Risk and Safety in Civil, Surveying and Environmental Engineering

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Contents of Today's Lecture

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Introduction to Classical Reliability Theory

During the second world war it became evident that the reliability of complex technical installations was a problem

As an example the modern warships at the time were only operational for attack/defence in about 60 % of the time

Similar effects were observed on the reliability of e.g. the V1 / V2 rocket systems – the first many launches were unsuccessful

These events were the real initiator of the reliability theory for technical components and systems

The purpose of reliability analysis is to be able to estimate the probabilities and the consequences which enter the logical trees used for systems analysis

Introduction to Classical Reliability Theory

Initially the reliability theory was developed for systems with a large number of (semi-) identical components subject to the same exposure conditions

- electrical systems (bulbs, switches, ..)

and later on to

- nuclear power installations (valves, pipes, pumps, ..)

- chemical plants (pipes, pressure vessels, valves, pumps,..)

- manufacturing plants (pumps, compressors, conveyers,..)

Introduction to Classical Reliability Theory

For such systems the probability of a componential failure may be assessed in a frequentistic manner –

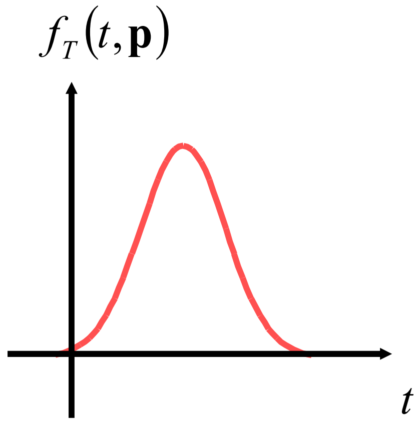
from observed failure rates

- number of failures per component operation hours

Due to the characteristics of the failure mechanism

- a steady deterioration as function of time/use

for the considered components the main concern was centered around the statistical modelling of the time till failure



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Introduction to Classical Reliability Theory

For technical systems such as structures and structural components the classical reliability analysis is of limited use because

- all structural components are unique

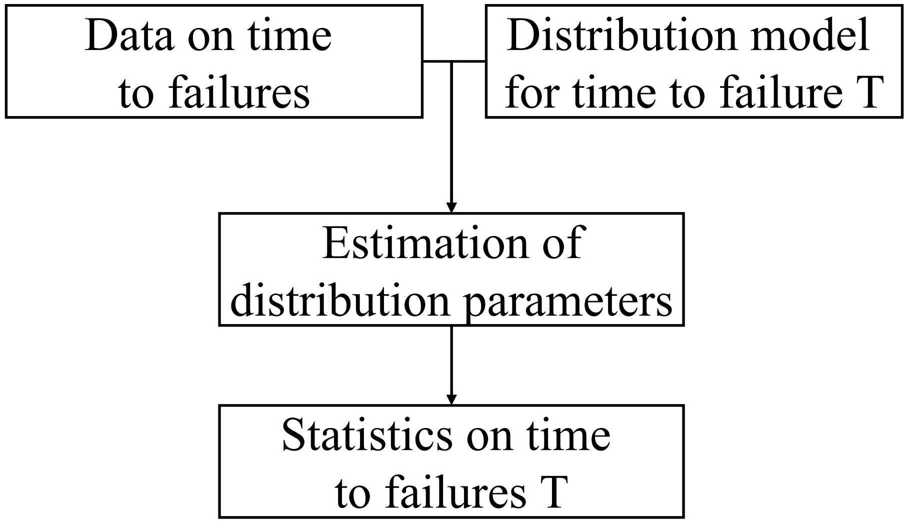
- the failure mechanism tends to be related to extreme load events exceeding the residual capacity of the component – not the direct effect of deterioration alone

For such systems a different approach is thus required, namely

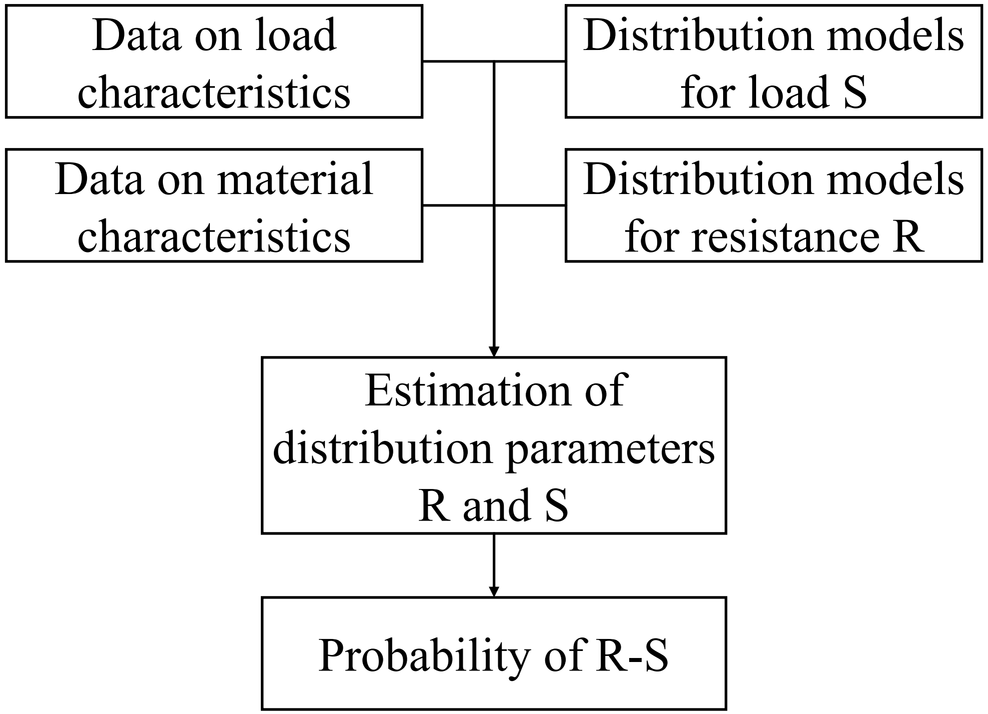
- an individual modelling of both the resistance as function of time and the loading as function of time

Introduction to Classical Reliability Theory

Classical Reliability Analysis



Structural Reliability Analysis



Reliability Analysis of Technical Components

The probability of failure for a components is defined by the

reliability function

$$R_T(t) = 1 - F_T(t) = 1 - P(T \leq t)$$

where T is the time till failure and $F_T(t)$ is the probability distribution function

If the probability density $f_T(t)$ function is known the reliability function may be defined by

$$R_T(t) = 1 - \int_0^t f_T(\tau)d\tau = \int_t^\infty f_T(\tau)d\tau$$

Reliability Analysis of Technical Components

The reliability function thus depends on the probability distribution function or the probability density function for the time till failure

Different types of probability distribution functions may be derived on the basis of physical arguments

An example is the Weibull distribution often used to describe the time till fatigue failure

Then the reliability function is given as

$$R_T(t) = 1 - F_T(t) = 1 - \left(1 - \exp\left[-\left(\frac{t}{k}\right)^{\beta}\right]\right) = \exp\left[-\left(\frac{t}{k}\right)^{\beta}\right], \qquad t \geq 0$$

Reliability Analysis of Technical Components

Having defined the reliability function the mean time till failure may be derived as

$$E\left[T\right]=\int_0^\infty \tau\cdot f_T(\tau)d\tau=\int_0^\infty R_T(\tau)d\tau$$

which may be seen from (integration by parts)

$$E\left[T\right]=\int_0^\infty \tau f_T(\tau)dt=\left[t\cdot F_T(t)\right]_0^\infty-\int_0^\infty F_T(\tau)d\tau$$

$$=\left[t\cdot(1-R_T(t))\right]_0^\infty-\int_0^\infty(1-R_T(\tau))d\tau$$

$$=\left[t\right]_0^\infty-\left[t\cdot R_T(t)\right]_0^\infty-\left[t\right]_0^\infty+\int_0^\infty R_T(\tau)d\tau$$

$$=\int_0^\infty R_T(\tau)d\tau-\left[t\cdot R_T(t)\right]_0^\infty=\int_0^\infty R_T(\tau)d\tau$$

provided that

$$\lim_{t\to\infty}\left[t\cdot R_T(t)\right]_0^\infty=0$$

Reliability Analysis of Technical Components

The probability that the component will fail in a given interval $\left[t, t+\delta \mathrm{t}\right]$ is given by

$$P(t < T \leq t + \delta\, t) = F_T(t+\delta \mathrm{t}) - F_T(t) = R_T(t) - R_T(t+\delta \mathrm{t})$$

The failure rate function *z(t)* is defined as the average rate of failures in a given time interval given that the component has not failed previously

$$z(t) = \frac{R_T(t) - R_T(t+\delta \mathrm{t})}{\delta\ t\, R_T(t)}$$

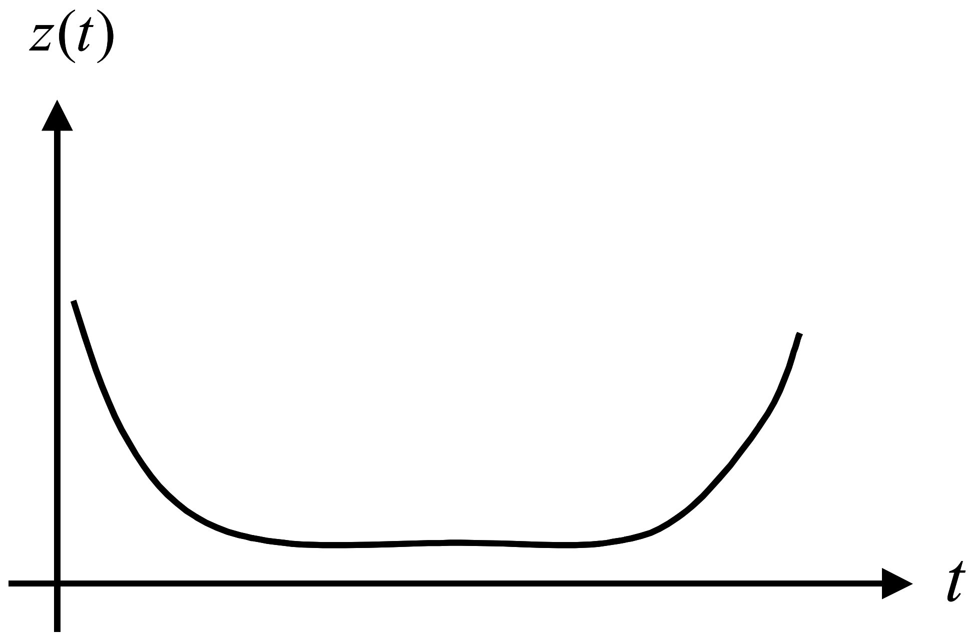
Reliability Analysis of Technical Components

The failure rate function typically has the following characteristics

First there is a rate of failure due to **birth defects**

Then there is a **steady state**

Then **deterioration starts** and the failure rate increases



Reliability Analysis of Technical Components

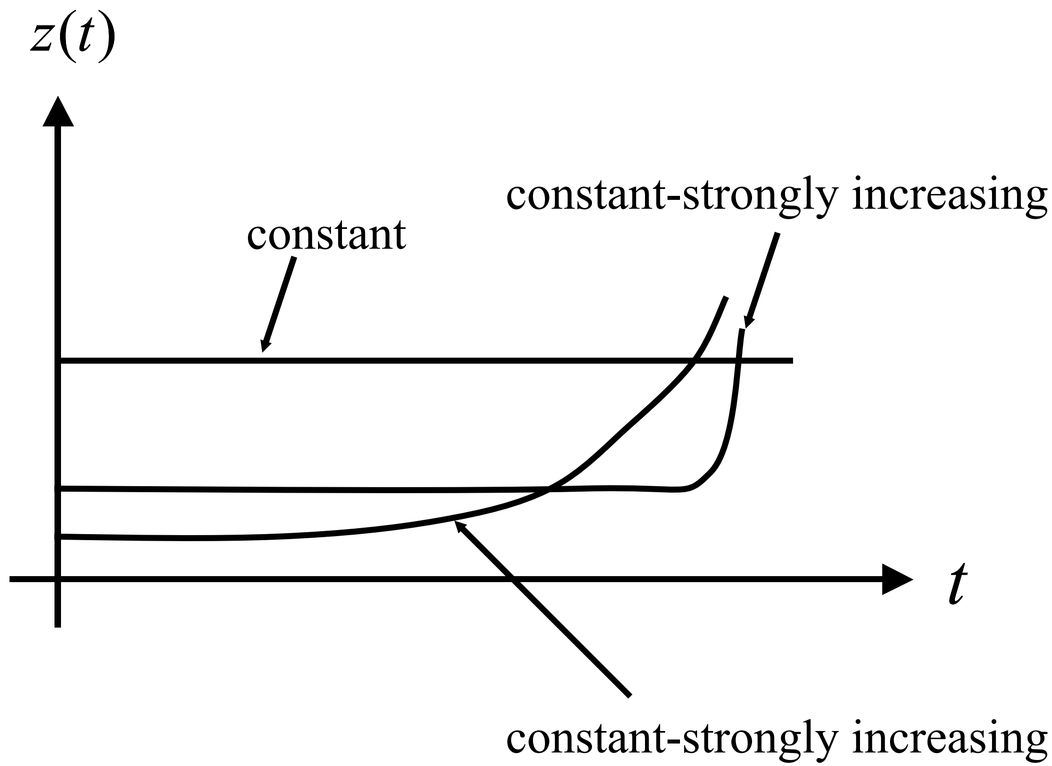
The failure rate function is important for the planning of maintenance and inspections

for constant failure rate functions there is no basis for inspection planning
- corrective maintenance

for constant-strongly increasing failure rate functions the time of failure is “known”

- preventive maintenance

for moderately increasing failure rate functions deterioration will be evident before failure
- condition control and condition based maintenance



Reliability Analysis of Technical Components

Failure rates may be assessed on the basis of observations of failures in a given time interval

$$z = \frac{n_f}{\tau \cdot n_i}$$

Preferably the failure rates are estimated by e.g. the method of maximum likelihood.

The uncertainty associated with the failure rate may then be quantified

$$f_Z^{'}(z)$$

Having additional information the uncertain failure rate may be updated

$$f_Z^{''}(z) = \frac{L(\boldsymbol{\tau}|z) \cdot f_Z^{'}(z)}{\int_0^{\infty} L(\boldsymbol{\tau}|z) \cdot f_Z^{'}(z)\, dz}$$

For exponential distributed failure times we have

$$L(\boldsymbol{\tau}|z) = \prod_{i=1}^{n} z \exp(-z \cdot \tau_i)$$

Example on Failure Rate Assessment

It is assumed that observed times till failures are available and the failure rate shall be determined

The probability density function for the time till failure is

$$f_T(t) = z\exp(-z \cdot t)$$

The log-Likelihood function is

$$l(\mathbf{t}|z) = \sum_{i=1}^{10}\left(\ln(z) - z \cdot t_i\right)$$

A failure rate equal to 0.95 is obtained

Pump	Time till failure
1	0.24
2	3.65
3	1.25
4	0.2
5	1.79
6	0.6
7	0.74
8	1.43
9	0.53
10	0.13

Example on Failure Rate Assessment

Assume now that only data on failure up till the first year are available

In this case a failure rate equal to 2.45 is obtained

Pump	Time till failure
1	**0.24**
2	3.65
3	1.25
4	**0.2**
5	1.79
6	**0.6**
7	**0.74**
8	1.43
9	**0.53**
10	**0.13**

Example on Failure Rate Assessment

Assume now that only data on failure up till the first year are available

By only observing a limited time interval in fact the sample can be considered to be sensored

However, by using a modified likelihood function

$$l(\mathbf{t}|z) = n_n \ln(1 - F_T(1)) + \sum_{i=1}^{n_f} \ln(z) - z \ \ t_i = -n_n z + \sum_{i=1}^{n_f} \ln(z) - z \ \ t_i$$

an estimate for the failure rate equal to 0.94 is achieved – very close to the correct

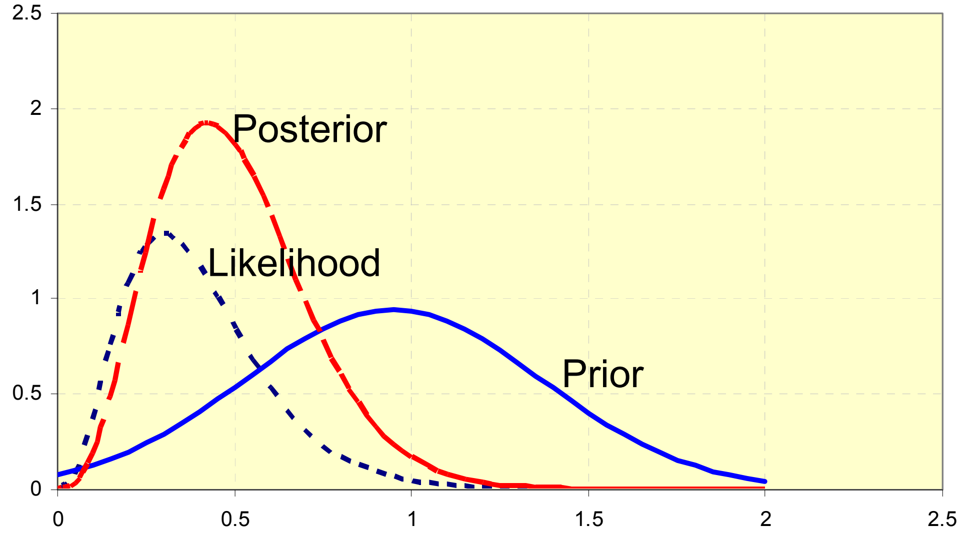
Pump	Time till failure
1	**0.24**
2	3.65
3	1.25
4	**0.2**
5	1.79
6	**0.6**
7	**0.74**
8	1.43
9	**0.53**
10	**0.13**

Example on Failure Rate Assessment

Using all data the MLM provides us also with the statistical uncertainty by which the failure rate is associated

It is assumed that 3 new observations are made available

Pump	Time till failure
1	3.2
2	3.5
3	3.3



Prior: normal distributed
Mean = 0.95
Std.Dev. = 0.42

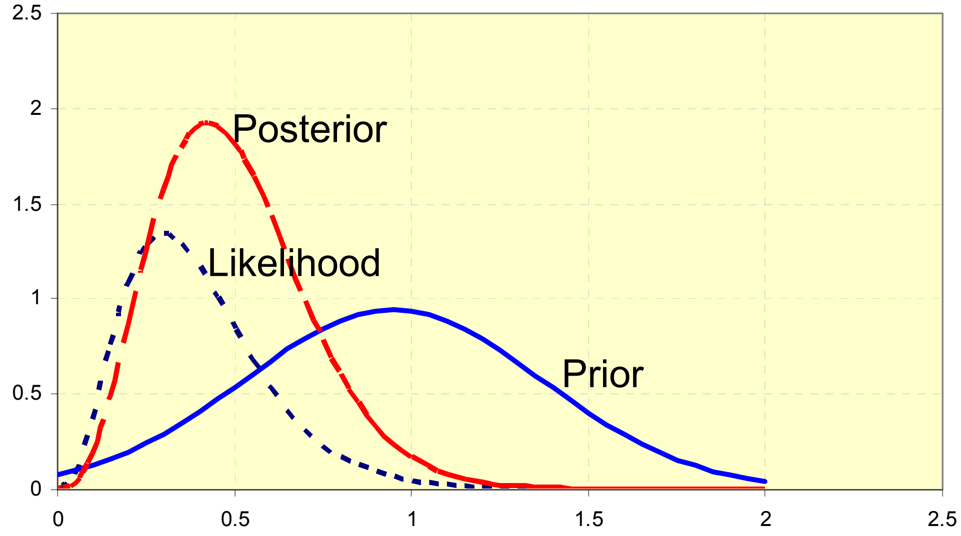
Example on Failure Rate Assessment

The likelihood of the 3 observations is

$$L(\mathbf{t}|z) = \prod_{i=1}^{3} z \exp(-z x_i)$$

The posterior probability density function for the failure rate is thus

$$f_Z''(z|\mathbf{t}) = \frac{1}{c} L(\mathbf{t}|z) f_Z'(z)$$



Prior: normal distributed
Mean = 0.95
Std.Dev. = 0.42

Example on Failure Rate Assessment

Finally the failure probability may be determined

$$P_F(T|z) = 1 - \exp(-zT)$$

However – as the failure rate is uncertain we must integrate out over the probability density function of the failure rate

$$P_F(T) = 1 - \int_0^1 \exp(-zT) f_Z(z) dz$$

Using the prior probability density function the failure probability is equal to 0.61

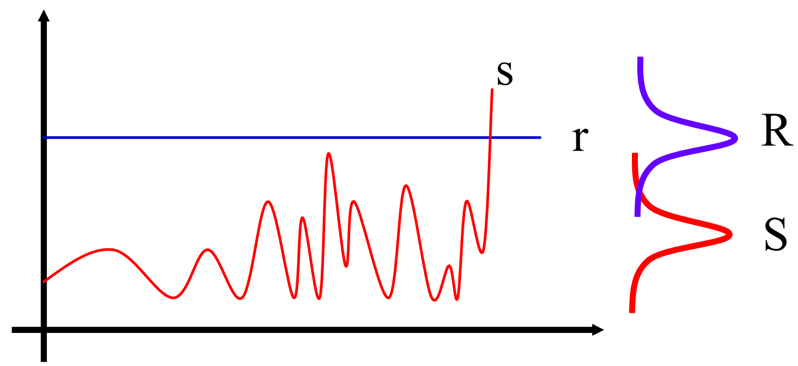
Using the posterior the failure probability is equal to 0.38

Reliability Analysis of Structural Components

Reliability of structures cannot be assessed through failure rates because

- Structures are unique in nature

- Structural failures normally take place due to extreme loads exceeding the residual strength



Therefore in structural reliablity models are established for resistances R and loads S individually and the structural reliability is assessed through

$$P_f = P(R - S \leq 0)$$

Reliability Analysis of Structural Components

If only the resistance is uncertain the failure probability may be assessed by

$$P_f = P(R \le s) = F_R(s) = P(R/s \le 0)$$

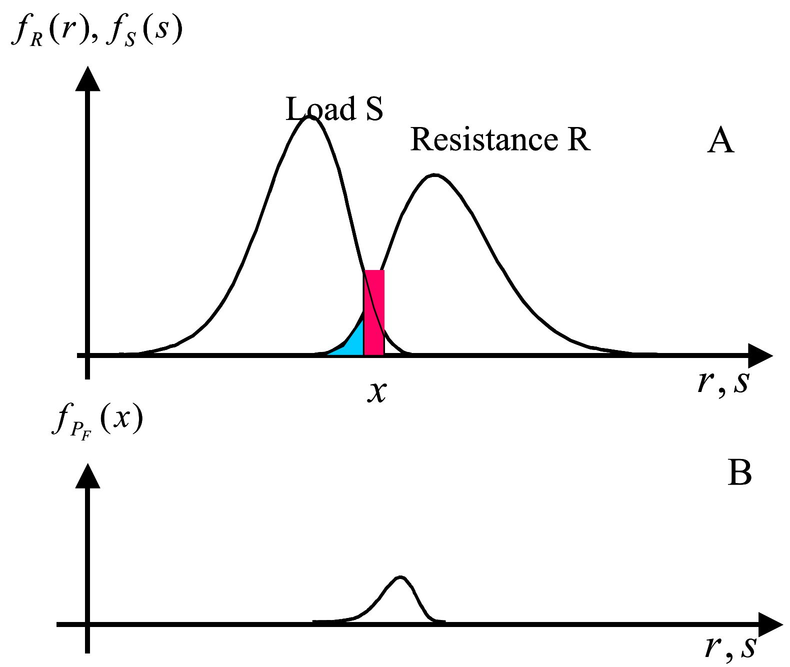
If also the load is uncertain we have

$$P_f = P(R \le S) = P(R - S \le 0) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx$$

where it is assumed that the load and the resistance are independent

This is called the

“Fundamental Case”



Reliability Analysis of Structural Components

In case R and S are normal distributed we have that the safety margin M is also normal distributed

$$M = R - S$$

Then the failure probability is

$$P_F = P(R - S \le 0) = P(M \le 0)$$

with the mean value of M

$$\mu_M = \mu_R - \mu_S$$

and standard deviation of M

$$\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2}$$

The failure probability is then

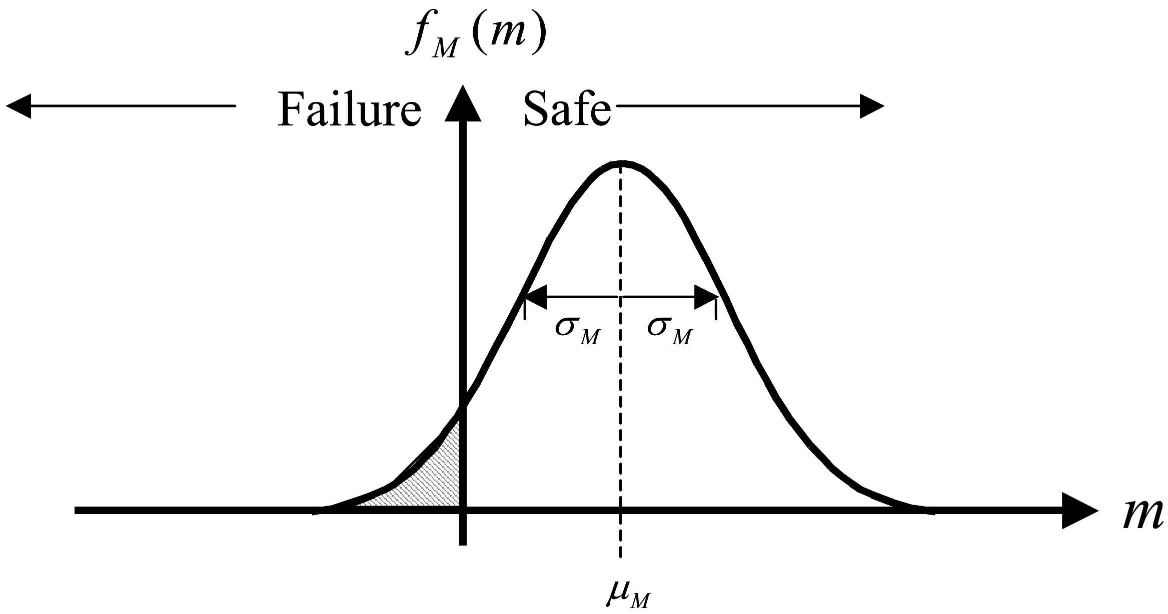
$$P_F = \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right) = \Phi(-\beta)$$

where the reliability index is

$$\beta = \mu_M / \sigma_M$$

Reliability Analysis of Structural Components

The normal distributed safety margin M



Reliability Analysis of Structural Components

In the general case the resistance and the load may be defined in terms of functions

$$R = f_1(\mathbf{X})$$
$$S = f_2(\mathbf{X})$$

where X are basic random variables

and the safety margin as

$$M = R - S = f_1(\mathbf{X}) - f_2(\mathbf{X}) = g(\mathbf{X})$$

where g(x) is called the

limit state function

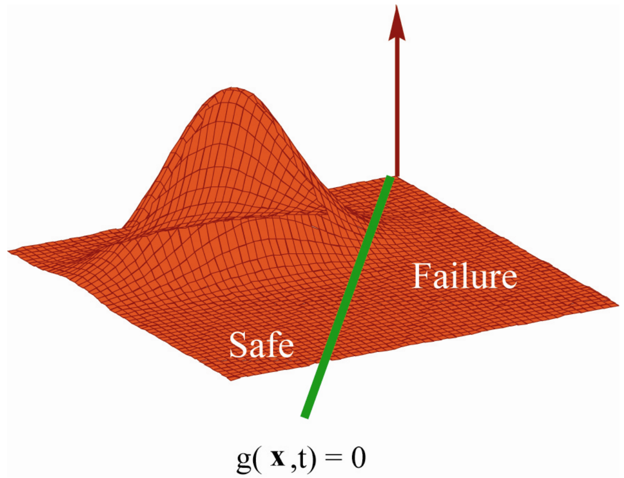
failure occurs when

$$g(\mathbf{x}) \leq 0$$

Reliability Analysis of Structural Components

Setting g(x)=0 defines a (n-1) dimensional surface in the space spanned by the n basic variables X

$$g(\mathbf{x}) = 0$$



This is the failure surface separating the sample space of X into a safe domain and a failure domain

The failure probability may in general terms be written as

$$P_F = \int_{g(x)\leq 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}$$