

## Exercises Tutorial 3

Statistics and Probability Theory
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## Exercise 3.1 (Descriptive Statistics)

Two sets of data are provided, each of which represents the daily traffic flow in Rosengartenstrasse in Zurich during the month of April 2001
Direction 1 corresponds to driving towards Bucheggplatz, while direction 2 corresponds to driving towards Escher Wyss Platz.

| Date | Direction 1 | Direction 2 |
| :---: | :---: | :---: |
| 01.04.2001 | 32618 | 24609 |
| 02.04.2001 | 33380 | 29965 |
| 03.04.2001 | 34007 | 30629 |
| 04.04.2001 | 33888 | 30263 |
| 05.04.2001 | 35237 | 31405 |
| 06.04.2001 | 35843 | 31994 |
| 07.04.2001 | 33197 | 26846 |
| 08.04.2001 | 30035 | 22762 |
| 09.04.2001 | 32158 | 30366 |
| 10.04.2001 | 33406 | 29994 |
| 11.04.2001 | 34576 | 30958 |
| 12.04.2001 | 34013 | 30680 |
| 13.04.2001 | 24846 | 19735 |
| 14.04.2001 | 28252 | 21145 |
| 15.04.2001 | 25365 | 17805 |
| 16.04.2001 | 24862 | 18123 |
| 17.04.2001 | 32472 | 28117 |
| 18.04.2001 | 33245 | 28858 |
| 19.04.2001 | 33788 | 29080 |
| 20.04.2001 | 34076 | 30313 |
| 21.04.2001 | 29976 | 23141 |
| 22.04.2001 | 29224 | 20903 |
| 23.04.2001 | 32962 | 27746 |
| 24.04.2001 | 33937 | 29586 |
| 25.04.2001 | 33198 | 30788 |
| 26.04.2001 | 34455 | 31074 |
| 27.04.2001 | 35852 | 32384 |
| 28.04.2001 | 33091 | 26525 |
| 29.04.2001 | 30613 | 22828 |
| 30.04.2001 | 34425 | 28877 |



What do we want to know?

Which is the best way to know it? - plot, histogram, statistics etc.

For example,
if you are interested in: the change in the traffic of direction 1 during the month


What do we want to know?

Which is the best way to know it? - plot, histogram, statistics etc.

For example, if you are interested in: the relation between the traffic of direction 1 and that of direction 2,


What do we want to know?

Which is the best way to know it? - plot, histogram, statistics etc.

For example, if you are interested in: the relation between the traffic of direction 1 and that of direction 2, but you are not interested in the time element


Correlated!

What do we want to know?

Which is the best way to know it? - plot, histogram, statistics etc.

For example,
if you are interested in:
traffic volume of each direction



We will look today into...
how to represent and compare the properties of sets of data which you have
$>$ graphically
frequency distribution (histogram)
cumulative frequency distribution
>numerically
median
quantile
$>$ a summary plot
Tukey box plot
$>$ Correlation between data sets

You can use excel, matlab and/or other programming/statistics software....BUT

Make sure ALWAYS to insert functions by yourself or check that the functions provided by the used program agree with those of the used script!

## Exercise 3.1

Provide frequency distributions and cumulative frequency distributions of the observed data. What is your first impression of the data? Try to make comparison between the two directions.

| Date | Direction 1 | Direction 2 |
| :---: | :---: | :---: |
| 01.04.2001 | 32618 | 24609 |
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| 03.04.2001 | 34007 | 30629 |
| 04.04.2001 | 33888 | 30263 |
| 05.04.2001 | 35237 | 31405 |
| 06.04.2001 | 35843 | 31994 |
| 07.04.2001 | 33197 | 26846 |
| 08.04.2001 | 30035 | 22762 |
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| 13.04.2001 | 24846 | 19735 |
| 14.04 .2001 | 28252 | 21145 |
| 15.04.2001 | 25365 | 17805 |
| 16.04.2001 | 24862 | 18123 |
| 17.04.2001 | 32472 | 28117 |
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| 19.04.2001 | 33788 | 29080 |
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| 23.04.2001 | 32962 | 27746 |
| 24.04.2001 | 33937 | 29586 |
| 25.04.2001 | 33198 | 30788 |
| 26.04.2001 | 34455 | 31074 |
| 27.04.2001 | 35852 | 32384 |
| 28.04.2001 | 33091 | 26525 |
| 29.04.2001 | 30613 | 22828 |
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## Steps

1. sort the data
2. select the number of intervals
3. count the data in each interval
4. draw the frequency distribution
5. draw the cumulative frequency distribution


Step 2 (select the number of intervals)
No general rule but suggestion - (script Equation (C.8))

$$
k=1+3.3 \log _{10} n
$$

$k$ is the number of the intervals, $n$ is the number of the data.
In this case, $n=30$

$$
k=1+3.3 \log _{10} 30=5.87 \approx 6 \text { intervals }
$$

For direction 1,

$$
\begin{aligned}
\operatorname{minimum} & =24846 \\
\max & =35852
\end{aligned}
$$

we may select the intervals as follows:

## Step 3 (count the data in each interval)

3. count the data in each interval
4. draw the frequency distribution
5. draw the cumulative frequency distribution

Direction 1
24846
24862
25365
28252
29224
29976
30935
30613
32158
32472
32618
32618
32962
32962
33091
33091
33197
33197
33198
33245
33380
33406
33788
33888
33937
34007
34013
34076
34425
34455
34576
35237
35843
35852

| Count |  | Interval (Number of cars *10 ${ }^{3}$ ) | Interval Midpoint (Number of cars *10 ${ }^{3}$ ) | Number of observations |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 24.5-26.5 | 25.5 | 3 |
|  |  | 26.5-28.5 | 27.5 | 1 |
|  |  | 28.5-30.5 | 29.5 | 3 |
|  |  | 30.5-32.5 | 31.5 | 3 |
|  |  | 32.5-34.5 | 33.5 | 16 |
|  |  | 34.5-36.5 | 35.5 | 4 |

Step 4 (draw the frequency distribution)

## Steps

1. sort the data
2. select the number of intervals

Step 4 (draw the frequency distribution)
3. count the data in each interval
4. draw the frequency distribution
5. draw the cumulative frequency distribution

## But first some calculations....

| - | Interval (Number of cars *10 ${ }^{3}$ ) | Interval Midpoint (Number of cars *10 ${ }^{3}$ ) | Number of observations | Frequency \% |
| :---: | :---: | :---: | :---: | :---: |
| ${ }_{0}$ | 24.5-26.5 | 25.5 | 3 | 10.000 |
| \% | 26.5-28.5 | 27.5 | 1 | 3.333 |
| $\stackrel{\text { ¢ }}{ }$ | 28.5-30.5 | 29.5 | 3 | 10.000 |
| - | 30.5-32.5 | 31.5 | 3 | 10.000 |
|  | 32.5-34.5 | 33.5 | 16 | 53.333 |
|  | 34.5-36.5 | 35.5 | 4 | 13.333 |


|  |
| :---: |
|  |
|  |
|  |
| Step 4 (draw the frequency distribution) |
|  |


|  | Interval (Number of cars *10 ${ }^{3}$ ) | Interval Midpoint (Number of cars *10 ${ }^{3}$ ) | Number of observations | Frequency \% |
| :---: | :---: | :---: | :---: | :---: |
|  | 24.5-26.5 | 25.5 | 3 | 10.000 |
|  | 26.5-28.5 | 27.5 | 1 | 3.333 |
|  | 28.5-30.5 | 29.5 | 3 | 10.000 |
|  | 30.5-32.5 | 31.5 | 3 | 10.000 |
|  | 32.5-34.5 | 33.5 | 16 | 53.333 |
|  | 34.5-36.5 | 35.5 | 4 | 13.333 |

Draw


|  | Steps |
| :--- | :--- | :--- |
| Step 5 (draw the cumulative frequency distribution) | sort the data <br> select the number of intervals <br> 2. |
| 3. <br> count the data in each interval |  |
| 4. <br> draw the frequency distribution |  |
| 5. draw the cumulative frequency distribution |  |


|  | Interval (Number of cars *10 ${ }^{3}$ ) | Interval <br> Midpoint (Number of cars *10 ${ }^{3}$ ) | Number of observations | Frequency \% | Cumulative frequency |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 24.5-26.5 | 25.5 | 3 | 10.000 | 0.100 |
|  | 26.5-28.5 | 27.5 | 1 | 3.333 | 0.133 |
|  | 28.5-30.5 | 29.5 | 3 | 10.000 | 0.233 |
|  | 30.5-32.5 | 31.5 | 3 | 10.000 | 0.333 |
|  | 32.5-34.5 | 33.5 | 16 | 53.333 | 0.867 |
|  | 34.5-36.5 | 35.5 | 4 | 13.333 | 1.000 |



|  | Interval (Number of cars *10 ${ }^{3}$ ) | Interval Midpoint (Number of cars *10 ${ }^{3}$ ) | Number of observations | Frequency \% | Cumulative frequency |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 24.5-26.5 | 25.5 | 3 | 10.000 | 0.100 |
|  | 26.5-28.5 | 27.5 | 1 | 3.333 | - 0.133 |
|  | 28.5-30.5 | 29.5 | 3 | 10.000 | 0.233 |
|  | 30.5-32.5 | 31.5 | 3 | 10.000 | 0.333 |
|  | 32.5-34.5 | 33.5 | 16 | 53.333 | 0.867 |
|  | 34.5-36.5 | 35.5 | 4 | 13.333 | 1.000 |



Cumulate


## Answer 3.1

Do the same for direction 2.

What can we know from these plots?

Direction 1


Direction 2



These figures give nice overviews of the data!

## Answer 3.1

a. When we have in hand all observations:
prefer to plot the cumulative distribution plot using the quantiles of the data!
b. If we have in hand only the intervals observed and the frequency of observations within each interval
a. is not possible so...plot the cumulative frequency!
a.

b.


## Quantiles

A quantile is related to a given percentage $\alpha$, for which $\alpha \%$ of all observations in the data set have smaller values.
e.g. the 0.65 quantile of a given data set of observations corresponds to the observation for which $65 \%$ of all observations in the data set have smaller values

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| :---: | :---: | :---: |
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| 05.04.2001 | 35237 | 31405 |
| 06.04.2001 | 35843 | 31994 |
| 07.04.2001 | 33197 | 26846 |
| 08.04.2001 | 30035 | 22762 |
| 09.04.2001 | 32158 | 30366 |
| 10.04.2001 | 33406 | 29994 |
| 11.04.2001 | 34576 | 30958 |
| 12.04.2001 | 34013 | 30680 |
| 13.04.2001 | 24846 | 19735 |
| 14.04.2001 | 28252 | 21145 |
| 15.04.2001 | 25365 | 17805 |
| 16.04.2001 | 24862 | 18123 |
| 17.04.2001 | 32472 | 28117 |
| 18.04.2001 | 33245 | 28858 |
| 19.04.2001 | 33788 | 29080 |
| 20.04.2001 | 34076 | 30313 |
| 21.04.2001 | 29976 | 23141 |
| 22.04.2001 | 29224 | 20903 |
| 23.04.2001 | 32962 | $\xrightarrow{27746} \mathrm{Q}=0.74$ |
| 24.04.2001 | 33937 | 29586 Q |
| 25.04.2001 | 33198 | 30788 |
| 26.04.2001 | 34455 | 31074 |
| 27.04.2001 | 35852 | 32384 |
| 28.04.2001 | 33091 | 26525 |
| 29.04.2001 | 30613 | 22828 |
| 30.04.2001 | 34425 | 28877 |

## Quantiles

A quantile is related to a given percentage $\alpha$, for which $\alpha \%$ of all observations in the data set have smaller values.
e.g. the 0.65 quantile of a given data set of observations corresponds to the observation for which $65 \%$ of all observations in the data set have smaller values

| Date | Direction 1 | Direction 2 |  |
| :---: | :---: | :---: | :---: |
| 01.04.2001 | 32618 | 24609 | 4 |
| 02.04.2001 | 33380 | 29965 |  |
| 03.04.2001 | 34007 | 30629 |  |
| 04.04.2001 | 33888 | 30263 |  |
| 05.04.2001 | 35237 | 31405 |  |
| 06.04.2001 | 35843 | 31994 |  |
| 07.04.2001 | 33197 | 26846 |  |
| 08.04.2001 | 30035 | 22762 |  |
| 09.04.2001 | 32158 | 30366 |  |
| 10.04.2001 | 33406 | 29994 |  |
| 11.04.2001 | 34576 | 30958 | $74 \%$ of the observations |
| 12.04.2001 | 34013 | 30680 |  |
| 13.04.2001 | 24846 | 19735 | Have a smaller value! |
| 14.04.2001 | 28252 | 21145 |  |
| 15.04.2001 | 25365 | 17805 |  |
| 16.04.2001 | 24862 | 18123 |  |
| 17.04.2001 | 32472 | 28117 |  |
| 18.04.2001 | 33245 | 28858 |  |
| 19.04.2001 | 33788 | 29080 |  |
| 20.04.2001 | 34076 | 30313 |  |
| 21.04.2001 | 29976 | 23141 |  |
| 22.04.2001 | 29224 | 20903 |  |
| 23.04.2001 | 32962 | 27746 | $Q=0.74$ |
| 24.04.2001 | 33937 | 29586 |  |
| 25.04.2001 | 33198 | 30788 |  |
| 26.04.2001 | 34455 | 31074 |  |
| 27.04.2001 | 35852 | 32384 |  |
| 28.04.2001 | 33091 | 26525 |  |
| 29.04.2001 | 30613 | 22828 |  |
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## Quantiles

A quantile is related to a given percentage $\alpha$, for which $\alpha \%$ of all observations in the data set have smaller values.
e.g. the 0.65 quantile of a given data set of observations corresponds to the observation for which $65 \%$ of all observations in the data set have smaller values

How to calculate it????

$$
Q_{i}=\frac{i}{n+1}, n: \text { total number of obervations }
$$

## Exercise 3.2

Use the Tukey box plot to provide a summary of the main features of the distribution of each data set. Plot the Tukey box plots on the same graph so that you are able to compare these features. Do you observe any symmetry in the data sets?

## Steps

1. calculate the median
2. calculate the $75 \%$ - and $25 \%$ - quantile.
3. calculate the adjacent values.
4. check for outside values
5. draw the Tukey box plot

## Steps

1. calculate the median
2. calculate the $75 \%$ - and $25 \%$ - quantile.

Step 1 (calculate the median)
3. calculate the adjacent values.
4. check for outside values
5. draw the Tukey box plot

Just take the central value (50\%-quantile).
but.....
if the number of data is even, this is not possible!

$$
\text { Median is } \frac{33198+33245}{2}=33221.5
$$

Steps

1. calculate the median
2. calculate the $75 \%$ - and $25 \%$ - quantile.

## Step 2 (calculate the quantiles)

3. calculate the adjacent values.
4. check for outside values
5. draw the Tukey box plot

Roughly speaking,


## Step 2 (calculate the quantiles) <br> More strictly speaking, <br> $Q_{i}=\frac{i}{n+1}, n$ : total number of obervations

| Direction 1 | i | i/31 |
| :--- | ---: | :--- |
| 24846 | 1 | 0.03 |
| 24862 | 2 | 0.06 |
| 25365 | 3 | 0.10 |
| 28252 | 4 | 0.13 |
| 29224 | 5 | 0.16 |
| 29976 | 6 | 0.19 |
| 30035 | 7 | 0.23 |
| 30613 | 8 | 0.26 |
| 32158 | 9 | 0.29 |
| 32472 | 10 | 0.32 |
| 32618 | 11 | 0.35 |
| 32962 | 12 | 0.39 |
| 33091 | 13 | 0.42 |
| 33197 | 14 | 0.45 |
| 33198 | 15 | 0.48 |
| 33245 | 16 | 0.52 |
| 33380 | 17 | 0.55 |
| 33406 | 18 | 0.58 |
| 33788 | 19 | 0.61 |
| 33888 | 20 | 0.65 |
| 33937 | 21 | 0.68 |
| 34007 | 22 | 0.71 |
| 34013 | 23 | 0.74 |
| 34076 | 24 | 0.77 |
| 34425 | 25 | 0.81 |
| 34455 | 26 | 0.84 |
| 34576 | 27 | 0.87 |
| 35237 | 28 | 0.90 |
| 35843 | 29 | 0.94 |
| 35852 | 30 | 0.97 |

Step 2 (calculate the quantiles)
Interpolation

Steps

1. calculate the median
2. calculate the $75 \%$ and $25 \%$ quantile.
Step 3 (calculate the adjacent values)
3. calculate the adjacent values.
4. check for outside values
5. draw the Tukey box plot

$$
\left.\begin{array}{l}
Q_{0.75}=34029 \\
Q_{0.25}=30469
\end{array}\right\} \begin{array}{r}
\text { Interquartile range } \\
r^{\prime} \equiv Q_{0.75}-Q_{0.25}=34029-30469=3560
\end{array}
$$

Upper adjacent value: largest observation $\leq(75 \%$ quantile $)+1.5 r$
$\qquad$

1. calculate the median 2. calculate the $75 \%$ and $25 \%$ quantile.

Step 3 (calculate the adjacent values)
3. calculate the adjacent values.
4. check for outside values
5. draw the Tukey box plot

$$
\left.\begin{array}{l}
Q_{0.75}=34029 \\
Q_{0.25}=30469
\end{array}\right\} r \equiv Q_{0.75}-Q_{0.25}=34029-30469=3560
$$

Lower adjacent value: smallest observation $\geq(25 \%$ quantile $)-1.5 r$
Direction 1
24846
24862
25365
28252
29224
29976
30035
30613
32158
32472
32618
32962
33091
33197
33198

|  |  | Steps |
| :---: | :---: | :---: |
|  |  | 1. calculate the median |
| Direction 1 24846 | Step 4 | 2. calculate the $75 \%$ and $25 \%$ quantile. |
| 24862 | (check for outside values) | 3. calculate the adjacent values. |
| 25365 |  | 5. draw the Tukey box plot |
| 28252 29224 | Outside values: |  |
| 29976 30035 | Outside the upper and lower adjacent values |  |
| 30613 | 24846 |  |
| 32158 32472 | 24862 |  |
| 32618 |  |  |
| 32962 |  |  |
| 33091 | summary |  |
| 33197 33198 | Upper adjacent value: 35852 |  |
| 33245 | 75\% quantile : 34029 |  |
| 33380 | Median : 33222 |  |
| 33788 | 25\% quantile : 30469 |  |
| 33888 | Lower adjacent value: 25365 |  |


|  | Steps |
| :--- | :--- |
| Step 4 | 1. calculate the median |
| (draw the Tukey box plot) | 2. calculate the $75 \%$ and $25 \%$ quantile. |
|  | 3. calculate the adjacent values. |
|  | 4. check for outside values |
|  | 5. draw the Tukey box plot |

summary
Upper adjacent value: 35852
75\% quantile
Median
25\% quantile
: 34029

Lower adjacent value: 25365

Outside values:

$$
\begin{aligned}
& 24846 \\
& 24862
\end{aligned}
$$



Direction 1

## Answer 3.2

Use the Tukey box plot to provide a summary of the main features of the distribution of each data set.

- median
- Adjacent values
- Upper and lower quartiles
- Outside values

Plot the Tukey box plots on the same graph so that you are able to compare these features.
???

Do you observe any symmetry in the data sets? ???

## Exercise 3.5

The data sets in Table 3.5.1 show the number of newcomers to the university and the number of total students at the university.
Estimate the correlation of these numbers using the following calculation sheet.

|  | Univ. A | Univ. B | Univ. C | Univ. D | Univ. E | Univ. F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Newcomer | 3970 | 732 | 499 | 1300 | 3463 | 2643 |
| Total students | 24273 | 5883 | 2847 | 5358 | 23442 | 17076 |

Table 3.5.1 Number of newcomers to the university and the number of total students at the university.

## Exercise 3.5

Estimate the correlation of these numbers using the following calculation sheet.

|  | Univ. A | Univ. B | Univ. C | Univ. D | Univ. E | Univ. F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Newcomer | 3970 | 732 | 499 | 1300 | 3463 | 2643 |
| Total students | 24273 | 5883 | 2847 | 5358 | 23442 | 17076 |

Table 3.5.1 Number of newcomers to the university and the number of total students at the university.

## What is known?

Newcomers: $X$
total students: $Y$
Number of newcomers: $\quad x_{i}, i=1, \ldots, 6$
Number of total students: $\quad y_{i}, i=1, \ldots, 6$
Number of observations/university: $n=6$

## Exercise 3.5

Estimate the correlation of these numbers using the following calculation sheet.

|  | Univ. A | Univ. B | Univ. C | Univ. D | Univ. E | Univ. F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Newcomer | 3970 | 732 | 499 | 1300 | 3463 | 2643 |
| Total students | 24273 | 5883 | 2847 | 5358 | 23442 | 17076 |

Table 3.5.1 Number of newcomers to the university and the number of total students at the university.

## What is known?

Newcomers:
total students:
Number of newcomers:
Number of total students:
Number of observations/university: $n=6$

What is required?
Correlation: $r_{X Y}=\frac{1}{n} \sum_{i=1}^{n} \frac{\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{s_{X} s_{Y}}$
Need to:
Calculate the sample mean values: $\bar{x} \quad \bar{y}$
Calculate sample standard deviations: $S_{X} S_{Y}$

## Exercise 3.5

Estimate the correlation of these numbers using the following calculation sheet.

|  | Univ. A | Univ. B | Univ. C | Univ. D | Univ. E | Univ. F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Newcomer | 3970 | 732 | 499 | 1300 | 3463 | 2643 |
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Table 3.5.1 Number of newcomers to the university and the number of total students at the university.

> Total number of students

From a first view are they correlated ???????

Give a rough estimation for the correlation coefficient!!!!

$$
-1 \leq r_{X Y} \leq 1
$$



## Solution 3.5

|  | Univ. A | Univ. B | Univ. C | Univ. D | Univ. E | Univ. F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Newcomer | 3970 | 732 | 499 | 1300 | 3463 | 2643 |
| Total students | 24273 | 5883 | 2847 | 5358 | 23442 | 17076 |

## What is known?

| Newcomers: | $X$ |
| :--- | :---: |
| total students: | $Y$ |
| Number of newcomers: | $x_{i}, i=1, \ldots, 6$ |
| Number of total students: | $y_{i}, i=1, \ldots, 6$ |

Number of total students
$y_{i}, i=1, \ldots, 6$
Table 3.5.1 Number of newcomers to the university and the number of total students at the university.
Number of observations/university: $n=6$
What is required?
Correlation: $r_{X Y}=\frac{1}{n} \sum_{i=1}^{n} \frac{\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{s_{X} s_{Y}}$

Need to:
Calculate the sample mean values: $\bar{x}=\frac{1}{n} \sum_{i=1}^{n} x_{i} \quad \bar{y}=\frac{1}{n} \sum_{i=1}^{n} y_{i}, ~$

Calculate sample standard deviations: $s_{X}=\frac{1}{n} \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right) \quad s_{Y}=\frac{1}{n} \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)$

## Solution 3.5

|  | Univ. A | Univ. B | Univ. C | Univ. D | Univ. E | Univ. F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Newcomer | 3970 | 732 | 499 | 1300 | 3463 | 2643 |
| Total students | 24273 | 5883 | 2847 | 5358 | 23442 | 17076 |

Table 3.5.1 Number of newcomers to the university and the number of total students at the university.

|  | $x_{i}$ | $y_{i}$ | $x_{i}-\bar{x}$ | $y_{i}-\bar{y}$ | $\left(x_{i}-\bar{x}\right)^{2}$ | $\left(y_{i}-\bar{y}\right)^{2}$ | $\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 3970 | 24273 | 1868.83 | 11126.5 | 3492538 | 123799002 | 20793574 |
| B | 732 | 5883 | -1369.17 | -7263.5 | 1874617 | 52758432 | 9944942 |
| C | 499 | 2847 | -1602.17 | -10299.5 | 2566938 | 106079700 | 16501516 |
| D | 1300 | 5358 | -801.17 | -7788.5 | 641868 | 60660732 | 6239887 |
| E | 3463 | 23442 | 1361.83 | 10295.5 | 1854590 | 105997320 | 14020755 |
| F | 2643 | 17076 | 541.83 | 3929.5 | 293583 | 15440970 | 2129134 |
| $\Sigma$ | 12607 | 78879 | - | - | 10724135 | 464736158 | 69629808 |
| $\Sigma / n$ | 2161.17 | 13146.5 | - | - | 1787356 | 77456026 | 11604968 |
| $\sqrt{\Sigma / n}$ | - | - |  |  |  |  |  |

## Solution 3.5

$$
r_{X Y}=\frac{1}{n} \sum_{i=1}^{n} \frac{\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{s_{X} s_{Y}}=\frac{11604968}{1337 \cdot 8801}=0.99
$$



## Exercise 3.4 (Group exercise- to be presented on 12.04.07)

Resistivity measurements help to predict the possible corrosion of bridge structures. During a general bridge inspection the data shown in Table 3.2 were obtained from resistivity measurements along the two bridge lanes (direction 1 and 2):
a. Draw two box plots for the data provided in Table 3.4.1 (direction 1 and direction 2). Show the main features of the box plots and write their values next to the corresponding points on the diagrams. Plot also the outside values, if any.
b. Tukey box plot is a helpful tool for assessing the symmetry of data sets.

Discuss the symmetry/skewness of the resistivity data for both lanes.
c. Choose a suitable number of intervals and plot the histogram for the resistivity data of direction 1.

## Exercise 3.4 (Group exercise- to be presented on 12.04.07)

a. Draw two box plots for the data provided in Table 3.4.1 (direction 1 and direction 2).

Show the main features of the box plots and write their values next to the corresponding points on the diagrams. Plot also the outside values, if any.
b. Tukey box plot is a helpful tool for assessing the symmetry of data sets. Discuss the symmetry/skewness of the resistivity data for both lanes.
c.

Choose a suitable number of intervals and plot the histogram for the resistivity data of direction 1.

What should be in the presentation of the solution?
$\mathrm{a}, \mathrm{b}$ and c !
An example of calculation where applicable e.g. features of the Tukey box plot etc....

Try to work with a simple calculator, diagrams can be on a transparency made by hand:

You can try for yourself to solve in e.g. excel or matlab or other.

