Small Exercise 1

In one experiment, measurements are performed on the dimensions ***a***, ***b*** and ***f*** in the figure shown below.

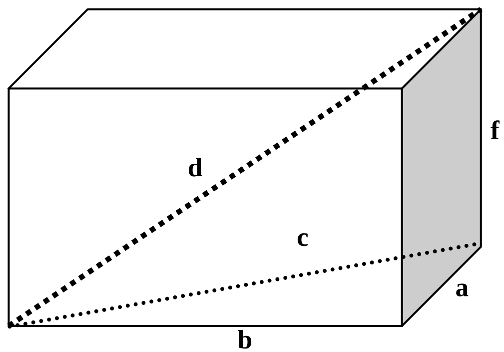
All the measurements are carried out independently with an error ε assumed to be standard normally distributed, that is, $\varepsilon_a = \varepsilon_b = \varepsilon_f = \varepsilon$

The error in d is then assumed to propagate according to $\varepsilon_d = \sqrt{\varepsilon_a^2 + \varepsilon_b^2 + \varepsilon_f^2}$

In another experiment, the dimension ***d*** is measured directly with an error ***e*** assumed to be standard normally distributed.

The ratio ***R*** $(R = e / \varepsilon_d)$ of the error in the measurement of ***d*** from the 2 experiments is now being studied.

What is the variance of the variable ***R*** ??



1

0

0.5

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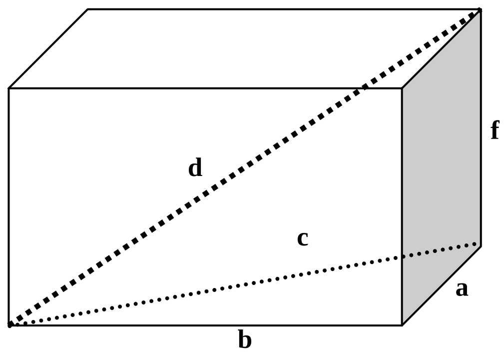

What is the variance of the variable ***R*** ??

Small Exercise 1

$$R = \frac{e}{\sqrt{\varepsilon_a^2 + \varepsilon_b^2 + \varepsilon_f^2}}$$

$$Define\ S = \sqrt{3}R = \sqrt{3}\frac{e}{\sqrt{\varepsilon_a^2 + \varepsilon_b^2 + \varepsilon_f^2}}$$

S **is a** ***t*****-distributed variable with 3 degrees of freedom.**

$$Var[S] = \sigma_S^2 = \frac{n}{n-2} \qquad \text{(Script E.9)}$$

$$Var[S] = \sigma_S^2 = \frac{3}{3-2} = 3$$

$$Var[S] = Var[\sqrt{3}R] = 3Var[R] \qquad (Var[cX] = c^2 Var[X] \qquad \text{Script D.18)}$$

$$Var[R] = \frac{1}{3}Var[S] = 1$$

Small Exercise 2

The annual maximum flow of a given river is known to have a standard deviation of 60 m^3/s.

Over the last 20 years, measurements of the annual maximum flow have been recorded and the following statistics have been obtained:

sample mean = 305 m^3/s

sample variance = 4250 $(m^3/s)^2$

What is the expected value of the sample variance ??

3420 $(m^3/s)^2$

4250 $(m^3/s)^2$

3600 $(m^3/s)^2$

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Small Exercise 2

The expected value of the sample variance S^2 is:

$$E\left[S^2\right] = \frac{(n-1)}{n}\sigma_X^2 \qquad \text{(Script E.19)}$$

where n = number of samples

σ_X = true standard deviation

$$E\left[S^2\right] = \frac{(n-1)}{n}\sigma_X^2$$

$$= \frac{(20-1)}{20}(60)^2 = 3420\ (m^3/s)^2$$

Small Exercise 3

The compressive strength of concrete from a certain plant is known to have a variance of 2.56 $(N/mm^2)^2$.

At a construction site which uses concrete from this plant, 64 samples are taken daily to estimate the compressive strength.

What is the largest error that one can expect to make with a probability of 0.9 when using the mean value from these samples to estimate the true mean value of the compressive strength ??

$\pm$ 0.09

$\pm$ 0.33

$\pm$ 0.21

Small Exercise 3

The compressive strength of concrete from a certain batching plant is known to have a variance of 2.56 $(N/mm^2)^2$.

At a construction site which uses concrete from this plant, 64 samples are taken daily to estimate the compressive strength.

What is the largest error that one can expect to make with a probability of 0.9 when using the mean value from these samples to estimate the true mean value of the compressive strength ??

$\pm$ 0.33

Small Exercise 3

$$P\left[-k_{\alpha/2} < \frac{\bar{X} - \mu_X}{\sigma_X \frac{1}{\sqrt{n}}} < k_{\alpha/2}\right] = 1 - \alpha \qquad \text{(Script E.22)}$$

$\alpha = 0.1$; $n = 64$; $\sigma_X = \sqrt{2.56} = 1.6$ N/mm^2

For $\alpha = 0.1$, $k_{\alpha/2} = 1.645$

$$P\left[-1.645 < \frac{\bar{X} - \mu_X}{1.6 \frac{1}{\sqrt{64}}} < 1.645\right] = 0.9$$

$$P\left[-0.33 < \bar{X} - \mu_X < 0.33\right] = 0.9$$

Largest error or largest difference between $\bar{X}$ and σ_X is $\pm$ 0.33