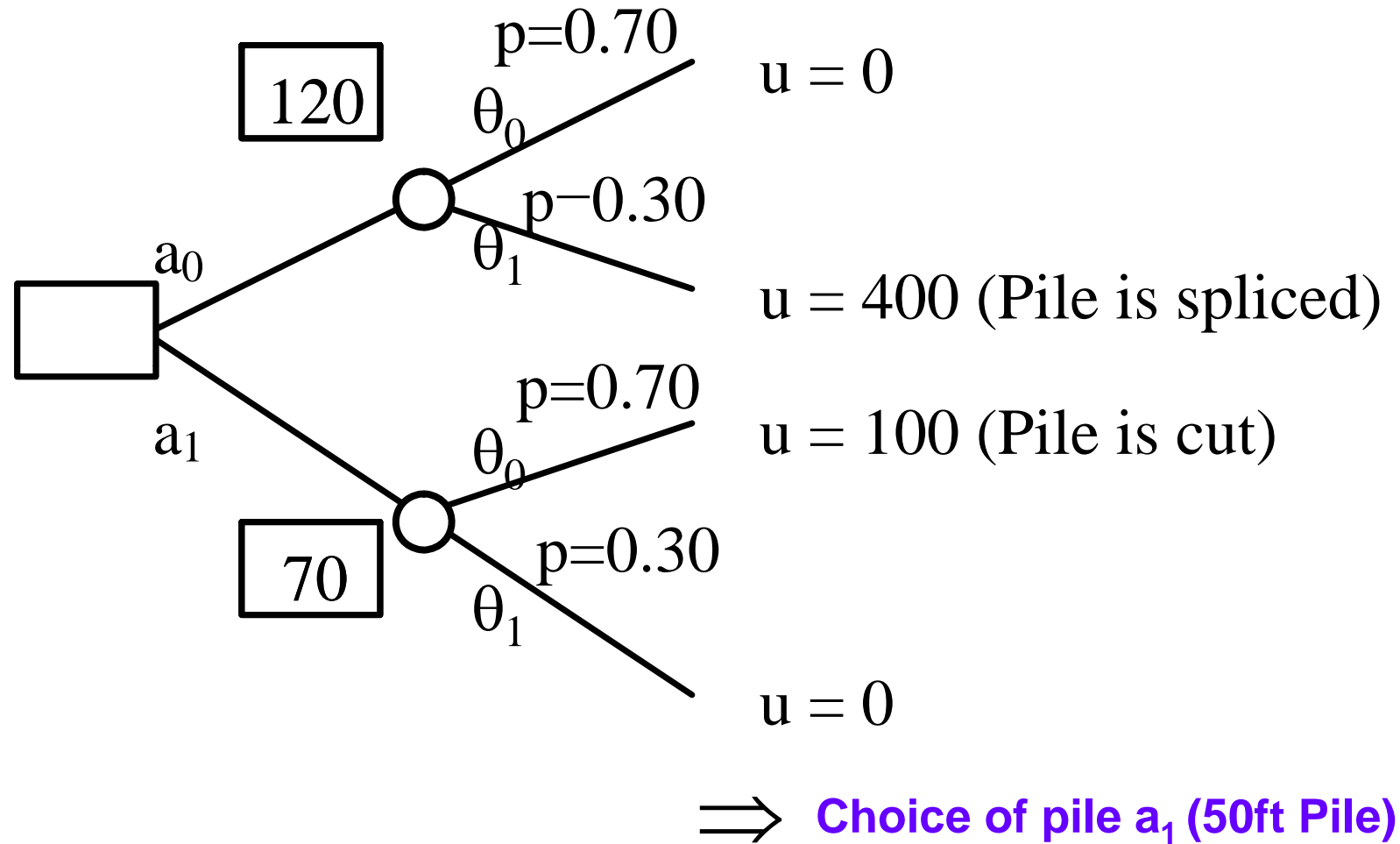
$$E'[u] = \min\{u[a_0], u[a_1]\}$$

$$= \min\{P'[\theta_0] \times u[\theta_0|a_0] + P'[\theta_1] \times u[\theta_1|a_0],$$

$$P'[\theta_0] \times u[\theta_0|a_1] + P'[\theta_1] \times u[\theta_1|a_1]\}$$

$$= \min\{0.7 \times 0 + 0.3 \times 400, 0.7 \times 100 + 0.3 \times 0\}$$

$$= \min\{120, 70\} = 70 \Rightarrow \text{Decision for } a_1 \text{ (50ft Pile)}$$

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$$p=0.70$$

$$\boxed{120}$$

$$\theta_0$$

$$u = 0$$

$$p-0.30$$

$$\theta_1$$

$$u = 400 \text{ (Pile is spliced)}$$

$$a_0$$

$$a_1$$

$$p=0.70$$

$$\theta_0$$

$$u = 100 \text{ (Pile is cut)}$$

$$\boxed{70}$$

$$p=0.30$$

$$\theta_1$$

$$u = 0$$

$$\Rightarrow$$ **Choice of pile a_1 (50ft Pile)**

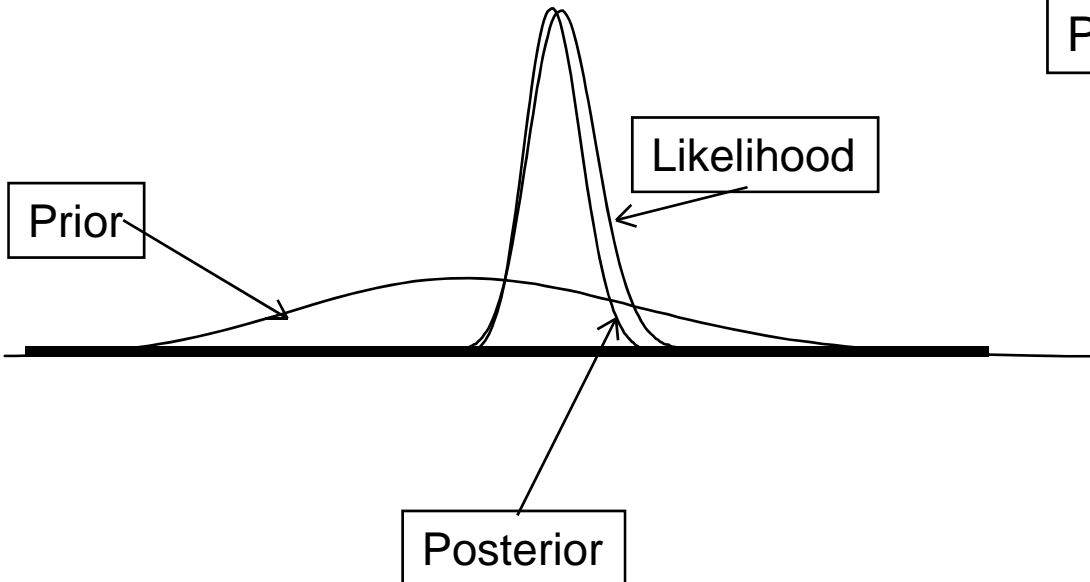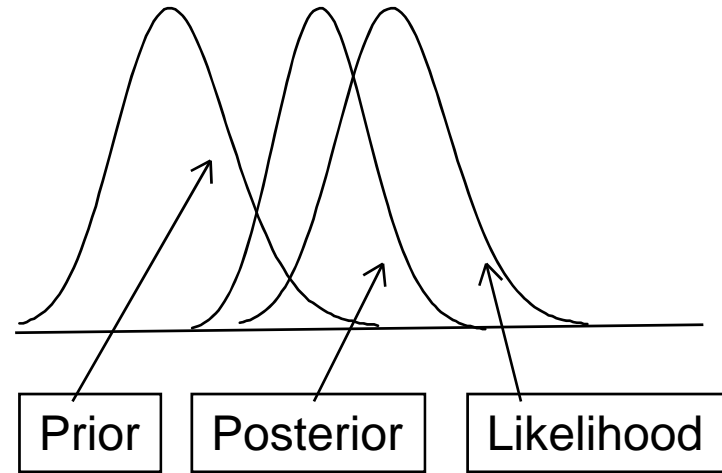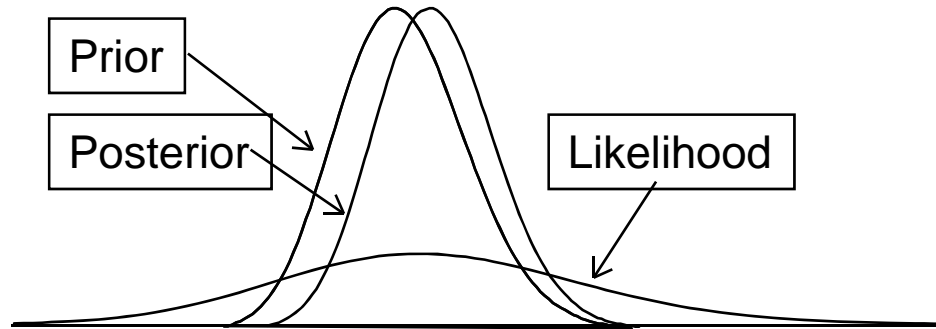**ETH** *Swiss Federal Institute of Technology*

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Posterior Analysis

$$P''(\theta_i) = \frac{P[z_k|\theta_i]P'[\theta_i]}{\sum_j P[z_k|\theta_j]P'[\theta_j]}$$

$$\begin{pmatrix} \text{Posterior probability of } \theta_i \\ \text{with given sample outcome} \end{pmatrix} = \begin{pmatrix} \text{Normalising} \\ \text{constant} \end{pmatrix} \times \begin{pmatrix} \text{Sample likelihood} \\ \text{given } \theta \end{pmatrix} \times \begin{pmatrix} \text{prior probability} \\ \text{of } \theta \end{pmatrix}$$

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Posterior Analysis

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Posterior Analysis

$$P''(\theta_i) = \frac{P[z_k|\theta_i]P'[\theta_i]}{\sum_j P[z_k|\theta_j]P'[\theta_j]}$$

Ultrasonic tests to determine the depth to bed rock

True state / Test result	θ_0 40 ft – depth	θ_1 50 ft – depth
z_0 - 40 ft indicated	0.6	0.1
z_1 - 50 ft indicated	0.1	0.7
z_2 - 45 ft indicated	0.3	0.2

Likelihoods of the different indications/test results given the various possible states of nature – ultrasonic test methods $P[z_i|\theta_j]$

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Posterior Analysis

It is assumed that a test gives a 45 ft indication

$$P''(\theta_i) = \frac{P[z_k|\theta_i]P'[\theta_i]}{\sum_j P[z_k|\theta_j]P'[\theta_j]}$$

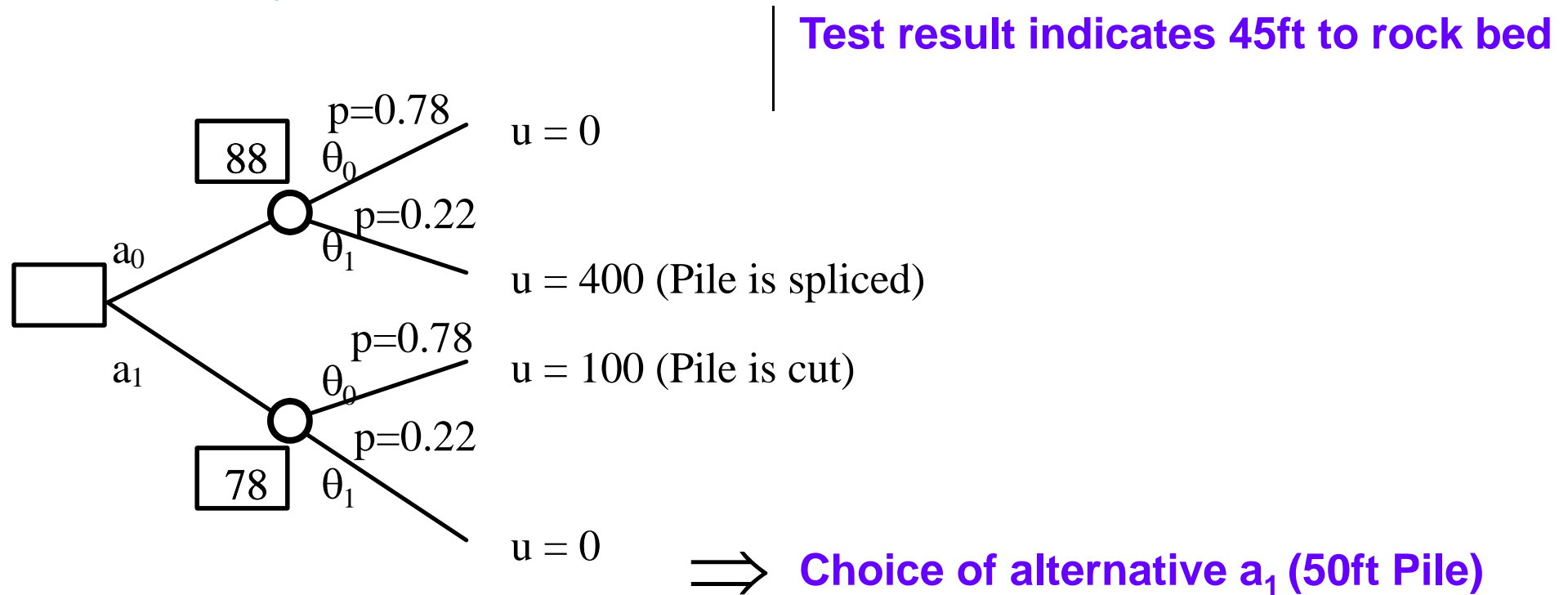$$P''[\theta_0] = P[\theta_0|z_2] \propto P[z_2|\theta_0]P[\theta_0] = 0.3 \; x \; 0.7 = 0.21$$

$$P''[\theta_1] = P[\theta_1|z_2] \propto P[z_2|\theta_1]P[\theta_1] = 0.2 \; x \; 0.3 = 0.06$$

$$P''[\theta_0|z_2] = \frac{0.21}{0.21 + 0.06} = 0.78$$

$$P''[\theta_1|z_2] = \frac{0.06}{0.21 + 0.06} = 0.22$$

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Posterior Analysis

Test result indicates 45ft to rock bed

$$\Rightarrow \text{ Choice of alternative } a_1 \text{ (50ft Pile)}$$

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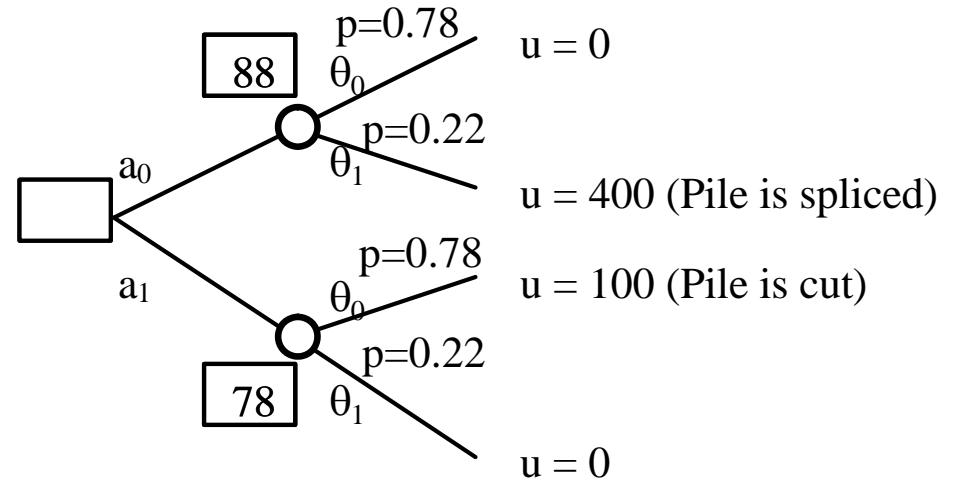
Posteriori Analysis

$$E''\left[u|z_2\right] = \min_j \left\{ E''\left[u(a_j)|z_2\right] \right\}$$

$$= \min\{P''[\theta_0] \times 0 + P''[\theta_1] \times 400, P''[\theta_0] \times 100 + P''[\theta_1] \times 0\}$$

$$= \min\{0.78 \times 0 + 0.22 \times 400, 0.78 \times 100 + 0.22 \times 0\}$$

$$= \min\{88, 78\} = 78$$

$\Longrightarrow$ **Choice of alternative a_1 (50ft Pile)**

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Pre-posterior Analysis

$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''[u|z_i] = \sum_{i=1}^{n} P'[z_i] \times \min_{j=1,m}\{E''[u(a_j)|z_i]\}$$

$$P'[z_i] = P[z_i|\theta_0] \times P'[\theta_0] + P[z_i|\theta_1] \times P'[\theta_1]$$

$$P'[z_0] = P[z_0|\theta_0] \times P'[\theta_0] + P[z_0|\theta_1] \times P'[\theta_1] = 0.6 \times 0.7 + 0.1 \times 0.3 = 0.45$$
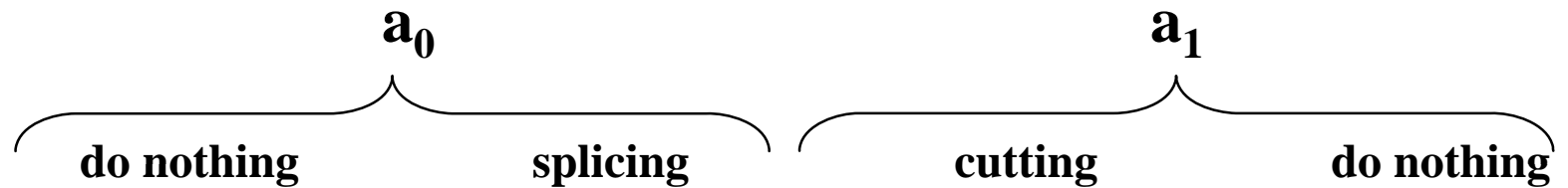
$$P'[z_1] = P[z_1|\theta_0] \times P'[\theta_0] + P[z_1|\theta_1] \times P'[\theta_1] = 0.1 \times 0.7 + 0.7 \times 0.3 = 0.28$$

$$P'[z_2] = P[z_2|\theta_0] \times P'[\theta_0] + P[z_2|\theta_1] \times P'[\theta_1] = 0.3 \times 0.7 + 0.2 \times 0.3 = 0.27$$

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Pre-posterior Analysis

$$E''[u|z_0] = \min_{j}\{E''[u(a_j)|z_0]\} =$$

$$\mathbf{a_0}$$

| do nothing | splicing | cutting | do nothing |

$$\min\{P''[\theta_0|z_0] \times 0 + P''[\theta_1|z_0] \times 400, P''[\theta_0|z_0] \times 100 + P''[\theta_1|z_0] \times 0\}$$

$$\min\{0.93 \times 0 + 0.07 \times 400, 0.93 \times 100 + 0.07 \times 0\} =$$

$$0.07 \times 400 + 0.93 \times 0 = 28$$

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Pre-posterior Analysis

$$E''\left[u|z_1\right] = \min_{j}\{E''\left[u(a_j)|z_1\right]\} =$$

$$\overbrace{\quad\text{do nothing}\qquad\qquad\text{splicing}\quad}^{\mathbf{a_0}} \qquad \overbrace{\quad\text{cutting}\qquad\qquad\text{do nothing}\quad}^{\mathbf{a_1}}$$

$$\min\{P''\left[\theta_0|z_1\right]\times 0 + P''\left[\theta_1|z_1\right]\times 400, \, P''\left[\theta_0|z_1\right]\times 100 + P''\left[\theta_1|z_1\right]\times 0\}$$

$$\min\{0.25\times 0 + 0.75\times 400, \, 0.25\times 100 + 0.75\times 0\} =$$

$$0.25\times 100 + 0.75\times 0 = 25$$

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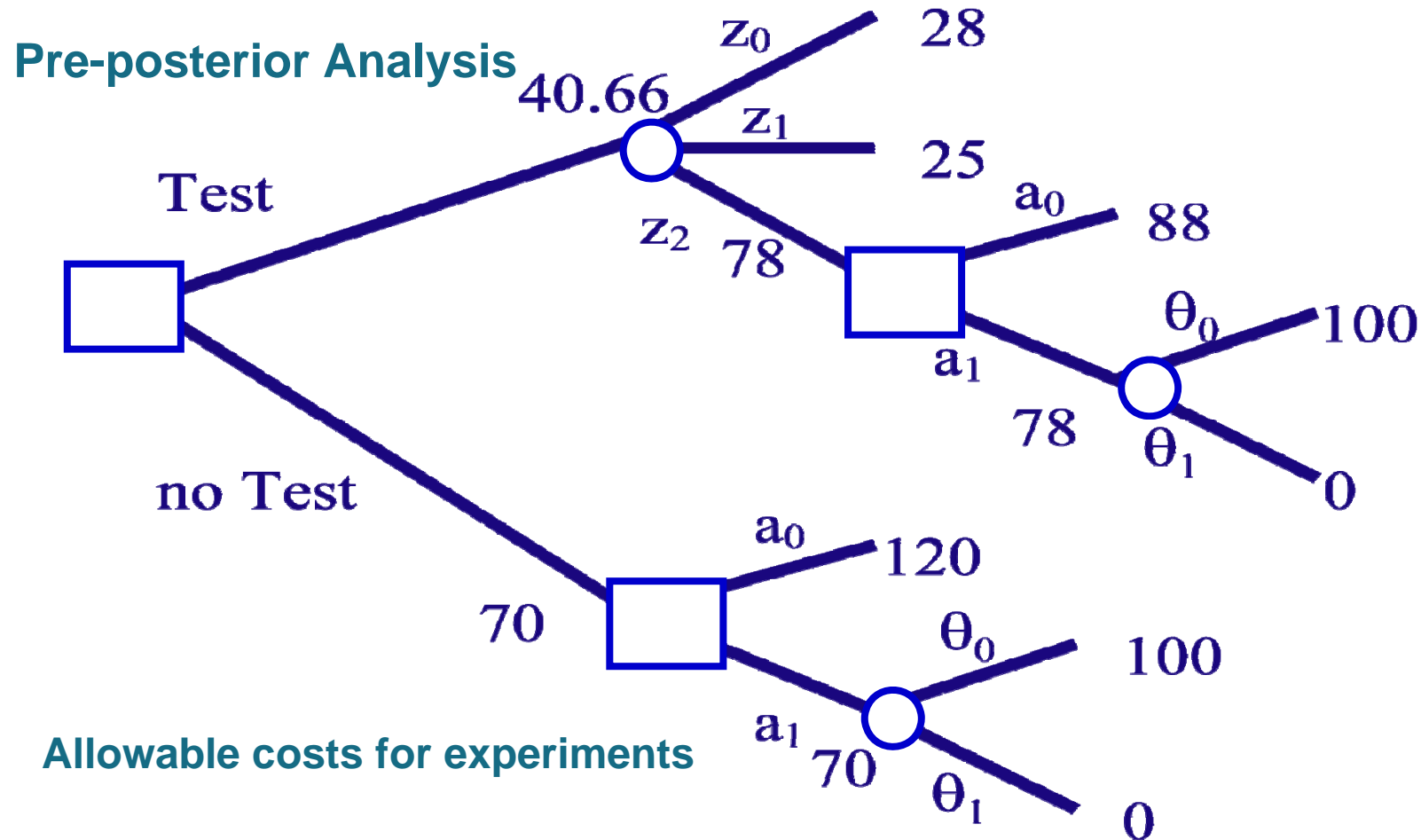
Pre-posterior Analysis

The minimum expected costs based on pre-posterior decision analysis
– not including costs of experiments

$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''[u|z_i] = 28 \times 0.45 + 25 \times 0.28 + 78 \times 0.27 = 40.66$$

Allowable costs for the experiment

$$E'[u] - E[u] = 70.00 - 40.66 = 29.34$$

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Pre-posterior Analysis

Allowable costs for experiments

z_0 28

z_1 25

z_2 78

40.66

Test

a_0 88

a_1 78

θ_0 100

θ_1 0

no Test   70

a_0 120

a_1 70

θ_0 100

θ_1 0

$$E'[u] - E[u] = 70.00 - 40.66 = 29.34$$

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