

Risk and Safety
in
Civil, Surveying and Environmental
Engineering

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Exercise 1

Consider a steel rod. The rod will fail if the applied load on the rod exceeds the resistance of the rod.

The resistance R of the rod is a Normal distributed random variable with parameters:

$$\mu_R = 350 \text{ MPa} \text{ and } \sigma_R = 35 \text{ MPa}$$

The applied load S on the rod is deterministic and equal to 200 MPa



We need to estimate the probability of failure of the rod

Exercise 1

1. **Safety margin** $M = R - S$
2. **We write:** $\text{FLIM}(1)\{M\} = R - S$
3. In “Stochastic modeler” provide the name, distribution and distribution parameters of R and S
4. In “Computation Options” choose FORM for the method of integration
5. Save the file with name “Exercise_1”
6. Run the reliability analysis

Exercise 2

Consider Exercise 1 again:

Exercise 2

Consider Exercise 1 again:

- ➔ Carry out a parametric study for the loading stress S
- use values ranging between **100 to 300 MPa with an interval of 10 MPa**
 - Go to “parametric study” and input the above information under lower and upper bound
 - Proceed as before and save the file as Exercise 2
 - Run the reliability analysis

Exercise 3

- Consider the same steel rod

The load S on the rod is now also Normal distributed with parameters:

$$\mu_S = 200 \text{ MPa} \text{ and } \sigma_S = 40 \text{ MPa}$$

- ➡ a. Estimate again the probability of failure of the rod using FORM
- ➡ b. Use Monte Carlo Simulation to do the above estimation

Exercise 3a

1. **Safety margin** $M = R - S$
2. **We write:** $\text{FLIM}(1)\{M\} = R - S$
3. **In “Stochastic modeler” provide the name, distribution and distribution parameters of R and S (both Normal distributed)**

$$\mu_R = 350 \text{ MPa} \text{ and } \sigma_R = 35 \text{ MPa}$$

$$\mu_S = 200 \text{ MPa} \text{ and } \sigma_S = 40 \text{ MPa}$$

Exercise 3a

1. **Safety margin** $M = R - S$
2. **We write:** $\text{FLIM}(1)\{M\} = R - S$
3. **In “Stochastic modeler” provide the name, distribution and distribution parameters of R and S (both Normal distributed)**

$$\mu_R = 350 \text{ MPa} \text{ and } \sigma_R = 35 \text{ MPa}$$

$$\mu_S = 200 \text{ MPa} \text{ and } \sigma_S = 40 \text{ MPa}$$

4. **In “Computation Options” choose FORM for the method of integration**
5. **Save the file with name “Exercise_3a”**
6. **Run the reliability analysis**

Exercise 3b

1. **Safety margin** $M = R - S$
2. **We write:** $\text{FLIM}(1)\{M\} = R - S$
3. **In “Stochastic modeler” provide the name, distribution and distribution parameters of R and S (both Normal distributed)**

$$\mu_R = 350 \text{ MPa} \text{ and } \sigma_R = 35 \text{ MPa}$$

$$\mu_S = 200 \text{ MPa} \text{ and } \sigma_S = 40 \text{ MPa}$$

4. **In “Computation Options” choose Crude Monte Carlo for the method of integration and give a number of simulations**
5. **Save the file with name “Exercise_3b”**
6. **Run the reliability analysis**

Exercise 4

- Consider again a steel rod

The resistance R of the rod is given by: $R = A \cdot f_y$

Where A is the cross-sectional area and f_y is the steel yield stress

- A is constant : $A = 10 \text{ mm}^2$
- f_y is Normal distributed with parameters: $\mu_{f_y} = 350 \text{ MPa}$ and $\sigma_{f_y} = 35 \text{ MPa}$

The load S is Normal distributed with parameters: $\mu_S = 1500 \text{ N}$ and $\sigma_S = 300 \text{ N}$

Estimate the probability of failure of the rod using FORM
carrying a parametric analysis for the cross sectional area of the rod
(from 10 mm^2 to 20 mm^2 , with step 2 mm^2)

Exercise 4

1. **Safety margin** $M = R - S$
2. **We write:** $\text{DEFFUNC}(1) = A * fy$
 $\text{FLIM}(1) \{M\} = \text{FUNC}(1) - S$
3. **In “Stochastic modeler” provide the name, distribution and distribution parameters of all the variables.**
4. **In “Computation Options” choose FORM for the method of integration**
5. **Save the file with name “Exercise_4”**
6. **Run the reliability analysis**

Exercise 5

- Consider Exercise 4 but:

the steel yield stress f_y

- is Normal distributed with parameters: μ_{f_y} and $\sigma_{\mu_{f_y}} = 35 \text{ MPa}$

where μ_{f_y} is Normal distributed: $\mu_{\mu_{f_y}} = 350 \text{ MPa}$ and $\sigma_{\mu_{f_y}} = 10 \text{ MPa}$

Remember that:

The resistance R of the rod is given by: $R = A \cdot f_y$

Where A is the cross-sectional area and f_y is the steel yield stress

A is constant : $A = 10 \text{ mm}^2$

The load S is normal distributed with parameters: $\mu_S = 1500 \text{ N}$ and $\sigma_S = 300 \text{ N}$

Estimate the probability of failure of the rod using FORM

Exercise 5

1. **Safety margin** $M = A * f_y - S$
2. **We write:** $FLIM(1)\{M\} = A * f_y - S$
RF07(1)=mfy
3. In “Stochastic modeler” provide the name, distribution and distribution parameters of all the variables. Watch out how to define $f_y!!!$
4. In “Computation Options” choose FORM for the method of integration
5. Save the file with name “Exercise_5”
6. Run the reliability analysis

Exercise 6

- Remember Exercise 7 (weekly exercises)

The time to corrosion initiation is given by:

$$T_{I_{\text{mod}}} = \frac{d^2}{4D} \left(\text{erf}^{-1} \left(1 - \frac{C_{CR}}{C_S} \right) \right)^{-2}$$

where:

d : the concrete cover depth to the reinforcement

D : the chloride diffusion coefficient

C_{CR} : the critical chloride concentration which when exceeded at the level of the reinforcement corrosion will initiate

C_S : the chloride concentration at the surface of the concrete

$\text{erf}^{-1}(\cdot)$: error function expressed as: $\text{erf}^{-1}(x) = \frac{\Phi^{-1}\left(\frac{x}{2} + \frac{1}{2}\right)}{\sqrt{2}}$

Exercise 6

- Remember Exercise 7 (weekly exercises)

The time to corrosion initiation is given by:

$$T_{I_{\text{mod}}} = \frac{d^2}{4D} \left(\text{erf}^{-1} \left(1 - \frac{C_{CR}}{C_S} \right) \right)^{-2}$$

Model uncertainty Ξ :

$$T_I = \Xi T_{I_{\text{mod}}}$$

Statistical characteristics:

Description	Representation	Units	Distribution	μ	σ
Cover thickness	d	mm	Log-normal	55	11
Diffusion coefficient	D	mm ² / yr	Log-normal	40	10
Surface concentration	C_S	Water % of concrete	Log-normal	0.4	0.08
Critical concentration	C_{CR}	Water % of concrete	Log-normal	0.15	0.05
Propagation time	T_P	years	Log-normal	7.5	1.88
Model uncertainty	Ξ	-	Log-normal	1	0.05

Exercise 6

- Remember Exercise 7
 - a. Estimate the probability of corrosion initiation after $t= 50$ years
 - b. Estimate the profile of corrosion initiation from year 1 to year 100

Exercise 6

a. Estimate the probability of corrosion initiation after $t= 50$ years

1. *Formulate safety margin M_{Cl} :*

Exercise 6

a. Estimate the probability of corrosion initiation after $t= 50$ years

1. Formulate safety margin M_{Cl} : $M_{Cl}=X_i \cdot T_{i_mod-t}$

Exercise 6

a. Estimate the probability of corrosion initiation after $t= 50$ years

1. Formulate safety margin M_{CI} $M_{CI}=X_i*Ti_{mod}-t$

2. Write the expression in Comrel and declare the distributions and their statistics

Error function $erf^{-1}(\cdot)$

$$erf^{-1}(x) = \frac{\Phi^{-1}\left(\frac{x}{2} + \frac{1}{2}\right)}{\sqrt{2}}$$

$$T_{I_{mod}} = \frac{d^2}{4D} \left(erf^{-1} \left(1 - \frac{C_{CR}}{C_S} \right) \right)^{-2}$$

DEFFUNC(1)(X)=IF(X<0,0,ICPHI(X/2+0.5)/SQRT(2))

Exercise 6

a. Estimate the probability of corrosion initiation after $t= 50$ years

1. Formulate safety margin M_{CI}
2. Write the expression in Comrel and declare the distributions and their statistics

DEFFUNC(1)(X)=IF(X<0,0,ICPHI(X/2+0.5)/SQRT(2))

$$\operatorname{erf}^{-1}(x) = \frac{\Phi^{-1}\left(\frac{x}{2} + \frac{1}{2}\right)}{\sqrt{2}}$$

$$M_{CI} = X_i * T_{i_mod} - t$$

$$T_{i_mod} = \frac{d^2}{4D} \left(\operatorname{erf}^{-1} \left(1 - \frac{C_{CR}}{C_S} \right) \right)^{-2}$$

Exercise 6

a. Estimate the probability of corrosion initiation after $t= 50$ years

1. Formulate safety margin M_{CI}
2. Write the expression in Comrel and declare the distributions and their statistics

DEFFUNC(1)(X)=IF(X<0,0,ICPHI(X/2+0.5)/SQRT(2))

FLIM(1){CI}=XI*((FUNC(1)(1-Ccr/Cs))^(-2)*d^2/(4*D))-t

3. Save as Exercise_6a

4. Run the reliability analysis

$$\operatorname{erf}^{-1}(x) = \frac{\Phi^{-1}\left(\frac{x}{2} + \frac{1}{2}\right)}{\sqrt{2}}$$

$$M_{CI} = X_i * T_{i_mod} - t$$

$$T_{i_mod} = \frac{d^2}{4D} \left(\operatorname{erf}^{-1} \left(1 - \frac{C_{CR}}{C_S} \right) \right)^{-2}$$

Exercise 6

b. Estimate the profile of corrosion initiation from year 1 to year 100

Exercise 6

b. Estimate the profile of corrosion initiation from year 1 to year 100

- 1. Keep Exercise_6a*
- 2. In “Parametric study” give for t , the lower bound equal to 1 and upper bound equal to 100*
- 3. Save as Exercise_6b*
- 4. Run the reliability analysis*

Exercise 6

Estimate the probability of visual corrosion after 50, 70 and 90 years, following Exercise 7 of the weekly exercises!