Testat examination 1
Statistics and probability theory

SS 2006

Prof. Dr. M.H. Faber

ETH Zurich

Tuesday, 16. Mai 2006
08:00 – 09:45

Surname: ....

Name: ....

Stud. Nr.: ....

Course of studies: ....

Date and duration:

Tuesday, 16. May 2006
Start: 8:00 Uhr
Duration: 90 minutes

Aids:

- Non-programmable pocket calculator
- No communication medium (e.g. cell phone)
- 4 pages (DIN A4 one-sided) original handwritten summary

Hints:

- Pease control first, if you have received all the materials (listed under: Contents).

- Please locate your Legi on your desk at the outside side.
-
- Please put your **name on every sheet of paper,** at the bottom left side**.**
-
- Use only the provided sheets of paper.

- Put **all materials** back in the envelope after the examination and do not leave your seats until all examination papers are collected.

- **Do not open** the paper fastener.

Contents

- General information and exercises (11 pages).
- 1 sheet of paper (checkered)

Part 1: „Multiple Choice“

In answering the following multiple choice questions it should be noted that for some of the questions several answers may be correct. Tick **ALL** correct alternatives in every question as: ☑

1.1 In probability theory the probability, $P(A)$, of an event A can take any value within the following boundaries:

$0 \le P(A) \le 1$ ☑

$-1 \le P(A) \le 1$ ☐

$-\infty \le P(A) \le \infty$ ☐

1.2 Which one(s) of the following expressions is(are) correct?

The probability of the union of two events A and B is equal to the sum of the probability of event A and the probability of event B, given that the two events are mutually exclusive. ☑

The probability of the union of two events A and B is equal to the probability of the sum of event A and event B, given that the two events are mutually exclusive. ☐

The probability of the intersection of two events A and B is equal to the product of the probability of event A and the probability of event B, given that the two events are mutually exclusive. ☐

The probability of the intersection of two events A and B is equal to the product of the probability of event A and the probability of event B, given that the two events are independent. ☑

1.3 Within the theory of sample spaces and events, which one(s) of the following statements is(are) correct?

An event A is defined as a subset of a sample space Ω. ☑

A sample space Ω is defined as a subset of an event A. ☐

1.4 The 0.75 quantile of a data set corresponds to a value of the data set for which:

75% of the data values in the data set are smaller. ☑

75% of the data values in the data set are larger. ☐

1.5 Two data sets of the realizations of the same random variable X are available. The estimated sample means and sample standard deviations are as given in the following: $\overline{x_1} = 28$, $\overline{x_2} = 21$, $s_1 = s_2 = 7$.

Which one(s) of the following statements is(are) correct?

The dispersion of the first data set is larger. ☐

The dispersion of the second data set is larger. ☑

The two data sets exhibit the same dispersion. (* see end of multiple choice) ☑

1.6 If the intersection of two events, A and B corresponds to the empty set $\varnothing$, i.e. $A \cap B = \varnothing$, the two events are:

Mutually exclusive. ☑

Independent. ☐

Empty events. ☐

1.7 The probability of the intersection of two mutually exclusive events is equal to:

The product of the probabilities of the individual events. ☐

The sum of the probabilities of the individual events. ☐

The difference between the probabilities of the individual events. ☐

One (1). ☐

Zero (0). ☑

None of the above. ☐

1.8 The probability of the union of two not mutually exclusive events A and B is given as: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. It is provided that the probability of event A is equal to 0.1, the probability of event B is 0.1 and the probability of event B given event A, i.e. $P(B|A)$ is 0.8. Which result is correct?

$P(A \cup B) = -0.6$ ☐

$P(A \cup B) = 0.12$ ☑

$P(A \cup B) = 0.04$ ☐

1.9 For an event A in the sample space Ω, event $\overline{A}$ represents the complementary event of event A. Which one(s) of the following hold?

$A \cup \overline{A} = \Omega$ ☑

$A \cap \overline{A} = \Omega$ ☐

$A \cup \overline{A} = \varnothing$ ☐

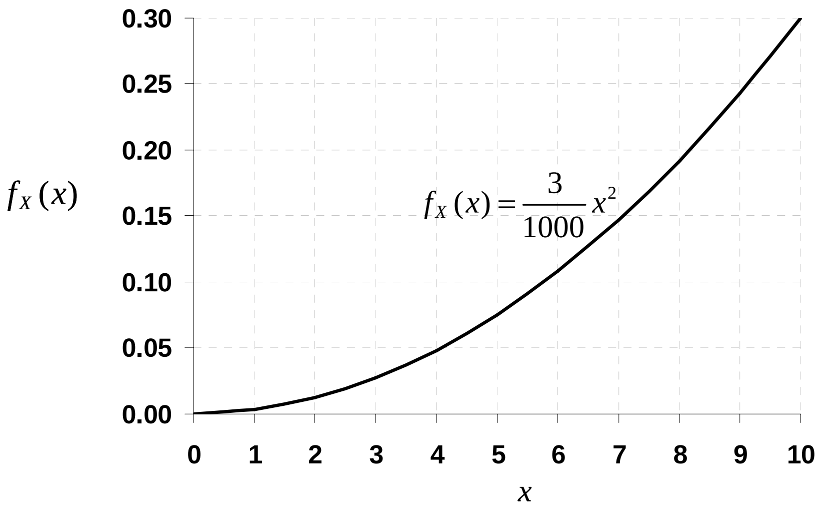
1.10 Probability distribution functions may be defined in terms of their moments. If X is a continuous random variable which one(s) of the following is(are) correct?

The first moment of X corresponds to its mean value, μ_X. ☑

The second moment of X corresponds to its mean value, μ_X. ☐

The second central moment of X corresponds to its variance, σ_X^2. ☑

1.11 The probability density function of a continuous random variable X is illustrated in the following diagram.



The probability of X exceeding the value of 5 is equal to:

$P(X > 5) = 0.875$ ☑

$P(X > 5) = 0.055$ ☐

$P(X > 5) = 0.125$ ☐

1.12 The variance of a continuous random variable X can be expressed as: $Var(X) = E\left[(X-\mu_X)^2\right]$, where μ_X is the mean value of X and $E[\cdot]$ is the expectation operator. Based on this expression which one(s) of the following expressions is(are) correct?

$Var(X) = E\left[X^2\right] - \mu_X{}^2$ ☑

$Var(X) = \mu_X - E\left[X^2\right]$ ☐

$Var(X) = X^2 - \mu_X$ ☐

1.13 Reverend Thomas Bayes was working with the mathematical modeling of how to combine a-priori knowledge with observations (evidence). At approximately the same time a well known philosopher was working on the same problem from a meta-physical perspective. The philosopher was:

I. Kant. ☑

A. Schopenhauer. ☐

F.W. Nietzsche. ☐

None of the above. ☐

1.14 Imagine that you have thrown a dice and that the dice is still hidden by a cup. What kind(s) of uncertainty is(are) associated with the outcome of the dice?

Aleatory uncertainty. ☐

Statistical uncertainty. ☐

Inherent random variability. ☐

None of the above. ☑

1.15 At a given location in Switzerland it has been observed that on average 4 avalanches occur per year. The annual probability of a house being hit by an avalanche on this location is thus:

Equal to one (1). ☐

Larger than one (1). ☐

None of the above. ☑

1.16 The convolution integral in probability describes how the probability density function for the sum of two random variables can be established. However, assumption(s) for its derivation is(are) that:

The random variables are normally distributed. ☐

The random variables are independent. ☑

The random variables are continuous. ☑

None of the above. ☐

1.17 Which one(s) of the following statements is(are) meaningful:

The probability of a big earthquake for the region around Zurich is close to 0.02. ☐

Strong winds occur in Ireland with a probability of 0.7. ☐

The probability of getting struck by lighting is equal to 0.1, if you stand under a tree. ☐

None of the above. ☑

1.18 A given random variable is assumed to follow a normal distribution. Which parameter(s) is(are) sufficient to define the probability distribution function of the random variable:

The variance and the standard deviation. ☐

The standard deviation and the mean value. ☑

The mode and the coefficient of variation. ☑

None of the above. ☐

1.19 Which one(s) of the following features is(are) characteristics of a normal distribution function?

The variance is equal to the coefficient of variation. ☐

The mode is equal to the median. ☑

The skewness is equal to zero. ☑

None of the above. ☐

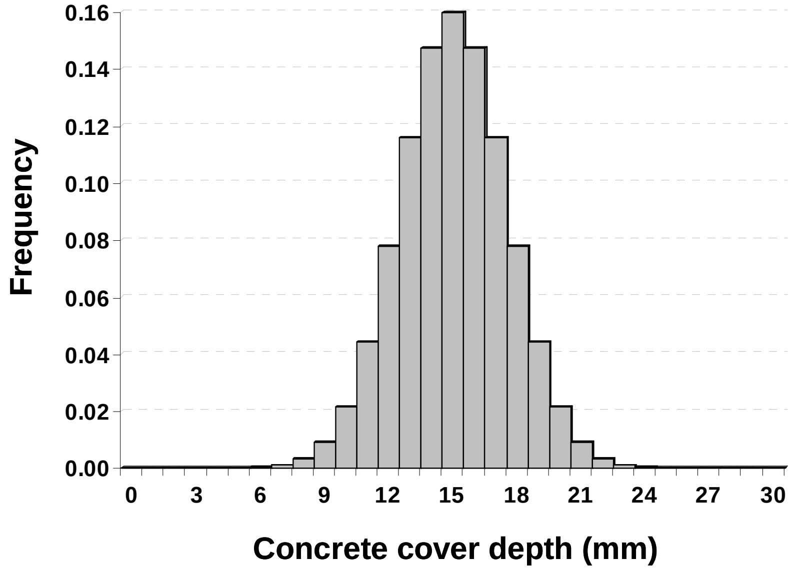
1.20 The median of a data set corresponds to:

The lower quartile of the data set. ☐

The 0.5 quantile of the data set. ☑

The upper quartile of the data set. ☐

1.21 Measurements were taken of the concrete cover depth of a bridge column. The following symmetrical histogram results from the plot of the measured values:



If X represents the random variable for the concrete cover depth, which one(s) of the following statements is(are) correct?

The sample mean, $\bar{x}$, is equal to 0.16 mm. ☐

The sample mean, $\bar{x}$, is equal to 15 mm. ☑

The mode of the data set is equal to 15 mm. ☑

1.22 The commutative, associative and distributive laws describe how to:

Operate with probabilities. ☐

Operate with intersections of sets. ☑

Operate with unions of sets. ☑

None of the above. ☐

1.23 Which one(s) of the following statements is(are) correct for a uniformly distributed random variable?

- [ ] The expected value of the random variable is equal to 1.
- [ ] The probability distribution function is constant over the definition space.
- [x] The probability density function is constant over the definition space.
- [ ] None of the above.

<u>* Explanation on multiple choice question 1.5:</u>

Here one of the two marked answers can be taken as correct. The correct answer depends on the interpretation of the word “dispersion”. If you consider the coefficient of variation as a measure of dispersion, then the “dispersion of the second data set is larger”. If you consider the standard deviation as a measure of dispersion then “the two data sets exhibit the same dispersion”.

Part 2: Exercise - Solution

Given:

D_1 : the proposal is accepted and the project will be funded.
D_2 : the proposal should be revised by the Professor and resubmitted to SNF.
D_3 : the proposal is not accepted and hence no funding is provided.

$P(D_1)=0.45$, $P(D_2)=0.35$, $P(D_3)=0.2$.

a. Complete the table.

SNF final decision D_i	**Dr. Beispiel's indicative assessment,** I_j		
	$I_j = D_1$	$I_j = D_2$	$I_j = D_3$
D_1	**0.86**	**0.1**	**0.04**
D_2	**0.2**	**0.74**	**0.06**
D_3	**0**	**0.1**	**0.9**

b. Using the Bayes' Theorem the probability that the final decision made by SNF ist he same with the indicative assessment of Dr. Beispiel is:

$$P\left(D_2 \middle| I = D_2\right) = \frac{P(I = D_2 | D_2)P(D_2)}{\sum_{i=1}^{3} P(I = D_2 | D_i)P(D_i)} = \frac{P(I = D_2 | D_2)P(D_2)}{P(I = D_2 | D_1)P(D_1) + P(I = D_2 | D_2)P(D_2) + P(I = D_2 | D_3)P(D_3)}$$

$$\frac{0.74 \cdot 0.35}{(0.1 \cdot 0.45) + (0.74 \cdot 0.35) + (0.1 \cdot 0.2)} = 0.799$$

Glossary

Annual probability	Jährliche Wahrscheinlichkeit
Avalanches	Lawinen
Average	Durchschnitt
Bridge column	Brückenpfeiler
Coefficient of variation	Variationskoeffizient
Complementary event	Komplementärereignis
Concrete cover depth	Betonüberdeckung
Continuous	Kontinuierlich
Decision	Entscheidung
Derivation	Herleitung
Dice	Würfel
Discrete	Diskret
Earthquake	Erdbeben
Event	Ereignis
Expected value	Erwartungswert
Experts	Experten
Funded	Finanziert
Independent	Unabhängig
Integral	Integral
Intersection	Durchschnitt (von Mengen)
Mean value	Mittelwert
Measurements	Messungen
Median	Median
Mode	Modus
Moments	Momente
Mutually exclusive	Gegenseitig ausschliessend
Observations	Beobachtungen
Probability	Wahrscheinlichkeit
Probability density function	Wahrscheinlichkeitsdichtefunktion
Probability distribution function	Wahrscheinlichkeitsverteilungsfunktion
Proposal	Projektvorschlag
Quantile	Quantil
Random variable	Zufallsvariable
Research	Forschung
Review	Bewertung
Revised	Überarbeitet
Sample mean	Stichprobenmittelwert
Sample space	Stichprobenraum
Standard deviation	Standardabweichung
Submit	einreichen
Subset	Teilmenge
Swiss National Foundation (SNF)	Schweizerischer Nationalfonds
Uncertainty	Unsicherheit
Uniformly distributed random variable	Gleichverteilte Zufallsvariable
Variance	Varianz