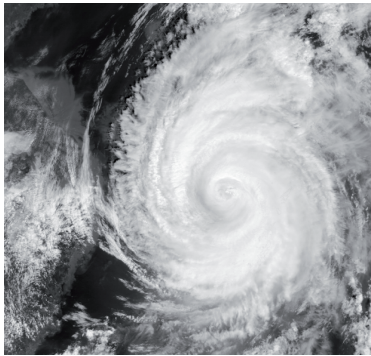# Risk and Safety in Civil Engineering

Exercises

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Exercise 1:

Exercise 1.1- Multiple choice questions:

In the following multiple choice questions it should be noted that for some of the questions several answers may be correct. Tick the correct alternatives in every question.

1.1 In probability theory the probability, $P(A)$, of an event A can take any value within the following boundaries:

$0 \leq P(A) \leq 1$ ☐

$-1 \leq P(A) \leq 1$ ☐

$-\infty \leq P(A) \leq \infty$ ☐

1.2 Which one(s) of the following expressions is(are) correct?

The probability of the union of two events A and B is equal to the sum of the probability of event A and the probability of event B, given that the two events are mutually exclusive. ☐

The probability of the union of two events A and B is equal to the probability of the sum of event A and event B, given that the two events are mutually exclusive. ☐

The probability of the intersection of two events A and B is equal to the product of the probability of event A and the probability of event B, given that the two events are mutually exclusive. ☐

The probability of the intersection of two events A and B is equal to the product of the probability of event A and the probability of event B, given that the two events are independent. ☐

1.3 Within the theory of sample spaces and events, which one(s) of the following statements is(are) correct?

An event A is defined as a subset of a sample space Ω. ☐

A sample space Ω is defined as a subset of an event A. ☐

1.4 If the intersection of two events, A and B corresponds to the empty set $\varnothing$, i.e. $A \cap B = \varnothing$, the two events are:

Mutually exclusive. ☐

Independent. ☐

Empty events. ☐

1.5 The probability of the intersection of two mutually exclusive events is equal to:

The product of the probabilities of the individual events. ☐

The sum of the probabilities of the individual events. ☐

The difference between the probabilities of the individual events. ☐

One (1). ☐

Zero (0). ☐

None of the above. ☐

1.6 The probability of the union of two not mutually exclusive events A and B is given as: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. It is provided that the probability of event A is equal to 0.1, the probability of event B is 0.1 and the probability of event B given event A, i.e. $P(B|A)$ is 0.8. Which result is correct?

$P(A \cup B) = -0.6$ ☐

$P(A \cup B) = 0.12$ ☐

$P(A \cup B) = 0.04$ ☐

1.7 For an event A in the sample space Ω, event $\overline{A}$ represents the complementary event of event A. Which one(s) of the following hold?

$A \cup \overline{A} = \Omega$ ☐

$A \cap \overline{A} = \Omega$ ☐

$A \cup \overline{A} = \varnothing$ ☐

1.8 Probability distribution functions may be defined in terms of their moments. If X is a continuous random variable which one(s) of the following is(are) correct?

The first moment of X corresponds to its mean value, μ_X. ☐

The second moment of X corresponds to its mean value, μ_X. ☐

The second central moment of X corresponds to its variance, σ_X^2. ☐

1.9 The probability density function of a continuous random variable X is illustrated in the following diagram.

$$f_X(x) = \frac{3}{1000} x^2$$
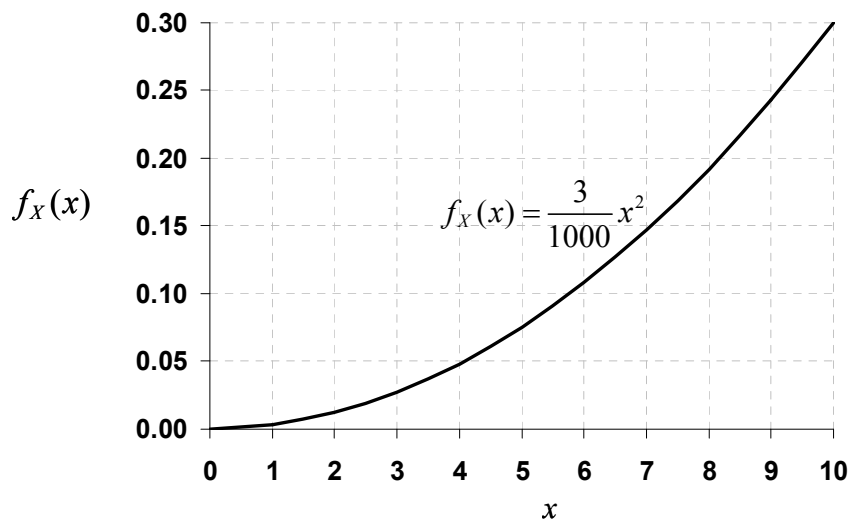
The probability of X exceeding the value of 5 is equal to:

$P(X > 5) = 0.875$ ☐

$P(X > 5) = 0.055$ ☐

$P(X > 5) = 0.125$ ☐

1.10 At a given location in Switzerland it has been observed that on average 4 avalanches occur per year. The annual probability of a house being hit by an avalanche on this location is thus:

Equal to one (1). ☐

Larger than one (1). ☐

None of the above. ☐

1.11 The variance of a continuous random variable X can be expressed as: $Var(X) = E\left[(X - \mu_X)^2\right]$, where μ_X is the mean value of X and $E[\cdot]$ is the expectation operator. Based on this expression which one(s) of the following expressions is(are) correct?

$Var(X) = E\left[X^2\right] - \mu_X^2$ ☐

$Var(X) = \mu_X - E\left[X^2\right]$ ☐

$Var(X) = X^2 - \mu_X$ ☐

1.12 Imagine that you have thrown a dice and that the dice is still hidden by a cup. What kind(s) of uncertainty is(are) associated with the outcome of the dice?

Aleatory uncertainty. ☐

Statistical uncertainty. ☐

Inherent random variability. ☐

None of the above. ☐

1.13 The convolution integral in probability describes how the probability density function for the sum of two random variables can be established. However, assumption(s) for its derivation is(are) that:

The random variables are normally distributed. ☐

The random variables are independent. ☐

The random variables are continuous. ☐

None of the above. ☐

1.14 Which one(s) of the following statements is(are) meaningful:

The probability of a big earthquake for the region around Zurich is close to 0.02. ☐

Strong winds occur in Ireland with a probability of 0.7. ☐

The probability of getting struck by lighting is equal to 0.1, if you stand under a tree. ☐

None of the above. ☐

1.15 A given random variable is assumed to follow a normal distribution. Which parameter(s) is(are) sufficient to define the probability distribution function of the random variable:

The variance and the standard deviation. ☐

The standard deviation and the mean value. ☐

The mode and the coefficient of variation. ☐

None of the above. ☐

1.16 Which one(s) of the following features is(are) characteristics of a normal distribution function?

The variance is equal to the coefficient of variation. ☐

The mode is equal to the median. ☐

The skewness is equal to zero. ☐

None of the above. ☐

1.17 The median of a data set corresponds to:

The lower quartile of the data set. ☐

The 0.5 quantile of the data set. ☐

The upper quartile of the data set. ☐

1.18 The commutative, associative and distributive laws describe how to:

Operate with probabilities. ☐

Operate with intersections of sets. ☐

Operate with unions of sets. ☐

None of the above. ☐

1.19 Measurements were taken of the concrete cover depth of a bridge column. The following symmetrical histogram results from the plot of the measured values:
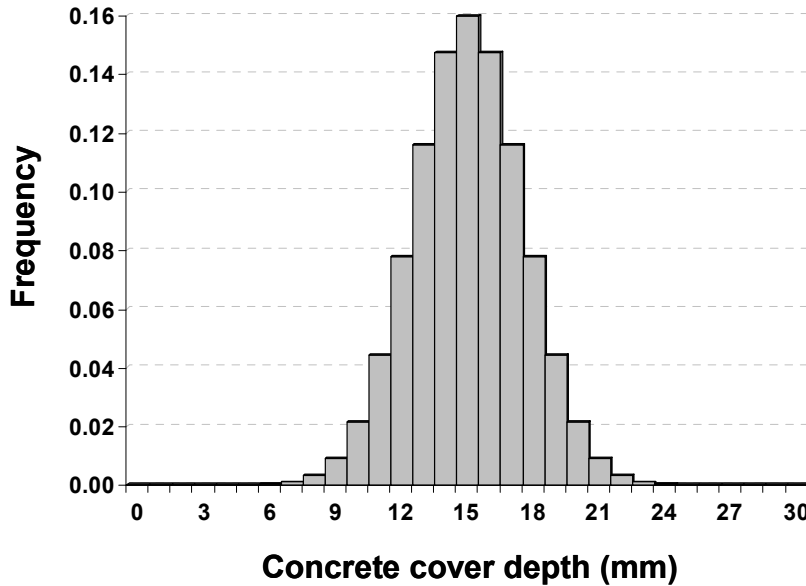
If X represents the random variable for the concrete cover depth, which one(s) of the following statements is(are) correct?

The sample mean, $\bar{x}$, is equal to 0.16 mm. ☐

The sample mean, $\bar{x}$, is equal to 15 mm. ☐

The mode of the data set is equal to 15 mm. ☐

1.20 After the completion of a concrete structure an engineer tests the null hypothesis that the mean value of the concrete cover depth corresponds to design assumptions. Measurements of the concrete cover depth are taken and after performing the hypothesis test the engineer accepts the null hypothesis. In its early years in service the structure shows signs of deterioration that can be explained only in the case that the design assumptions are not fulfilled. Which of the following statement(s) is(are) correct?

The engineer has performed a Type I error. ☐

The engineer has performed a Type II error. ☐

The engineer has performed a Type I and a Type II error. ☐

None of the above. ☐

1.21 Which one(s) of the following statements is(are) correct for a uniformly distributed random variable?

The expected value of the random variable is equal to 1. ☐

The probability distribution function is constant over the definition space. ☐

The probability density function is constant over the definition space. ☐

None of the above. ☐

1.22 According to the central limit theorem which of the following statement(s) hold?

The probability distribution function of the sum of a number of independent random variables approaches the normal distribution as the number of the variables increases. ☐

The probability distribution function of the product of a number of independent random variables approaches the normal distribution as the number of the variables increases. ☐

None of the above. ☐

1.23 The maximum likelihood method enables engineers to estimate the distribution parameters of a random variable on the basis of data. Which of the following statement(s) is(are) correct for the maximum likelihood method?

It provides point estimates of the distribution parameters. ☐

It provides information about the uncertainty associated with the estimated parameters. ☐

It provides no information about the uncertainty associated with the estimated parameters. ☐

None of the above. ☐

1.24 Consider a number of log-normally distributed and independent random variables. Which of the following statement(s) hold?

The probability distribution function of the sum of the random variables approaches the log-normal distribution as the number of the variables increases. ☐

The probability distribution function of the sum of the random variables approaches the normal distribution as the number of the variables increases. ☐

None of the above. ☐

1.25 It is given that the operational life (until breakdown) T of a diesel engine has an exponential distribution, $F_T(t) = 1 - e^{-\lambda t}$, with parameter λ and mean value, $\mu_T = 1/\lambda$, equal to 10 years. The engine is inspected every 2 years and if a problem is observed it is fully repaired. The probability that the engine breaks down before the first inspection is equal to:

$P(T \leq 2\,years) = 0.181$. ☐

$P(T \leq 2\,years) = 0.819$. ☐

$P(T \leq 2\,years) = 0.0067$. ☐

None of the above. ☐

1.26 From past experience it is known that the shear strength of soil can be described by a log-normal distribution. 15 samples of soil are taken from a site and an engineer wants to use the data in order to estimate the parameters of the log-normal distribution. The engineer:

may use a probability paper to estimate the parameters of the log-normal distribution. ☐

may use the maximum likelihood method to estimate the parameters of the log-normal distribution. ☐

may use the method of moments to estimate the parameters of the log-normal distribution. ☐

None of the above. ☐

1.27 Based on experience it is known that the concrete compressive strength may be modeled by a normally distributed random variable X, with mean value $\mu_X = 30MPa$ and standard deviation $\sigma_X = 5MPa$. The compressive strength of 20 concrete cylinders is measured. An engineer wants to test the null hypothesis H_o that X follows a normal distribution with the above given parameters. He/she carries out a Chi-square test by dividing the sample into $k = 4$ intervals. He/she calculates a Chi-square sample statistic equal to $\varepsilon_m^2 = 0.5$. Which of the following statement(s) is(are) correct?

The engineer cannot reject the null hypothesis H_o at the 5% significance level. ☐

The engineer can reject the null hypothesis H_o at the 5% significance level. ☐

The engineer can accept the null hypothesis H_o at the 10% significance level. ☐

None of the above. ☐

1.28 Consider a simply supported timber beam. The beam will fail if the applied central moment exceeds the bending strength of the beam. The bending strength R of the beam and the annual maximum of the applied central moment L are modeled by uncorrelated normally distributed variables with parameters: $\mu_R = 30kNm$, $\sigma_R = 5kNm$, $\mu_L = 10kNm$, $\sigma_L = 2kNm$. Which of the following statement(s) is(are) correct? (***HINT***: *If M represents the linear safety margin then the probability of failure is given by:* $P_F = P(M \leq 0) = \Phi(-\beta)$. *$\beta$ is the so-called reliability index given as:* $\beta = \dfrac{\mu_M}{\sigma_M}$, *where μ_M and σ_M are the mean and standard deviation of the safety margin respectively.*)

The reliability index of the timber beam corresponding to a one year reference period is equal to 3.71. ☐

The probability of failure of the timber beam in a year is equal to $1.04 \cdot 10^{-4}$. ☐

The reliability index of the timber beam corresponding to a one year reference period is equal to 4.08. ☐

The probability of failure of the timber beam in a year is equal to $2.25 \cdot 10^{-5}$. ☐

1.29 In a mediterranean city there are on average 5 snowfalls a year. Assume that the occurrence of snowfalls X follows a Poisson process with distribution function $F_X(x) = \dfrac{(\nu)^x e^{-\nu}}{x!}$ and with mean ν. Which of the following statement(s) is(are) correct?

The probability of exactly 5 snowfalls in the next year is equal to 0.175. ☐

The probability of exactly 5 snowfalls in the next year is equal to 1. ☐

The probability of no snowfall in the next year is equal to 0.774. ☐

The probability of no snowfall in the next year is equal to 0.0067. ☐
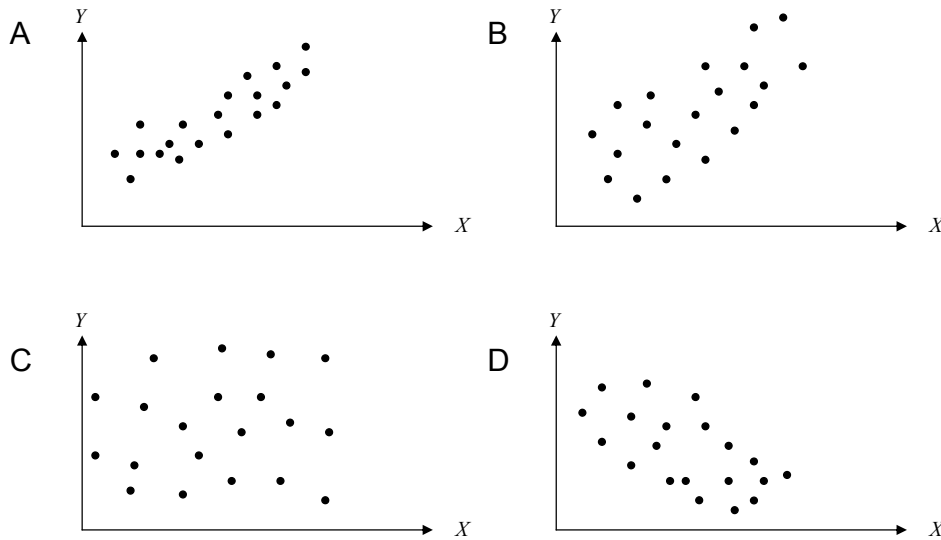

Exercise 2:

Exercise 2.1:

Risk represents the expected consequences of a hazardous event and may be expressed by the probability of occurrence of this event multiplied by the associated consequences.

Which of the following events is associated with a higher risk?

Event	1	2	3
Event probability	10%	1%	20%
Consequences	100 SFr	500 SFr	100 SFr

Exercise 2.2:

Comment on the correlation between the plotted pairs of data for all four cases. Provide an approximate value or range for the correlation coefficient.

Exercise 2.3:

A non destructive test method is carried out to examine whether the reinforcement of a concrete structural component is corroded or not. From a number of past experiments, it is known that the probability that the reinforcement is corroded is 6%. If the reinforcement is corroded, this will be always indicated by the test. However, there is a 14% probability that the test will indicate that the reinforcement is corroded even if it is not (false indication).

What is the probability that the reinforcement is corroded given a positive indication of corrosion from the test? Calculate the required probability using the Bayes' theorem.

Exercise 2.4:

We throw an ideal dice and consider the following events:
Case 1).　　A: "The outcome is an even number"
　　　　　　B: "The outcome is a number dividable by 3"

Case 2)　　A: "The outcome is an even number"
　　　　　　B: "The outcome is a prime number"
Calculate the probability that both events (A and B) occur simultaneously for each case.

Exercise 2.5:

Research in ETH is often funded by the Swiss National Foundation of research (SNF). The normal procedure is that a Professor submits a proposal for a new project. Experts working for SNF read the proposal and they may come to one of the following decisions:

D_1 : the proposal is accepted and the project will be funded

D_2 : the proposal should be revised by the Professor and resubmitted to SNF

D_3 : the proposal is not accepted and hence no funding is provided

Professor Muster works at ETH. During the past few years he has submitted many proposals to SNF. Based on experience Professor Muster assessed the probabilities associated with the possible final decisions of SNF as follows: $P(D_1) = 0.45$, $P(D_2) = 0.35$, $P(D_3) = 0.2$.

During a conference Professor Muster meets Dr. Beispiel who used to work in SNF. He was one of the experts who review the proposals in SNF and make the final decisions. At that time the Professor was about to submit a new proposal to SNF for possible funding. He kindly asked Dr. Beispiel to have a look at the proposal before submitting it to SNF. Dr. Beispiel could provide him with an indication of what decision he could expect from SNF. Of course Dr. Beispiel cannot say with certainty what will be the final SNF decision. The probabilities associated with Dr. Beispiel's opinion and the final decision of SNF, $P(I_j = D_j | D_i)$ are provided in the following table:

SNF final decision D_i	Dr. Beispiel's indicative decision, I_j		
	$I_j = D_1$	$I_j = D_2$	$I_j = D_3$
D_1	0.86	0.1	
D_2	0.2		0.06
D_3		0.1	0.9

a.　　Complete the above table.

b.　　Dr. Beispiel suggested to Professor Muster that if he was still working in SNF he would decide that the proposal should be revised and resubmitted. What is the probability that the final decision of SNF is the same as the decision of Dr. Beispiel?

Exercise 2.6:

a.   In Table 2.1 measurements of the mean yearly temperature are provided for the last 20 years in Zurich and globally. Plot the corresponding Tukey box plots and discuss the symmetry and skewness of the temperature distribution for both data sets.

Nr.	Zurich [°C]	Global [°C]
1	8.23	13.99
2	8.33	14.04
3	8.39	14.05
4	8.87	14.06
5	9.11	14.11
6	9.24	14.12
7	9.25	14.13
8	9.42	14.16
9	9.43	14.18
10	9.45	14.22
11	9.48	14.22
12	9.50	14.25
13	9.54	14.25
14	9.59	14.32
15	9.63	14.35
16	9.70	14.39
17	10.23	14.42
18	10.37	14.42
19	10.39	14.44
20	10.47	14.47

Table 2.1: Data of the annual maximum precipitation per hour.

b.   Plot the histogram and cumulative distribution function for both data sets. Show on the histogram the mean value, the median and the mode of each data set.

Exercise 2.7:

Table 2.2 provides the annual maximum values of precipitation per hour (rainfall), x_i, $i = 1, 2, .., n$, observed in the area of Zurich during the last $n = 9$ years. The data are given in ascending order.

i	Annual maximum precipitation per hour x_i (mm/hour)
1	73
2	85
3	93
4	98
5	100
6	108
7	115
8	121
9	130

Table 2.2: Data of the annual maximum precipitation per hour.

The modeling of the annual maximum precipitation per hour is of interest for the planning of safety measures. An environmental engineer believes that a Gumbel distributed random variable X is suitable for the modeling of the annual maximum precipitation per hour. Answer the following questions.

a.   Accepting that the Gumbel distribution is suitable for the modeling of the random variable X, use the method of moments to calculate the distribution parameters α and u. Table 2.3 provides the expression of the Gumbel distribution function and its first two moments.

	Gumbel distribution
Probability distribution function	$F_X(x) = \exp\left(-\exp\left(-\alpha\left(x-u\right)\right)\right)$
Moments	First order: $u + \dfrac{0.5772}{\alpha}$
	Second order: $\left(u + \dfrac{0.5772}{\alpha}\right)^2 + \dfrac{\pi^2}{6\alpha^2}$

Table 2.3: The Gumbel distribution function and its first two moments.

b.   Plot the data given in Table 2.2 on the provided probability paper (Figure 2.1). You may use Table 2.4 for the necessary calculations. (**_HINT:_** *The observed cumulative distribution function may be evaluated as* $F_{X,o}(x_i) = \dfrac{i}{n+1}$, *where* n *is the total number of observations*).

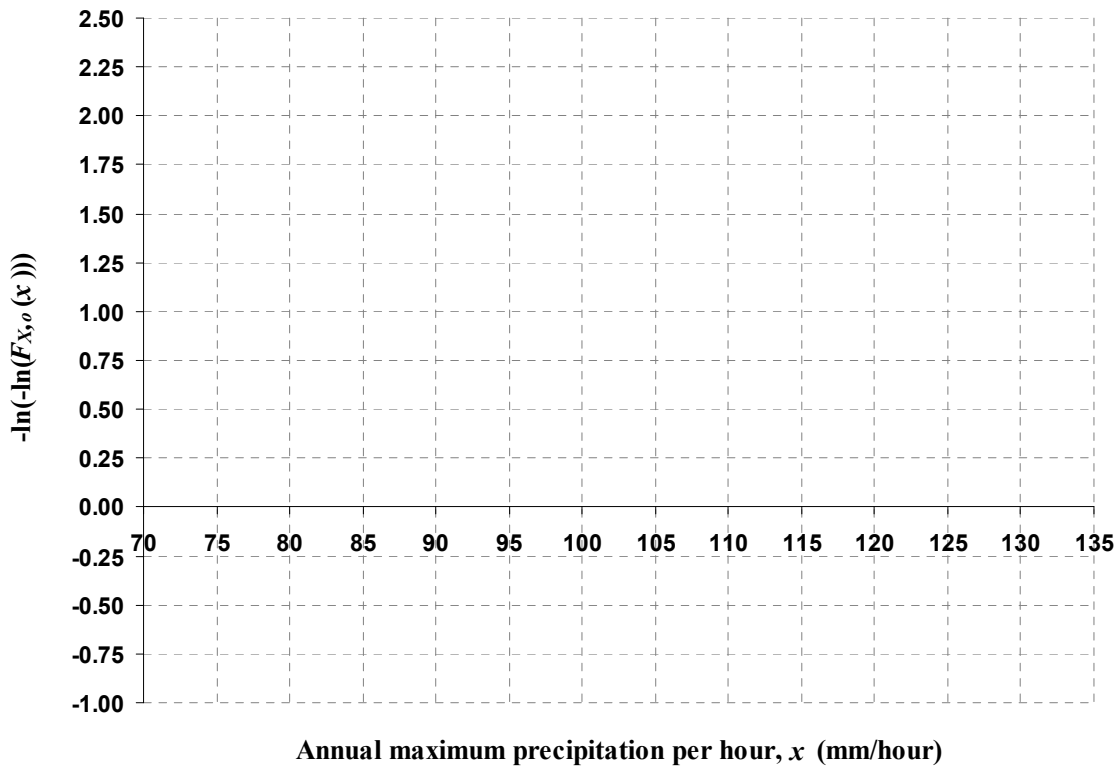| i	Annual maximum precipitation per hour x_i (mm/hour)	$F_{X,o}(x_i) = \dfrac{i}{n+1}$	$-\ln(-\ln(F_{X,o}(x_i)))$
1	73		
2	85		
3	93		
4	98		
5	100		
6	108		
7	115		
8	121		
9	130		

Table 2.4: Calculations for the probability paper.

Figure 2.1: Probability paper for the Gumbel distribution.

c. According to the resulting plot made in Figure 2.1, is the belief of the engineer, in regard to the choice of the Gumbel distribution function, correct?

d. Assuming that the answer in c. is yes, obtain the Gumbel distribution parameters α and u from the probability paper. Compare the values with the ones calculated using the method of moments. What do you observe? Why is this occurring?

Exercise 3:

Exercise 3.1:

After heavy snowfall, you need to decide whether to clean up a roof from the snow or not. There is though also the possibility to carry out firstly a test to check the melting status of the snow on the roof. The test basically provides an indication, I, of the wetness of the snow on the roof.

Therefore the decision problem is focused on either clean up the roof, do not clean up the roof or carry out first a test. In the following some information is provided to enable in the decision making.

The cost associated with carrying out a test is equal to 1000 CHF. The clean up of the roof can be made from the local fire department. This option is associated with a cost equal to 4000 CHF. In the case of collapse of the roof to the snow load the associated cost is equal to 1.000.000 CHF.

The probability of collapse of the roof has been estimated using First Order Reliability Methods (FORM). If the snow is dry, SD, the probability of collapse is: $P_f(SD) = 10^{-3}$. If the snow is wet, SW, the probability of collapse is: $P_f(SW) = 6.2 \cdot 10^{-3}$. In case where there is no snow, SN, on the roof the probability of collapse is equal to: $P_f(SN) = 5 \cdot 10^{-4}$.

The drawback of carrying out a test is that the test can provide a correct indication of the wetness, W, of the snow only in 75% of situations, $P(I_W|W) = 0.75$. You have a lot of experience in the assessment of snow wetness and based on this experience you believe that the probability that the snow on the roof is wet is equal to 0.6, $P(SW) = 0.6$.

The decision tree is provided in Figure 3.1. Estimate the branch probabilities (where necessary since some have been already provided) and fill in the boxes the costs associated with the different events. Which decision is the most beneficial one in terms of cost?
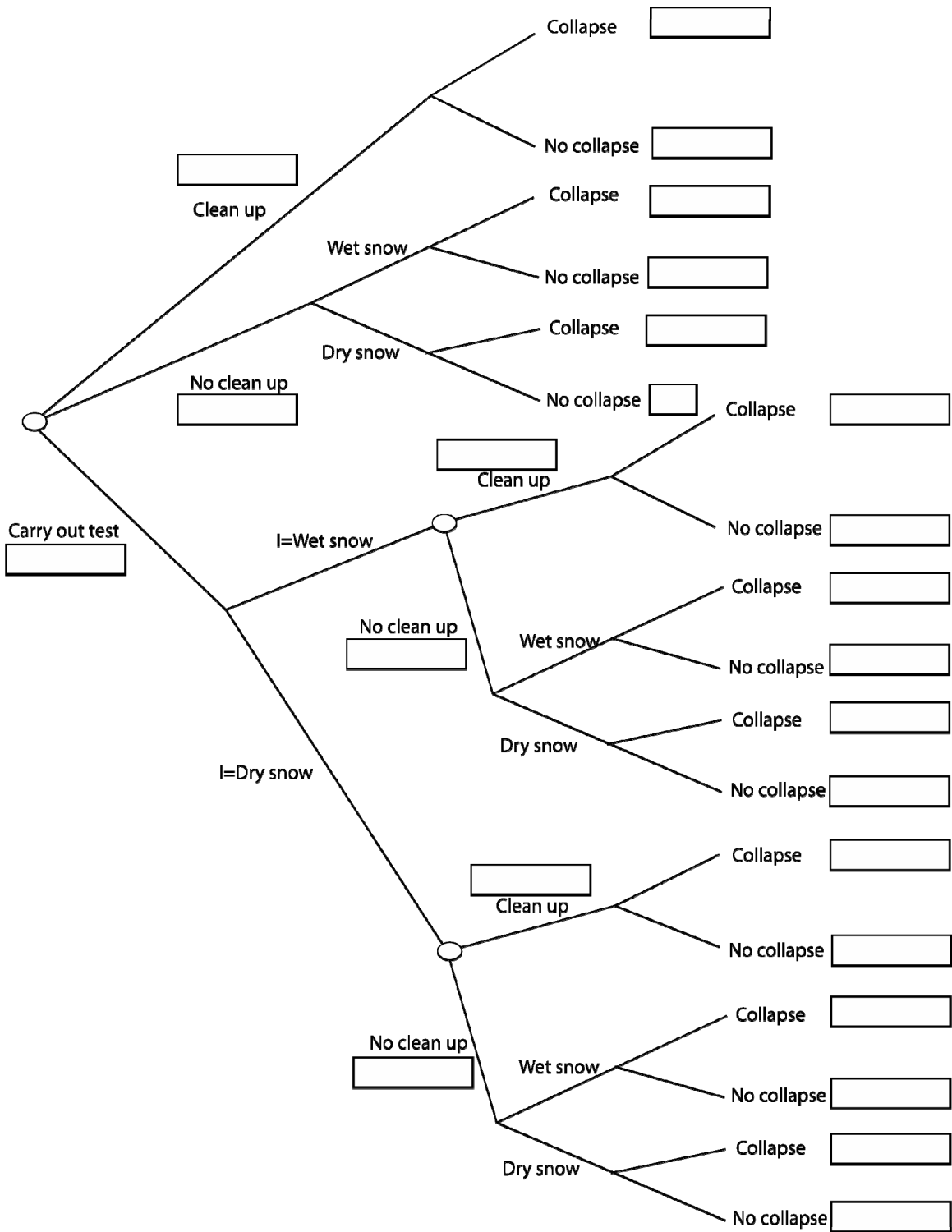
Collapse

No collapse

Collapse

Wet snow

No collapse

Collapse

Dry snow

No collapse

Clean up

Collapse

No collapse

No clean up

I=Wet snow

No clean up

Collapse

Wet snow

No collapse

Collapse

Dry snow

No collapse

Clean up

Carry out test

I=Dry snow

Clean up

Collapse

No collapse

No clean up

Collapse

Wet snow

No collapse

Collapse

Dry snow

No collapse

Figure 3.1. Event tree.

Exercise 3.2:

An oil producer needs to make the decision of opening up, a_1, or not, a_2, a borehole at a location where there is an old oil well. However, he is not sure of whether such an action is advantageous or not. The well maybe dry, D, it may be contaminated, C, (i.e. will have oil but and other substances also) or it may contain oil, O. Those events have a – priori probabilities equal to 50%, 30% and 20% respectively.

The benefit associated with the decision of opening up the borehole are estimated as the difference between the cost of opening the borehole and the cost benefit from the condition of the well (i.e. the income, if any, gained from the oil in the well). If the well is dry the benefit associated with opening up the borehole is -90.000 CHF. If the well is contaminated the benefit is equal to 50000 CHF and if the well has oil the benefit is equal to 170000 CHF.

a.   Carry out a pre-posterior decision analysis (using an appropriate event tree) and determine whether it is beneficial to open up the borehole.

b.   In order to receive some additional information regarding the state of the well the engineer can carry out a probe test that will cost 10000 CHF. The test can provide the following indications about the state of the well, WS :

- I_D : the well is dry
- I_C : the well is contaminated
- I_O : the well has oil

The information provided from the test is uncertain. The probabilities associated with a correct or false indication from the test about the state of the well given the true state of the well are given in the following table, $P(I_{WS}|WS)$

Indications	State of the well		
	D : dry	C : contaminated	O : oil
I_D	0.6	0.3	0.1
I_C	0.1	0.3	0.5
I_O	0.3	0.4	0.4

Construct an appropriate decision tree and estimate whether it is beneficial to carry out a test. Carry out then the posterior analysis to determine the benefit associated with opening up a borehole.

Exercise 4:

Exercise 4.1:

On a construction site the work is carried out in three phases involving different works. Every work is associated with a potential delay which may result in a delay of the whole construction project. In the following table the works carried out during each phase are shown together with the probability of delay associated with them.

	Works	Probability of delay
Phase 1	Form working	0.2
	Bending of Reinforcement	0.1
Phase 2	Installation of reinforcement	0.05
Phase 3	Concrete delivering and concreting	0.15

a. Estimate the probability for delay of the whole construction by using a fault tree analysis.

b. A possible delay during the construction will result in the construction project not being finished in autumn. Bad weather in the winter months will affect the work activities. The probabilities associated with the effect of the weather to the construction work are as follows:

   - 50% probability, that the construction work will not be affected
   - 20% probability, that there may be problems in concreting due to frost
   - 30% probability, that the construction work may stop due to heavy rainfall

In case of a delay of the project there is a penalty of 100000 CHF. In order to enable work in case of frost, the concrete could be enhanced with anti-frost additives at a cost of 20000 CHF. In case that the construction works must stop due to heavy rainfall the associated cost is equal to 50000 CHF.

Compute the costs in case of a delay using an event tree analysis.

c. A second form working team is appointed and the probability of a delay due to form working drops to 0.05. The cost of this extra team is 12000 CHF. Estimate again the costs of a delay as in 4.1.b, and discuss the decision to appoint a second team.

Exercise 5:

Exercise 5.1:

Consider the steel rod provided in the following.
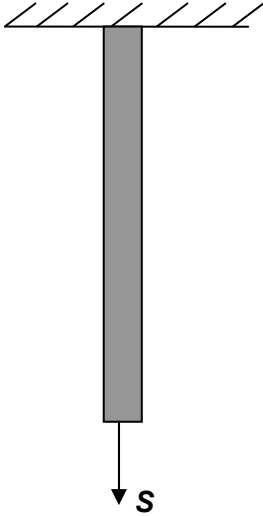
The rod carries a deterministic load, S =35 KN. The resistance, R, of the rod is given by the following product: $R = A \cdot f_y$, where A is the area of the rod, equal to 100 mm² and f_y is the yield stress modelled as a normal distributed random variable with mean $\mu_{f_y} = 425 KN/m^2$ and standard deviation $\sigma_{f_y} = 25 KN/m^2$.

Formulate a proper safety margin, M, for the steel rod and estimate the rod's reliability.

Draw the probability density function of the safety margin and indicate the safe and failure regions.

Exercise 6:

Exercise 6.1:

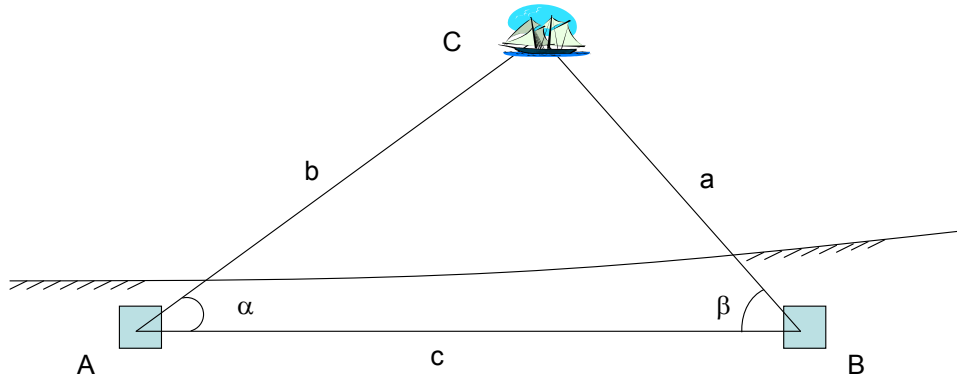The position of a ship is measured by two fixed points A and B located at the coast, see Figure 6.1.

Figure6.1:   Position determination of a ship

Angles α and β have been measured from the basis line AB at the same time. Determine the error in b if the following information is provided:

$$c = 6 \ km \pm 0.005 \ km$$
$$\alpha = 0.813 \ rad \pm 0.011 \ rad$$
$$\beta = 1.225 \ rad \pm 0.011 \ rad$$

where, for instance, $c = 6 \ km \pm 0.005 \ km$ means that the mean value of c is 6km and the stadnard deviation of c is 0.005km.

Exercise 6.2:
A boy wants to buy a TV game, which will be sold in the near future. The price of the TV game is not yet announced, but based on available information, he assumes that the price, A, follows the normal distribution with μ_A = 50 CHF, σ_A = 10 CHF. On the other hand he has 20 CHF at this moment, and until the sale of the TV game, he expects to get some money, B, from his parents. He assumes that this money follows the normal distribution with μ_B = 20 CHF, σ_B = 5 CHF.

Calculate the probability that he will fail to buy the TV game using the following steps:

a) Formulate an appropriate limit state function.
b) Reformulate the limit state function in the space of the standard normal variables.
c) Calculate the probability of failure.

Exercise 6.3:

Let X_1 and X_2 follow the standard normal distribution, and let the limit state function be expressed as:

$$g(X_1, X_2) = 2(X_1 - 1)^2 + X_2 - 3$$

Calculate the probability of failure $P[g(X_1, X_2) < 0]$.

Exercise 6.4:

Consider the simply supported beam in Figure 6.2. The beam has a rectangular cross-section and is supported over a span of 3 m. The beam is to carry a uniform distributed load of w KN/m with mean value equal to 5 KN/m and standard deviation equal to 1 KN/m. The beam depth h is also uniform along the span with a mean of 100 mm and standard deviation of 5 mm. Both variables are assumed normal distributed.

What is the probability that the central deflection will exceed 8 mm when the beam is loaded? (It is assumed that the beam's self weight is negligible).

Hint: The central deflection of a uniform beam under the uniformly distributed load may be calculated by:

$$w = \frac{5ql^4}{384EI}$$

where $I = \dfrac{b \cdot h^3}{12}$ and it is assumed that $E = 205 \, KN/mm^2$, $b = 50 \, mm$ and $l = 3 \, m$.
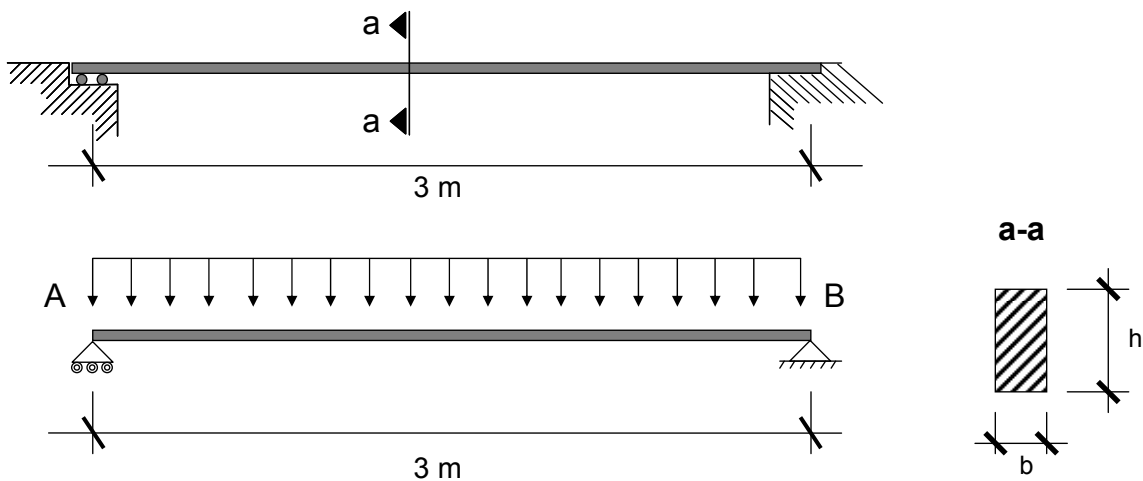

Figure 6.2:   Simply supported beam

Exercise 7:

Exercise 7.1:

Consider a concrete structure exposed to an alpine environment. The structure's degradation is due to chloride induced corrosion of the reinforcement. Corrosion itself may be seen as a process with two main phases, namely corrosion initiation and propagation.

The time until corrosion initiation can be modeled as:

$$T_{I_{mod}} = \frac{d^2}{4D}\left(erf^{-1}\left(1 - \frac{C_{CR}}{C_S}\right)\right)^{-2}$$

where:
d : the concrete cover depth to the reinforcement
D : the chloride diffusion coefficient
C_{CR} : the critical chloride concentration which when exceeded at the level of the reinforcement corrosion will initiate
C_S : the chloride concentration at the surface of the concrete

$erf^{-1}(.)$:error function expressed as: $erf^{-1}(x) = \dfrac{\Phi^{-1}\left(\dfrac{x}{2} + \dfrac{1}{2}\right)}{\sqrt{2}}$

Due to the simplifications associated with the modeling of the phenomenon of corrosion initiation a model uncertainty Ξ is introduced in the model such as:

$$T_I = \Xi T_{I_{mod}}$$

The time until corrosion is visual (a stage in the propagation phase) is simply:
$$T_{CV} = \Xi T_{I_{mod}} + T_P$$
where
T_P : the propagation time.

The statistical characteristics of the random variables in the above expressions are provided in Table 7.1.

Description	Representation	Units	Distribution	μ	σ
Cover thickness	d	mm	Log-normal	55	11
Diffusion coefficient	D	mm^2 / yr	Log-normal	40	10
Surface concentration	C_S	Water % of concrete	Log-normal	0.4	0.08
Critical concentration	C_{CR}	Water % of concrete	Log-normal	0.15	0.05
Propagation time	T_P	years	Log-normal	7.5	1.88
Model uncertainty	Ξ	-	Log-normal	1	0.05

Table 7.1:   Statistical characteristics of the random variables

Set up a program in Matlab to estimate the probability of corrosion initiation and of visual corrosion after $t = 50 years$ using Monte Carlo Simulation.

Hints:
- Formulate appropriate safety margins for the cases of corrosion initiation and visual corrosion.
- All necessary distribution functions are available in Matlab.
- To check up your program use initially a small number of simulations and then run the program for a sufficient number of simulations.
- If you are not familiar with Matlab try solving the exercise using Visual Basic for Applications. Keep though in mind that for large numbers of simulations it can get very slow).

Exercise 8:

Exercise 8.1:

The resistance of a structural member is given by the equation:
$$r(t) = r_o \cdot g(t)$$
where r_o is the initial resistance.

The limit state function, which compares the crack size to a defined critical crack size is:
$$g(t) = \frac{a_{cr} - a(t)}{a_{cr} - a_o}$$

The crack length at time t, $a(t)$, is given by:
$$a(t) = \left(a_o^{(2-m)/2} + \frac{2-m}{2} \cdot C \cdot \pi^{m/2} \Delta\sigma^m \cdot v \cdot t \right)^{2/(2-m)}$$
where:

α	:	crack length	v	:	number of stress cycles per time unit
α_o	:	initial crack length	t	:	time
α_{cr}	:	critical crack length	m	:	material constant
$\Delta\sigma$	:	fatigue stress range	C	:	material constant

For the computations, the following parameters should be used:

α_o	= 1 mm		v	=	1.000.000
α_{cr}	= 20 mm		m	=	3.6
$\Delta\sigma$	= 30 MPa		C	=	1.0E-14

The load, *S*, applied to the structural member is assumed to be ergodic and normal distributed with constant mean value equal to16 MPa and constant standard deviation equal to 8 MPa. The zero crossing frequency is given by $\omega_o = 1.000.000$ per year.

a.   Determine r_o, so that at time t_0 the reliability index β is 7. Assume r_o is deterministic and failures occur according to a Poisson process. Calculate the out-crossing rate $v^+(t)$ and the failure probability at 50 years $P_f(50)$.

b.   Assume that r_o is normal distributed with mean value 72 MPa and standard deviation 3.6 MPa. Calculate the failure probability at 50 years $P_f(50)$.

Exercise 9:

Exercise 9.1:

Consider the simply supported beam in Figure 9.1. The beam has a rectangular cross-section and is supported over a span of 3000 mm. The beam is to carry a uniform distributed load of q KN/m with mean value equal to 0.005 KN/mm and standard deviation equal to 0.001 KN/mm. The beam depth h is also uniform along the span with a mean of 100 mm and standard deviation of 5 mm. Both variables are assumed normal distributed.

Use COMREL to estimate the probability that the central deflection will exceed 8 mm when the beam is loaded? (It is assumed that the beam's self weight is negligible).

Hint: The central deflection of a uniform beam under the uniformly distributed load may be calculated by:

$$w = \frac{5ql^4}{384EI}$$

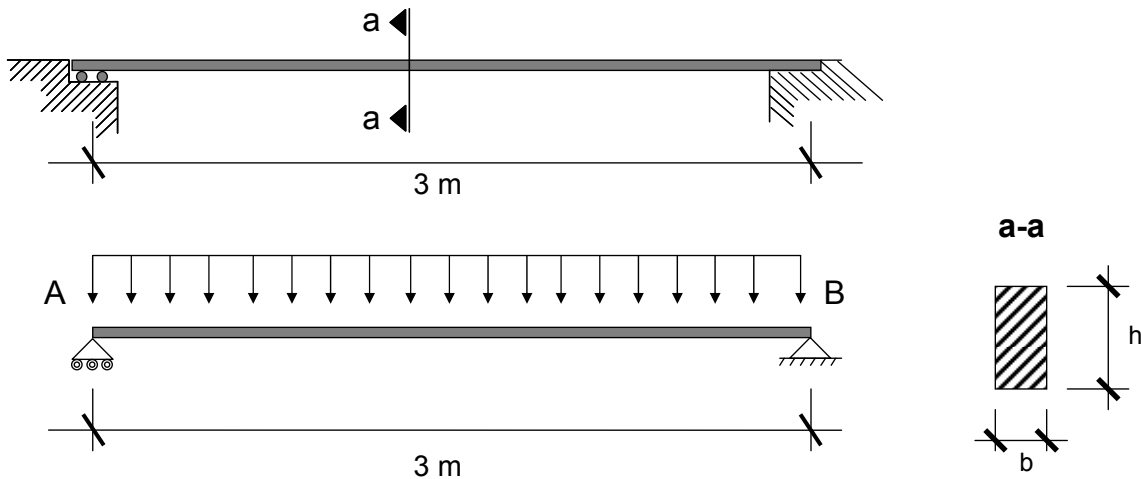where $I = \frac{b \cdot h^3}{12}$ and it is assumed that $E = 205\,KN/mm^2$, $b = 50\,mm$ and $l = 3000\,mm$.

Figure 6.2: Simply supported beam

Exercise 9.2:

Consider Exercise 9.1 but now the depth of the beam h has a normal distribution with mean μ_h and standard deviation $\sigma_h = \frac{\mu_h}{20}$. Use COMREL to estimate the probability that

the central deflection will exceed 10 mm for values of μ_h ranging from 80 to 120 mm (use a step of 10 mm).

Exercise 10:

Exercise 10.1:

Solve Exercise 8 by using system considerations.

It is suggested to subdivide the service life (T=50) into time components and to model failure within 50 years by means of a series system i.e. with 50 components.

Exercise 11:

Exercise 11.1:

Solve Exercise 4 using Bayesian Probabilistic Nets. (Software Hugin Lite)

Exercise 12:

Exercise 12.1:

Download from the web page of JCCS (http://www.jcss.ethz.ch) the free Software CodeCal and use it to solve Example 1 of the Lecture notes, Chapter 11.

Exercise 13:

Exercise 13.1:

Based on previous knowledge the bending strength of timber is assumed Normal distributed with standard deviation equal to $\sigma = 11\,MPa$ and uncertain mean μ assumed also Normal distributed with mean $\mu' = 43.8\,\text{MPa}$ and standard deviation $\sigma' = 3.5\,\text{MPa}$.

Assume now that 10 four point tests are carried out with the results shown in Table 13.1.

Based on the test results, update the prior probabilistic model for the mean value of the bending strength. Plot the prior and the posterior probability density functions.

Nr.	f_c [MPa]
1	31.5
2	38.3
3	36.6
4	38.2
5	35.1
6	44.9
7	50.8
8	42.5
9	39.6
10	36.9

Table 13.1　　Bending strength of the timber samples.

Exercise 13.2:

Revise the exercise presented in lecture 3 regarding the decision problem of establishing a well. The problem ranges through prior, posterior and pre-posterior analysis.

Exercise 13.3:

Steel channel profiles should be used to carry a ceiling panel. Channel profiles from U180 to U400 are available. The basic variables are modeled as given in the following table.
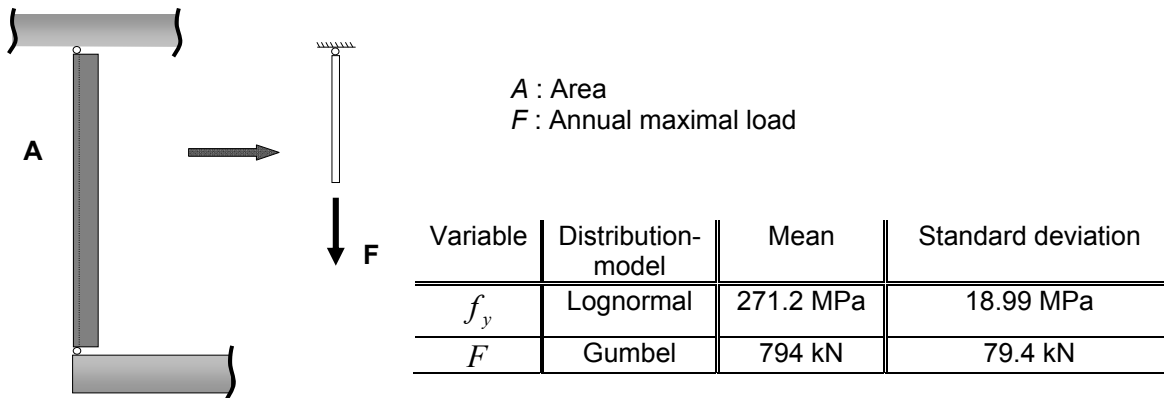
A : Area

F : Annual maximal load

Variable	Distribution-model	Mean	Standard deviation
f_y	Lognormal	271.2 MPa	18.99 MPa
F	Gumbel	794 kN	79.4 kN

Figure 13.1: Suspended ceiling panel

a Determine the reliability index β and the annual failure probability P_f related to the cross-sectional area A. Compute the failure rate λ assuming that failure follows a poisson process.

b Compute the optimal cross sectional area A^* for a time of 10 years.

$$U(A^*) = \max(B - C_T)$$

In order to capitalize the costs and revenues at the decision point a discount function $\delta(t)$ will be introduced.

$$\delta(t) = e^{-\gamma \cdot t} = \frac{1}{(1+r)^t}$$ where γ is the discount factor and r the interest rate.

The expected capitalized revenues B and costs C_T should be computed as follows:

$$B = \int_0^T b \cdot \delta(t) \cdot R(t) \cdot dt$$

$$C_T = C_B + C_F \qquad\qquad C_B,\ C_F: \text{construction costs, failure costs}$$

$$C_B = C_{B,0} + C_{B,1} \cdot A \qquad\qquad A \text{ in cm}^2$$

$$C_F = \int_0^T \overline{C_F} \cdot \delta(t) \cdot f(t) \cdot dt$$

Specifications: $C_{B,0} = 1$ monetary unit

$C_{B,1} = 0.1$ monetary units

$$\overline{C}_F = 30 \text{ monetary units}$$
$$b = 5 \text{ monetary units}$$
$$\gamma = 0.05$$
$$f(t) = \lambda \cdot e^{-\lambda \cdot t}$$
$$R(t) = e^{-\lambda \cdot t}$$

Hint: It is assumed, that the structural member is not reconstructed after failure, so that *R(t)* is equal to the structural reliability.

Exercise 14:

Exercise 14.1:

Consider a steel rod under pure tension. The rod will fail if the applied stress exceeds the steel yield stress. The resistance R of the rod and the loading strength of the rod S are assumed to be uncertain uncorrelated random variables. The yield strength is Log-normal distributed with a mean value of $\mu_R = 1\,MPa$ and coefficient of variation v_{f_y} which can take on values ranging from 0.05 to 0.8. The loading strength is modelled by a Gumbel distribution with a mean value of $\mu_S = 1\,N$ and coefficient of variation $v_S = 0.3$. The limit state function describing the condition of the rod is:

$$g(x \leq 0) = AR - S$$

where A is the rod's cross-sectional area in mm^2.

The expected value of persons N_{PE} affected by a failure may range from 1 to 1000. The probability of dying k given a rod failure is equal to one. The cost for steel depends on the decision alternative used for risk reduction, p. The costs are assumed to be 115 times the value of p. Hence it is:

$$k = 1$$
$$C_y(p) = 115p \;\; CHF$$

The benefit $B(p)$ from the use of the structural member and expected clean up costs C_U after the occurrence of a failure are:

$$C_U = 10.000\,CHF$$
$$B(p) = 100.000\,CHF$$

For values of the coefficient of variation R from 0.05 to 0.8 with a step of 0.02, estimate the values of the rod's cross-sectional area that maximize the utility of the owner. Examine the cases where the expected value of persons N_{PE} affected by a failure is equal to 10, 100 and 1000.