ibk 1



Exercises Tutorial 3

Statistics and Probability Theory

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Exercise 3.1 (Descriptive Statistics)

Two sets of data are provided, each of which represents the daily traffic flow in Rosengartenstrasse in Zurich during the month of April 2001 Direction 1 corresponds to driving towards Bucheggplatz, while direction 2 corresponds to driving towards Escher Wyss Platz.

)ate		Direction 1	Direction 2
	01.04.2001	32618	24609
	02.04.2001	33380	29965
	03.04.2001	34007	30629
	04.04.2001	33888	30263
	05.04.2001	35237	31405
	06.04.2001	35843	31994
	07.04.2001	33197	26846
	08.04.2001	30035	22762
	09.04.2001	32158	30366
	10.04.2001	33406	29994
	11.04.2001	34576	30958
	12.04.2001	34013	30680
	13.04.2001	24846	19735
	14.04.2001	28252	21145
	15.04.2001	25365	17805
	16.04.2001	24862	18123
	17.04.2001	32472	28117
	18.04.2001	33245	28858
	19.04.2001	33788	29080
	20.04.2001	34076	30313
	21.04.2001	29976	23141
	22.04.2001	29224	20903
	23.04.2001	32962	27746
	24.04.2001	33937	29586
	25.04.2001	33198	30788
	26.04.2001	34455	31074
	27.04.2001	35852	32384
	28.04.2001	33091	26525
	29.04.2001	30613	22828
	30.04.2001	34425	28877





Which is the best way to know it? - plot, histogram, statistics etc.

For example, if you are interested in: the change in the traffic of direction 1 during the month





Which is the best way to know it? - plot, histogram, statistics etc.

For example, if you are interested in: the relation between the traffic of direction 1 and that of direction 2,





Which is the best way to know it? – plot, histogram, statistics etc.

For example,

if you are interested in:

the relation between the traffic of direction 1 and that of direction 2, but you are not interested in the time element



Correlated!



Which is the best way to know it? - plot, histogram, statistics etc.

For example, if you are interested in: traffic volume of each direction



We will look today into...

how to represent and compare the properties of sets of data which you have

≻graphically

frequency distribution (histogram) cumulative frequency distribution

≻numerically median quantile

➤a summary plot Tukey box plot

Correlation between data sets

You can use excel, matlab and/or other programming/statistics software....BUT

Make sure ALWAYS to insert functions by yourself or check that the functions provided by the used program agree with those of the used script!



Provide frequency distributions and cumulative frequency distributions of the observed data. What is your first impression of the data? Try to make comparison between the two directions.

Date	Direction 1	Direction 2
01.04.2001	32618	24609
02.04.2001	33380	29965
03.04.2001	34007	30629
04.04.2001	33888	30263
05.04.2001	35237	31405
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15.04.2001	25365	17805
16.04.2001	24862	18123
17.04.2001	32472	28117
18.04.2001	33245	28858
19.04.2001	33788	29080
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29.04.2001	30613	22828
30.04.2001	34425	28877

Steps

- 1. sort the data
- 2. select the number of intervals
- 3. count the data in each interval
- 4. draw the frequency distribution
- 5. draw the cumulative frequency distribution



Step 1 (sort the data)

Date	Direction 1	Direction 2
01.04.2001	32618	24609
02.04.2001	33380	29965
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13.04.2001	24846	19735
14.04.2001	28252	21145
15.04.2001	25365	17805
16.04.2001	24862	18123
17.04.2001	32472	28117
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29.04.2001	30613	22828
30.04.2001	34425	28877

	Direction 1	Direction 2
	24846	17805
	24862	18123
	25365	19735
	28252	20903
	29224	21145
	29976	22762
	30035	22828
	30613	23141
	32158	24609
	32472	26525
	32618	26846
	32962	27746
	33091	28117
oort/ordor	33197	28858
Solloldel	33198	28877
	33245	29080
	33380	29586
in occording	33406	29965
in ascending	33788	29994
	33888	30263
order	33937	30313
01001	34007	30366
	34013	30629
	34076	30680
	34425	30788
	34455	30958
	34576	31074
	35237	31405
	35843	31994
	35852	32384

Steps

- 1. sort the data
- 2. select the number of intervals
- 3. count the data in each interval
- 4. draw the frequency distribution
- 5. draw the cumulative frequency distribution



	Steps
	1. sort the data
	2. select the number of intervals
	3. count the data in each interval
Stop 2 (coloct the number of intervale)	4. draw the frequency distribution
Step Z (select the number of intervals)	5. draw the cumulative frequency distribution

No general rule but suggestion - (script Equation (C.8)) $k = 1 + 3.3 \log_{10} n$

k is the number of the intervals, n is the number of the data.

In this case, n = 30 $k = 1 + 3.3 \log_{10} 30 = 5.87 \approx 6$ intervals

For direction 1, minimum = 24846 max = 35852

we may select the intervals as follows:

[24.5 26.5 28.5 30.5 32.5 34.5 36.5] (*1000)



Step 3 (count the data in each interval)

- 1.
- sort the data
- 2. select the number of intervals
- 3. count the data in each interval
- 4. draw the frequency distribution
- draw the cumulative frequency distribution 5.



35852

£	Interval (Number of cars *10 ³)	Interval Midpoint (Number of cars *10 ³)	Number of observations
uo	24.5-26.5	25.5	3
cti	26.5-28.5	27.5	1
re	28.5-30.5	29.5	3
Ō	30.5-32.5	31.5	3
	32.5-34.5	33.5	16
	34.5-36.5	35.5	4



- 1. sort the data
- 2. select the number of intervals
- 3. count the data in each interval
- 4. draw the frequency distribution
- 5. draw the cumulative frequency distribution

Step 4 (draw the frequency distribution)

But first some calculations....

7	Interval (Number of cars *10 ³)	Interval Midpoint (Number of cars *10 ³)	Number of observations	Frequency %
on	24.5-26.5	25.5	3	10.000
cti	26.5-28.5	27.5	1	3.333
ē	28.5-30.5	29.5	3	10.000
ā	30.5-32.5	31.5	3	10.000
	32.5-34.5	33.5	16	53.333
	34.5-36.5	35.5	4	13.333

Frequency% =
$$\frac{n_o}{n}$$
100
= $\frac{3}{30}$ 100 = 10 ×



2. select the number of intervals

sort the data

- 3. count the data in each interval
- 4. draw the frequency distribution
- 5. draw the cumulative frequency distribution



Step 4 (draw the frequency distribution)





- 1. sort the data
- 2. select the number of intervals
- 3. count the data in each interval
- 4. draw the frequency distribution
- 5. draw the cumulative frequency distribution

				Cun	nulate
۲	Interval (Number of cars *10 ³)	Interval Midpoint (Number of cars *10 ³)	Number of observations	Frequency %	Cumulative frequency
u	24.5-26.5	25.5	3	10.000	0.100
cti	26.5-28.5	27.5	1	3.333	0.133
ē	28.5-30.5	29.5	3	10.000	0.233
١	30.5-32.5	31.5	3	10.000	0.333
	32.5-34.5	33.5	16	53.333	0.867
	34.5-36.5	35.5	4	13.333	1.000

Step 5 (draw the cumulative frequency distribution)



- 1. sort the data
- 2. select the number of intervals
- 3. count the data in each interval
- 4. draw the frequency distribution
- 5. draw the cumulative frequency distribution

Step 5 (draw the cumulative frequency distrib	ution)	
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			Cumulate		
-	Interval (Number of cars *10 ³)	Interval Midpoint (Number of cars *10 ³)	Number of observations	Frequency %	Cumulative frequency
o	24.5-26.5	25.5	3	10.000	0.100
cti	26.5-28.5	27.5	1	3.333	+0.133
Le	28.5-30.5	29.5	3	10.000	0.233
ā	30.5-32.5	31.5	3	10.000	0.333
	32.5-34.5	33.5	16	53.333	0.867
	34.5-36.5	35.5	4	13.333	1.000



- 1. sort the data
- 2. select the number of intervals
- 3. count the data in each interval
- 4. draw the frequency distribution
- 5. draw the cumulative frequency distribution



Step 5 (draw the cumulative frequency distribution)

Answer 3.1

Do the same for direction 2.

What can we know from these plots?



These figures give nice overviews of the data!

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Answer 3.1

a. When we have in hand all observations:

prefer to plot the cumulative distribution plot using the quantiles of the data!

b. If we have in hand only the intervals observed and the frequency of observations within each interval

a. is not possible so...plot the cumulative frequency!





A quantile is related to a given percentage α , for which α % of all observations in the data set have smaller values.

e.g. the 0.65 quantile of a given data set of observations corresponds to the observation for which 65% of all observations in the data set have smaller values



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Date		Direction 1	Direction 2	
	01.04.2001	32618	24609	
	02.04.2001	33380	29965	
	03.04.2001	34007	30629	
	04.04.2001	33888	30263	
	05.04.2001	35237	31405	
	06.04.2001	35843	31994	
	07.04.2001	33197	26846	
	08.04.2001	30035	22762	
	09.04.2001	32158	30366	
	10.04.2001	33406	29994	
	11.04.2001	34576	30958	
	12.04.2001	34013	30680	
	13.04.2001	24846	19735	
	14.04.2001	28252	21145	
	15.04.2001	25365	17805	
	16.04.2001	24862	18123	
	17.04.2001	32472	28117	
	18.04.2001	33245	28858	
	19.04.2001	33788	29080	
	20.04.2001	34076	30313	
	21.04.2001	29976	23141	
	22.04.2001	29224	20903	
	23.04.2001	32962	27746	O = 0.74
	24.04.2001	33937	29586	Q -011
	25.04.2001	33198	30788	
	26.04.2001	34455	31074	
	27.04.2001	35852	32384	
	28.04.2001	33091	26525	
	29.04.2001	30613	22828	
	30.04.2001	34425	28877	



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e.g. the 0.65 quantile of a given data set of observations corresponds to the observation for which 65% of all observations in the data set have smaller values

Date	Direc	ction 1	Directi	on 2		
01.04	.2001	32618		24609		k
02.04	.2001	33380		29965		
03.04	.2001	34007		30629		
04.04	.2001	33888		30263		
05.04	.2001	35237		31405		
06.04	.2001	35843		31994		
07.04	.2001	33197		26846		
08.04	.2001	30035		22762		
09.04	.2001	32158		30366		
10.04	.2001	33406		29994		
11.04	.2001	34576		30958		74% of the observations
12.04	.2001	34013		30680		7470 of the obset validity
13.04	.2001	24846		19735		Have a smaller value!
14.04	.2001	28252		21145		
15.04	.2001	25365		17805		
16.04	.2001	24862		18123		
17.04	.2001	32472		28117		
18.04	.2001	33245		28858		
19.04	.2001	33788		29080		
20.04	.2001	34076		30313		
21.04	.2001	29976		23141		
22.04	.2001	29224		20903		
23.04	.2001	32962	<	27746	>	O=0.74
24.04	.2001	33937		29586		
25.04	.2001	33198		30788		
26.04	.2001	34455		31074		
27.04	.2001	35852		32384		
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A quantile is related to a given percentage α , for which α % of all observations in the data set have smaller values.

e.g. the 0.65 quantile of a given data set of observations corresponds to the observation for which 65% of all observations in the data set have smaller values

How to calculate it????

 $Q_i = \frac{i}{n+1}$, *n*: total number of observations



Use the Tukey box plot to provide a summary of the main features of the distribution of each data set. Plot the Tukey box plots on the same graph so that you are able to compare these features. Do you observe any symmetry in the data sets?

Steps

- 1. calculate the median
- 2. calculate the 75%- and 25%- quantile.
- 3. calculate the adjacent values.
- 4. check for outside values
- 5. draw the Tukey box plot



Step 1 (calculate the m	nedian)	Ste 1. 2. 3. 4. 5.	calculate the median calculate the 75%- and 25%- quantile calculate the adjacent values. check for outside values draw the Tukey box plot
Just take the central va	alue (50%-quantile).		
but	if the number of data	a is eve	en, this is not possible!
Direction 1 24846 24862 25365 28252 29224 29976 30035 30613 32158 32472 32618 32962 33091 33197 33198 33245 33380 33406 33788 3388 33937 34007 34013 34076 34425 34455 34576 35237 35843 35852	In that case, take the then take the average Median is $\frac{33198 + 2}{2}$	e two va ge. - 33245	alues around the center, $\frac{1}{2} = 33221.5$



Step 2 (calculate the quantiles)

Roughly speaking,

Steps

- 1. calculate the median
- 2. calculate the 75%- and 25%- quantile.
- 3. calculate the adjacent values.
- 4. check for outside values
- 5. draw the Tukey box plot



Step 2 (calculate the quantiles)

More strictly speaking,

$$Q_i = \frac{i}{n+1}$$
, *n*: total number of observations

Direction 1	i	i/31	
24846	1	0.03	
24862	2	0.06	
25365	3	0.10	
28252	4	0.13	
29224	5	0.16	
29976	6	0.19	
30035	7	0.23	
30613	8	0.26	
32158	9	0.29	
32472	10	0.32	
32618	11	0.35	
32962	12	0.39	
33091	13	0.42	
33197	14	0.45	
33198	15	0.48	
33245	16	0.52	
33380	17	0.55	
33406	18	0.58	
33788	19	0.61	
33888	20	0.65	
33937	21	0.68	
34007	22	0.71	
34013	23	0.74	750/
34076	24	0.77	15%
34425	25	0.81	
34455	26	0.84	
34576	27	0.87	
35237	28	0.90	
35843	29	0.94	
35852	30	0.97	



Step 2 (calculate the quantiles)







		Ste	eps
Step 3 (calculate the	e adjacent values)	1. 2. 3. 4. 5.	calculate the median calculate the 75% and 25% quantile. calculate the adjacent values. check for outside values draw the Tukey box plot
$Q_{0.75} = 34029$ $Q_{0.25} = 30469$	$\begin{cases} \text{Interquartile range} \\ r \equiv Q_{0.75} - Q_{0.25} = 34029 \end{cases}$	- 3	0469 = 3560

<u>Upper adjacent value</u>: largest observation \leq (75% *quantile*) + 1.5*r*

Upper adjacent value =



	Steps
	1. calculate the median
	2. calculate the 75% and 25% quantil
Ctop 2 (coloulate the ediceost values)	3. calculate the adjacent values.
Step 3 (calculate the adjacent values)	4. check for outside values
	5. draw the Tukey box plot

$$\left. \begin{array}{l} Q_{0.75} = 34029 \\ Q_{0.25} = 30469 \end{array} \right\} r \equiv Q_{0.75} - Q_{0.25} = 34029 - 30469 = 3560 \\ \end{array}$$

<u>Lower adjacent value</u>: smallest observation \geq (25% *quantile*) - 1.5*r*

Direction 1	
24846	
24862	
25365	
28252	
29224	
29976	
30035	lower adjacent value =
30613	,
32158	
32472	
32618	
32962	
33091	
33197	
33198	



		Ste	eps
Direction 1 24846	Step 4	1. 2.	calculate the median calculate the 75% and 25% quantile
24862 25365	(check for outside values)	3. 4. 5.	calculate the adjacent values. check for outside values draw the Tukey box plot
28252 29224	Outside values:		
29976 30035 30613	Outside the upper and lower adjacent values 24846		
32158 32472 32618	24862		
32962 33091	summary		
33197 33198 33245	Upper adjacent value: 35852 75% quantile : 34029		
33380 33406	Median : 33222		
33788 33888 33937	25% quantile : 30469 Lower adjacent value: 25365		
34007 34013 34076			
34425 34455 34576 35227			
35843			







Answer 3.2

Use the Tukey box plot to provide a summary of the main features of the distribution of each data set.

- median
- Adjacent values
- Upper and lower quartiles
- Outside values

Plot the Tukey box plots on the same graph so that you are able to compare these features. ???

Do you observe any symmetry in the data sets? ???





The data sets in Table 3.5.1 show the number of newcomers to the university and the number of total students at the university.

Estimate the correlation of these numbers using the following calculation sheet.

	Univ. A	Univ. B	Univ. C	Univ. D	Univ. E	Univ. F
Newcomer	3970	732	499	1300	3463	2643
Total students	24273	5883	2847	5358	23442	17076

 Table 3.5.1
 Number of newcomers to the university and the number of total students at the university.



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	Univ. A	Univ. B	Univ. C	Univ. D	Univ. E	Univ. F
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Total students	24273	5883	2847	5358	23442	17076

 Table 3.5.1
 Number of newcomers to the university and the number of total students at the university.

What is known?

Newcomers:	X
total students:	Y
Number of newcomers:	$x_i, i=1,,6$
Number of total students:	$y_{i}, i=1,,6$
Number of observations/university:	n=6

Estimate the correlation of these numbers using the following calculation sheet.

	Univ. A	Univ. B	Univ. C	Univ. D	Univ. E	Univ. F
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Number of newcomers:	$x_i, i=1,,6$
Number of total students:	$y_i, i=1,,6$
Number of observations/university:	<i>n</i> =6

What is required?

Correlation:
$$r_{XY} = \frac{1}{n} \sum_{i=1}^{n} \frac{(x_i - \bar{x})(y_i - \bar{y})}{s_X s_Y}$$

<u>Need to:</u> Calculate the sample mean values: \overline{x} \overline{y}

Calculate sample standard deviations: $s_X \ s_Y$



Estimate the correlation of these numbers using the following calculation sheet.

	Univ. A	Univ. B	Univ. C	Univ. D Univ. E		Univ. F
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Solution 3.5

	Univ. A	Univ. B	Univ. C	Univ. D	Univ. E	Univ. F
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What is known?

Newcomers:	X				
total students:	Y				
Number of newcomers:	$x_i, i=1,,6$				
Number of total students:	$y_{i}, i=1,,6$				
Number of observations/university: $n=6$					

 Table 3.5.1
 Number of newcomers to the university and the number of total students at the university.

What is required?

Correlation:
$$r_{XY} = \frac{1}{n} \sum_{i=1}^{n} \frac{(x_i - \overline{x})(y_i - \overline{y})}{s_X s_Y}$$

<u>Need to:</u> Calculate the sample mean values: $\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ $\overline{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$

Calculate sample standard deviations:
$$s_x = \frac{1}{n} \sum_{i=1}^n (x_i - \overline{x})$$
 $s_y = \frac{1}{n} \sum_{i=1}^n (x_i - \overline{x})$

Solution 3.5

	Univ. A	Univ. B	Univ. C	Univ. D	Univ. E	Univ. F
Newcomer	3970	732	499	1300	3463	2643
Total students	24273	5883	2847	5358	23442	17076

 Table 3.5.1
 Number of newcomers to the university and the number of total students at the university.

	X _i	y_i	$x_i - \overline{x}$	$y_i - \overline{y}$	$(x_i - \bar{x})^2$	$(y_i - \overline{y})^2$	$(x_i - \overline{x})(y_i - \overline{y})$
А	3970	24273	1868.83	11126.5	3492538	123799002	20793574
В	732	5883	-1369.17	-7263.5	1874617	52758432	9944942
С	499	2847	-1602.17	-10299.5	2566938	106079700	16501516
D	1300	5358	-801.17	-7788.5	641868	60660732	6239887
Е	3463	23442	1361.83	10295.5	1854590	105997320	14020755
F	2643	17076	541.83	3929.5	293583	15440970	2129134
Σ	12607	78879	-	-	10724135	464736158	69629808
Σ/n	2161.17	13146.5	-	-	1787356	77456026	11604968
$\sqrt{\Sigma/n}$	-	-	-	-	1336.92	8800.91	-



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Solution 3.5

$$r_{XY} = \frac{1}{n} \sum_{i=1}^{n} \frac{(x_i - \overline{x})(y_i - \overline{y})}{s_X s_Y} = \frac{11604968}{1337 \cdot 8801} = 0.99$$





3.4.1

Exercise 3.4 (Group exercise- to be presented on 12.04.07)

Resistivity measurements help to predict the possible corrosion of bridge structures. During a general bridge inspection the data shown in Table 3.2 were obtained from resistivity measurements along the two bridge lanes (direction 1 and 2):

- a. Draw two box plots for the data provided in Table 3.4.1 (direction 1 and direction 2). Show the main features of the box plots and write their values next to the corresponding points on the diagrams. Plot also the outside values, if any.
- b. Tukey box plot is a helpful tool for assessing the symmetry of data sets. Discuss the symmetry/skewness of the resistivity data for both lanes.
- c. Choose a suitable number of intervals and plot the histogram for the resistivity data of direction 1.



Exercise 3.4 (Group exercise- to be presented on 12.04.07)

- a. Draw two box plots for the data provided in Table 3.4.1 (direction 1 and direction 2). Show the main features of the box plots and write their values next to the corresponding points on the diagrams. Plot also the outside values, if any.
 b. Tukey box plot is a helpful tool for assessing the symmetry of data sets. Discuss the symmetry/skewness of the resistivity data for both lanes.
- c. Choose a suitable number of intervals and plot the histogram for the resistivity data of direction 1.

According to exercise 3.1!!!

What should be in the presentation of the solution?

a, b and c!!

An example of calculation where applicable e.g. features of the Tukey box plot etc....

Try to work with a simple calculator, diagrams can be on a transparency made by hand[©]

You can try for yourself to solve in e.g. excel or matlab or other.