Statistics and Probability Theory in Civil, Surveying and Environmental Engineering

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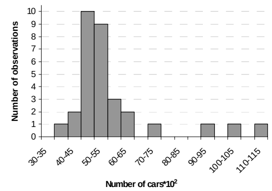
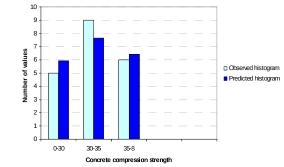
A Summary of the Lecture

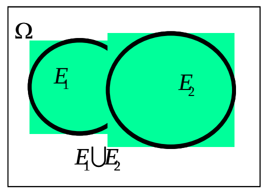
Graphical / numerical interpretation of data



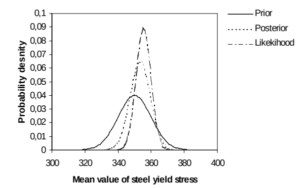
Verification and testing of models



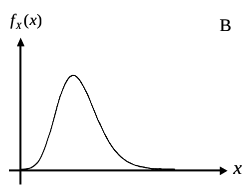
Basic probability theory



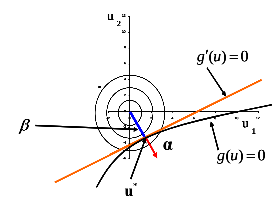
Bayesian modeling



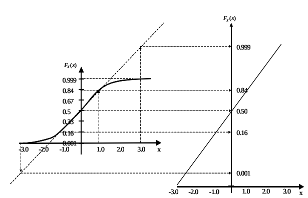
Distribution functions moments and extremes



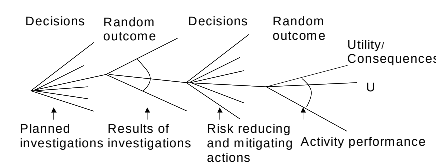
Basic reliability analysis



Modeling and Description of data



Basic decision analysis



Decision Analysis in Engineering

- **Introduction to Decision Theory**
  - The problem
  - The decision tree
  - Prior decision analysis
  - Posterior decision analysis
  - Pre-posterior decision analysis

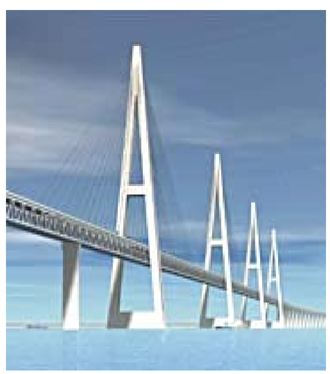
Decision Analysis in Engineering

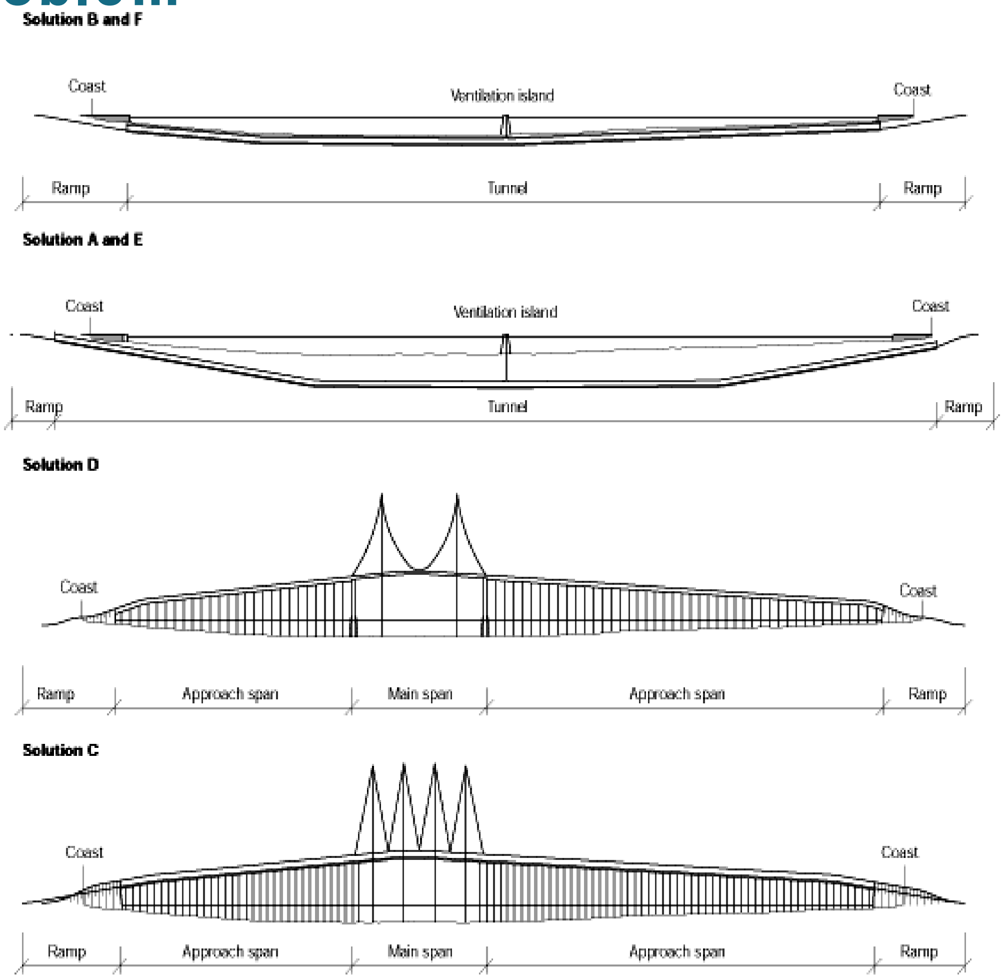
- **The basic engineering problem**

Several solutions may be identified

The available information is uncertain

A decision must be made !





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Approach

- **Formulation of the decision problem**
  - The decision maker and the preferences of the decision maker must be identified
  - Mapping of the decision process
  - All the possible decision alternatives must be identified
  - Identification of the contributing uncertainties

- **Identification of potential consequences and their utility (cost / benefit)**

- **Assessment of the probabilities of the consequences**

- **Comparison of the different decision alternatives based on their expected utilities**

- **Final decision making and reporting of the assumptions underlying the selected alternative**

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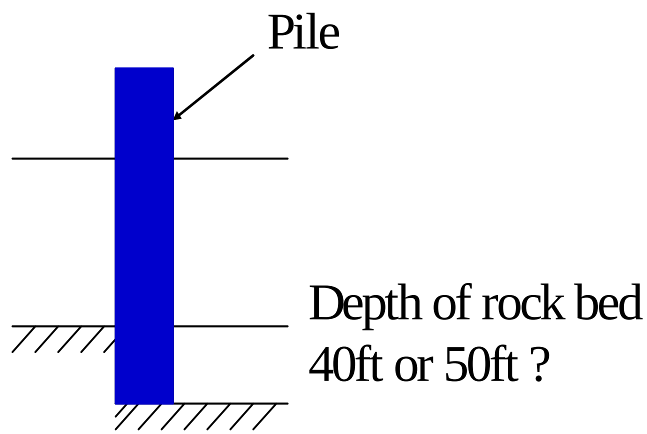
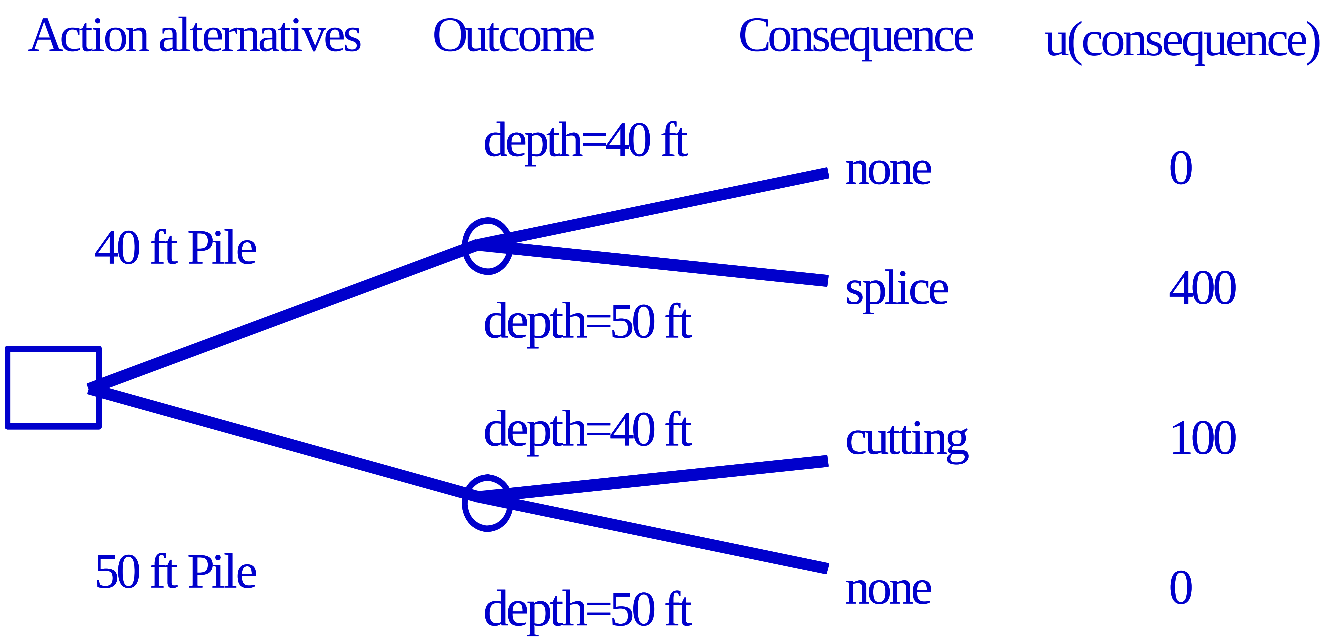
- **The decision tree**





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Assignment of utility

- The assignment of utility must reflect the preferences of the decision maker

- Utility functions may be defined as linear functions in monetary unity

- It is important to include all monetary consequences in the utility function

$$u(a_i) = \sum_{j=1}^{n} p_j \cdot u(K_j)$$

$u(a_i)\ldots$ Utility (cost/benefit) associated with action a_i

$p_j \cdot u(K_j)\ldots$ Expected utility associated with consequence K_j

$p_j\ldots$ Probability of the occurence of the consequence K_j

$u(K_j)\ldots$ Utility associated with the consequence K_j

$K_j\ldots$ A potential consequence associated with the action a_i

Decision Analysis in Engineering

The different types of decision analysis

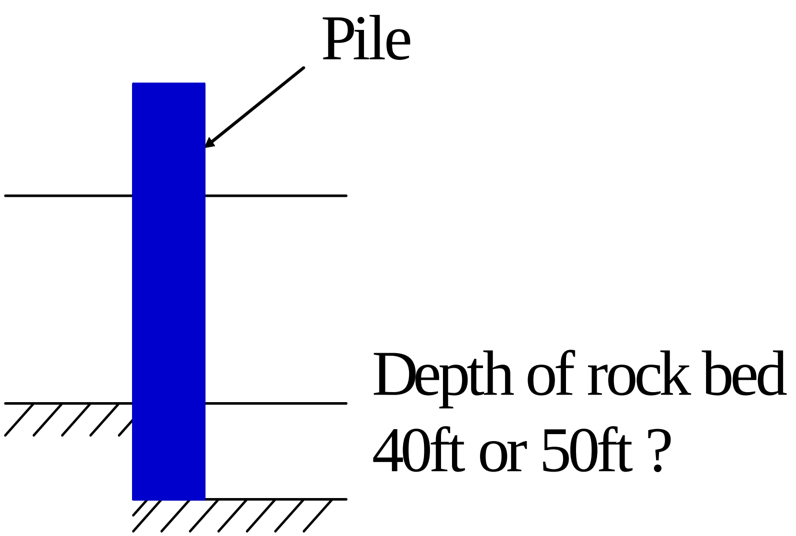
- **Prior**
- **Posterior**
- **Pre-posterior**

Illustrated on an example :

Question : What pile length should be applied ?

Alternatives :
a_0 : Choose a 40 ft pile
a_1 : Choose a 50 ft pile

States of nature
(depth to rock bed)
θ_0 : Rock bed in 40 ft
θ_1 : Rock bed at 50 ft



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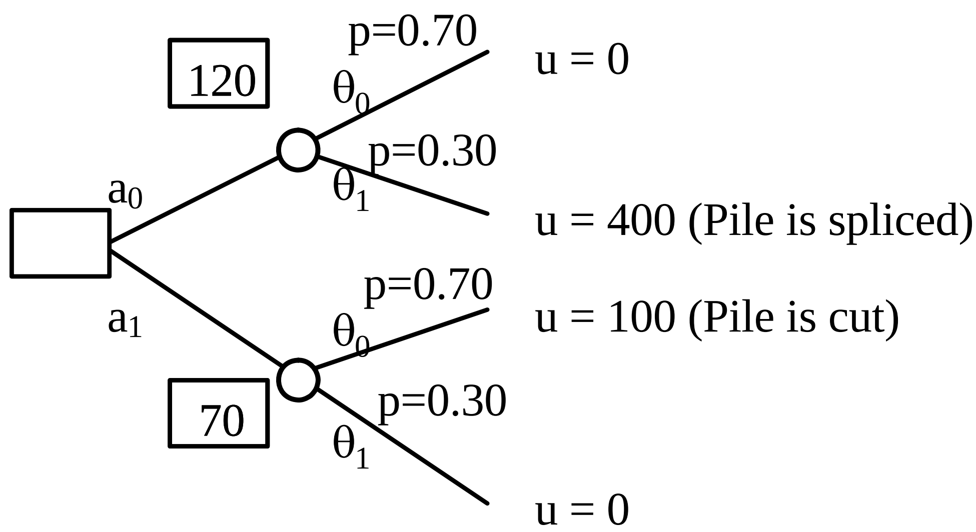
Prior Analysis

$P'[q_0] = 0.70$
$P'[q_1] = 0.30$



The expected utility is calculated to be equal to

$$E'[u] = \min\{u[a_0], u[a_1]\}$$
$$= \min\{P'[\theta_0]\times u[\theta_0|a_0] + P'[\theta_1]\times u[\theta_1|a_0],$$
$$P'[\theta_0]\times u[\theta_0|a_1] + P'[\theta_1]\times u[\theta_1|a_1]\}$$
$$= \min\{0.7\times 0 + 0.3\times 400,\ 0.7\times 100 + 0.3\times 0\}$$
$$= \min\{120, 70\} = 70 \quad \Rightarrow \text{Decision for } a_1 \text{ (50ft Pile)}$$

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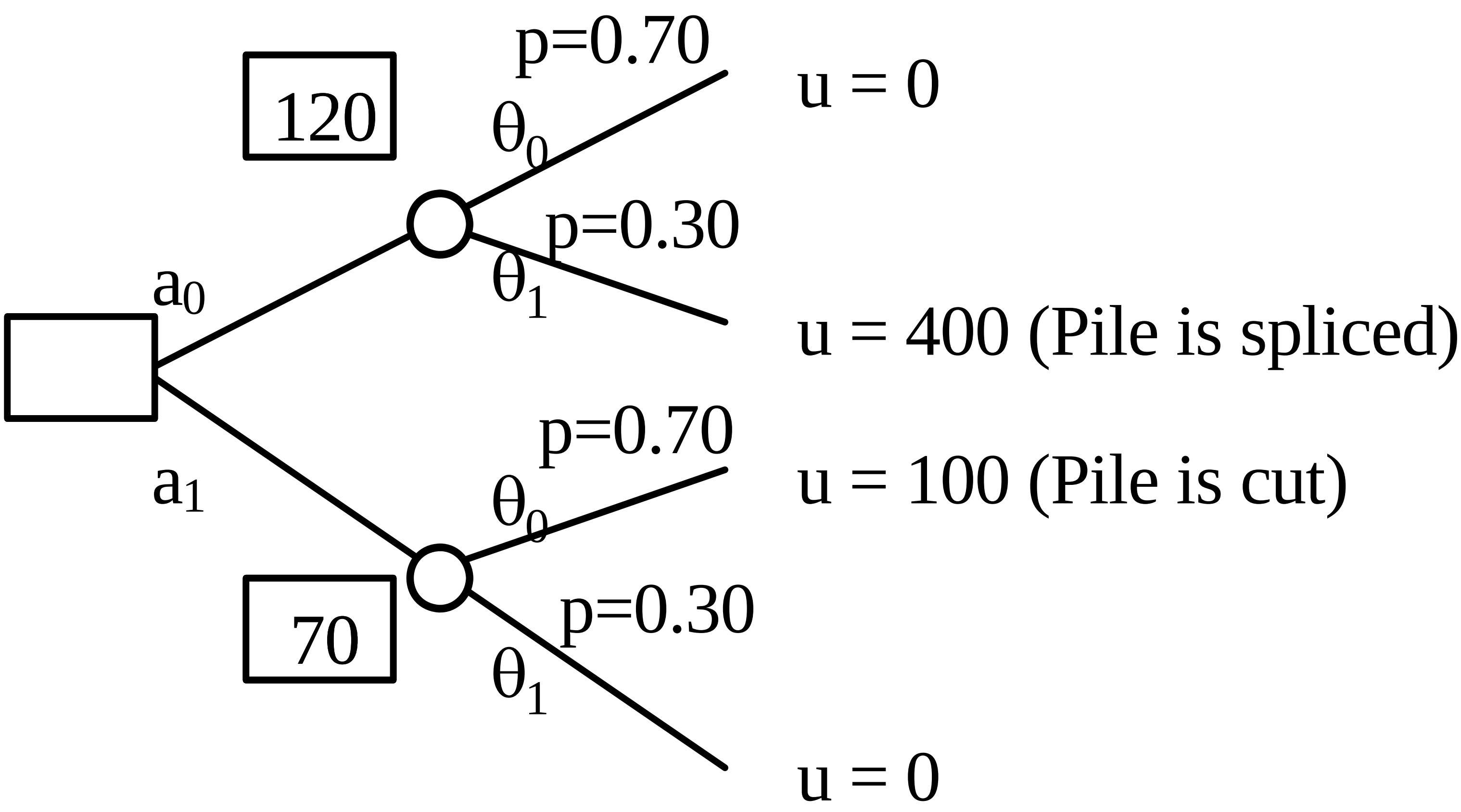

$\Rightarrow$ **Choice of pile a_1 (50ft Pfahl)**

Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

$$\begin{pmatrix}\text{Posterior probability of } \theta_i \\ \text{with given sample outcome}\end{pmatrix} = \begin{pmatrix}\text{Normalizing} \\ \text{constant}\end{pmatrix} \text{ x } \begin{pmatrix}\text{Samplelikelihood} \\ \text{given } \theta\end{pmatrix} \text{ x } \begin{pmatrix}\text{prior probability} \\ \text{of } \theta\end{pmatrix}$$

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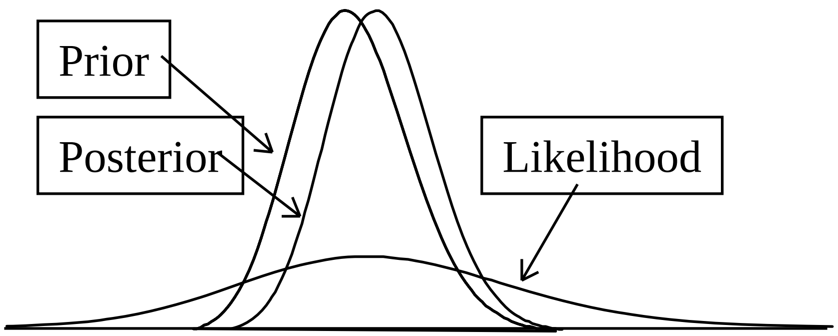
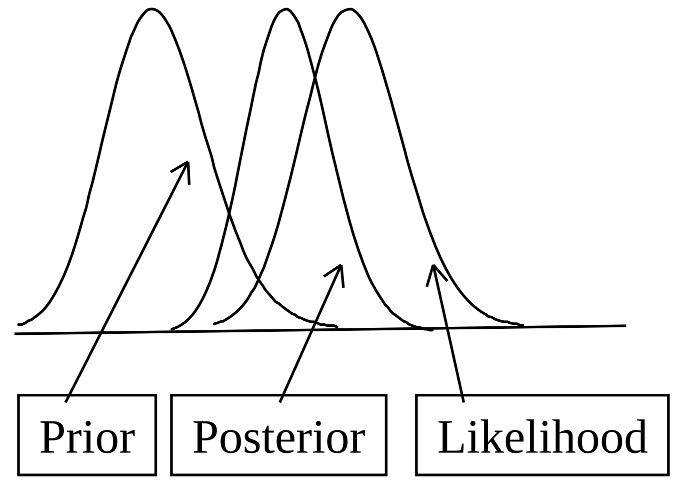
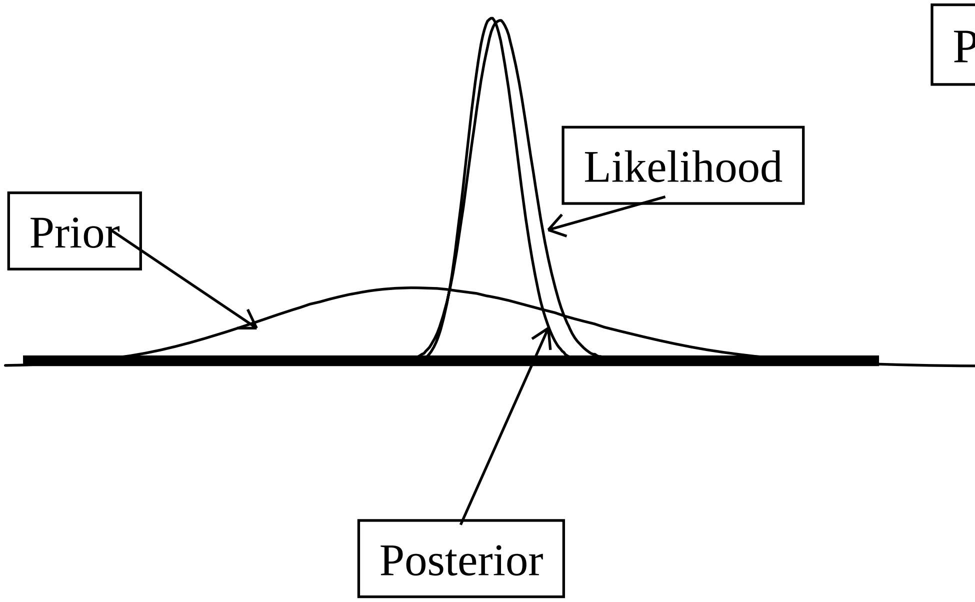
Posterior Analysis







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$$P''(\theta_i) = \frac{P[z_k|\theta_i]P'[\theta_i]}{\sum_j P[z_k|\theta_j]P'[\theta_j]}$$

Posterior Analysis

Ultrasonic tests to determine the depth to bed rock

True state / Test result	θ_0 40 ft – depth	θ_1 50 ft – depth
z_0 - 40 ft indicated	0.6	0.1
z_1 - 50 ft indicated	0.1	0.7
z_2 - 45 ft indicated	0.3	0.2

Likelihoods of the different indications/test results given the various possible states of nature – ultrasonic test methods

$$P[z_i|\theta_j]$$

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$$P''(\theta_i) = \frac{P[z_k|\theta_i]P'[\theta_i]}{\sum_j P[z_k|\theta_j]P'[\theta_j]}$$

Posterior Analysis

It is assumed that a test gives a 45 ft indication

$$P''[\theta_0] = P[\theta_0|z_2] \propto P[z_2|\theta_0]P[\theta_0] = 0.3\,x\,0.7 = 0.21$$

$$P''[\theta_1] = P[\theta_1|z_2] \propto P[z_2|\theta_1]P[\theta_1] = 0.2\,x\,0.3 = 0.06$$

$$P''[\theta_0|z_2] = \frac{0.21}{0.21+0.06} = 0.78$$

$$P''[\theta_1|z_2] = \frac{0.06}{0.21+0.06} = 0.22$$

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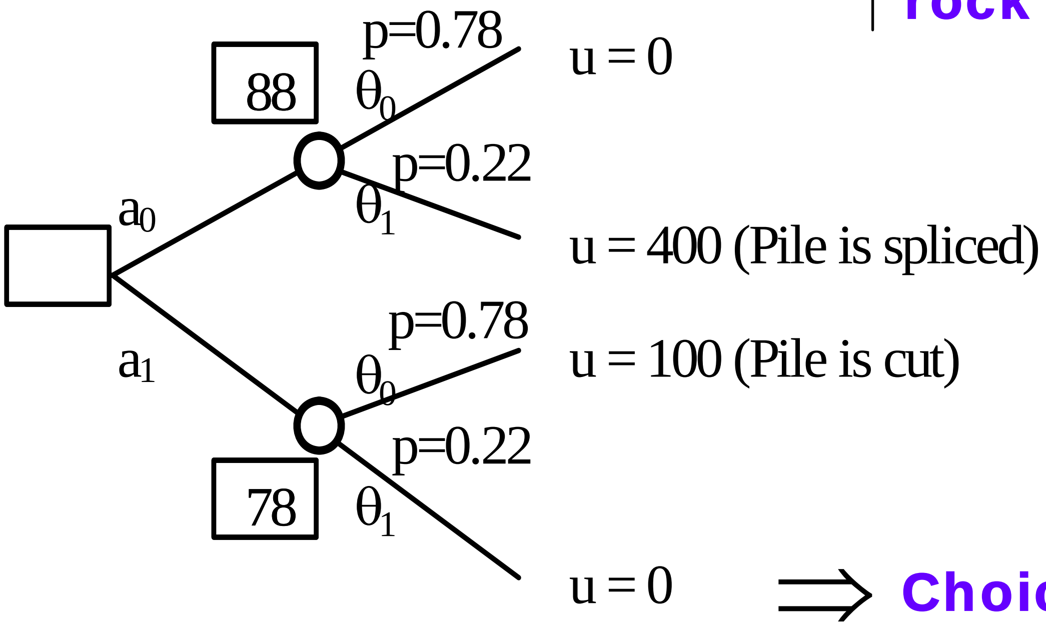
Posterior Analysis



Test result indicates 45ft to rock bed

$\Longrightarrow$ **Choice of alternative a_1 (50ft Pile)**

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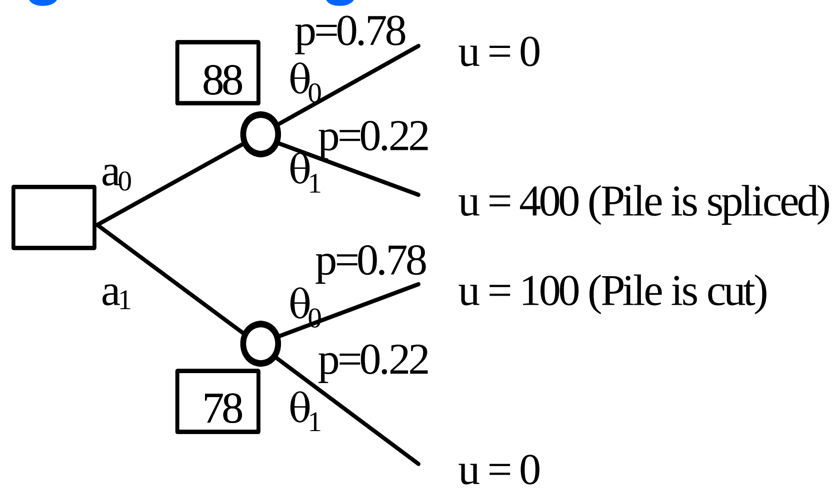
Posterior Analysis



$$E''\left[u \middle| z_2\right] = \min_j \{E''\left[u(a_j) \middle| z_2\right]\}$$

$$= \min\{P''\left[\theta_0\right] \times 0 + P''\left[\theta_1\right] \times 400, P''\left[\theta_0\right] \times 100 + P''\left[\theta_1\right] \times 0\}$$

$$= \min\{0.78 \times 0 + 0.22 \times 400, 0.78 \times 100 + 0.22 \times 0\}$$

$$= \min\{88, 78\} = 78$$

$\Rightarrow$ **Choice of alternative a_1 (50ft Pile)**

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Pre-posterior Analysis

$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''[u|z_i] = \sum_{i=1}^{n} P'[z_i] \times \min_{j=1,m}\{E''[u(a_j)|z_i]\}$$

$$P'[z_i] = P[z_i|\theta_0] \times P'[\theta_0] + P[z_i|\theta_1] \times P'[\theta_1]$$

$$P'[z_0] = P[z_0|\theta_0] \times P'[\theta_0] + P[z_0|\theta_1] \times P'[\theta_1] = 0.6 \times 0.7 + 0.1 \times 0.3 = 0.45$$

$$P'[z_1] = P[z_1|\theta_0] \times P'[\theta_0] + P[z_1|\theta_1] \times P'[\theta_1] = 0.1 \times 0.7 + 0.7 \times 0.3 = 0.28$$

$$P'[z_2] = P[z_2|\theta_0] \times P'[\theta_0] + P[z_2|\theta_1] \times P'[\theta_1] = 0.3 \times 0.7 + 0.2 \times 0.3 = 0.27$$

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Pre-posterior Analysis

$$E''\left[u\middle|z_0\right] = \min_j\left\{E''\left[u(a_j)\middle|z_0\right]\right\} =$$

$$\min\{\overbrace{\underbrace{P''\left[\theta_0\middle|z_0\right]\times 0}_{\textbf{do nothing}} + \underbrace{P''\left[\theta_1\middle|z_0\right]\times 400}_{\textbf{splicing}}}^{\mathbf{a_0}},\ \overbrace{\underbrace{P''\left[\theta_0\middle|z_0\right]\times 100}_{\textbf{cutting}} + \underbrace{P''\left[\theta_1\middle|z_0\right]\times 0}_{\textbf{do nothing}}}^{\mathbf{a_1}}\}$$

$$\min\{0.93\times 0 + 0.07\times 400,\ 0.93\times 100 + 0.07\times 0\} =$$

$$0.07\times 400 + 0.93\times 0 = 28$$

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Pre-posterior Analysis

$$E''\left[u \middle| z_1\right] = \min_j \{E''\left[u(a_j) \middle| z_1\right]\} =$$

$$\min\{\underbrace{\underset{\text{do nothing}}{P''\left[\theta_0 \middle| z_1\right] \times 0} + \underset{\text{splicing}}{P''\left[\theta_1 \middle| z_1\right] \times 400}}_{\mathbf{a_0}},\ \underbrace{\underset{\text{cutting}}{P''\left[\theta_0 \middle| z_1\right] \times 100} + \underset{\text{do nothing}}{P''\left[\theta_1 \middle| z_1\right] \times 0}}_{\mathbf{a_1}}\}$$

$$\min\{0.25 \times 0 + 0.75 \times 400,\ 0.25 \times 100 + 0.75 \times 0\} =$$

$$0.25 \times 100 + 0.75 \times 0 = 25$$

Pre-posterior Analysis

The minimum expected costs based on pre-posterior decision analysis – not including costs of experiments

$$E[u] = \sum_{i=1}^{n} P'\left[z_i\right] \times E''\left[u \middle| z_i\right] = 28 \times 0.45 + 25 \times 0.28 + 78 \times 0.27 = 40.66$$

Allowable costs for the experiment

$$E'[u] - E[u] = 70.00 - 40.66 = 29.34$$

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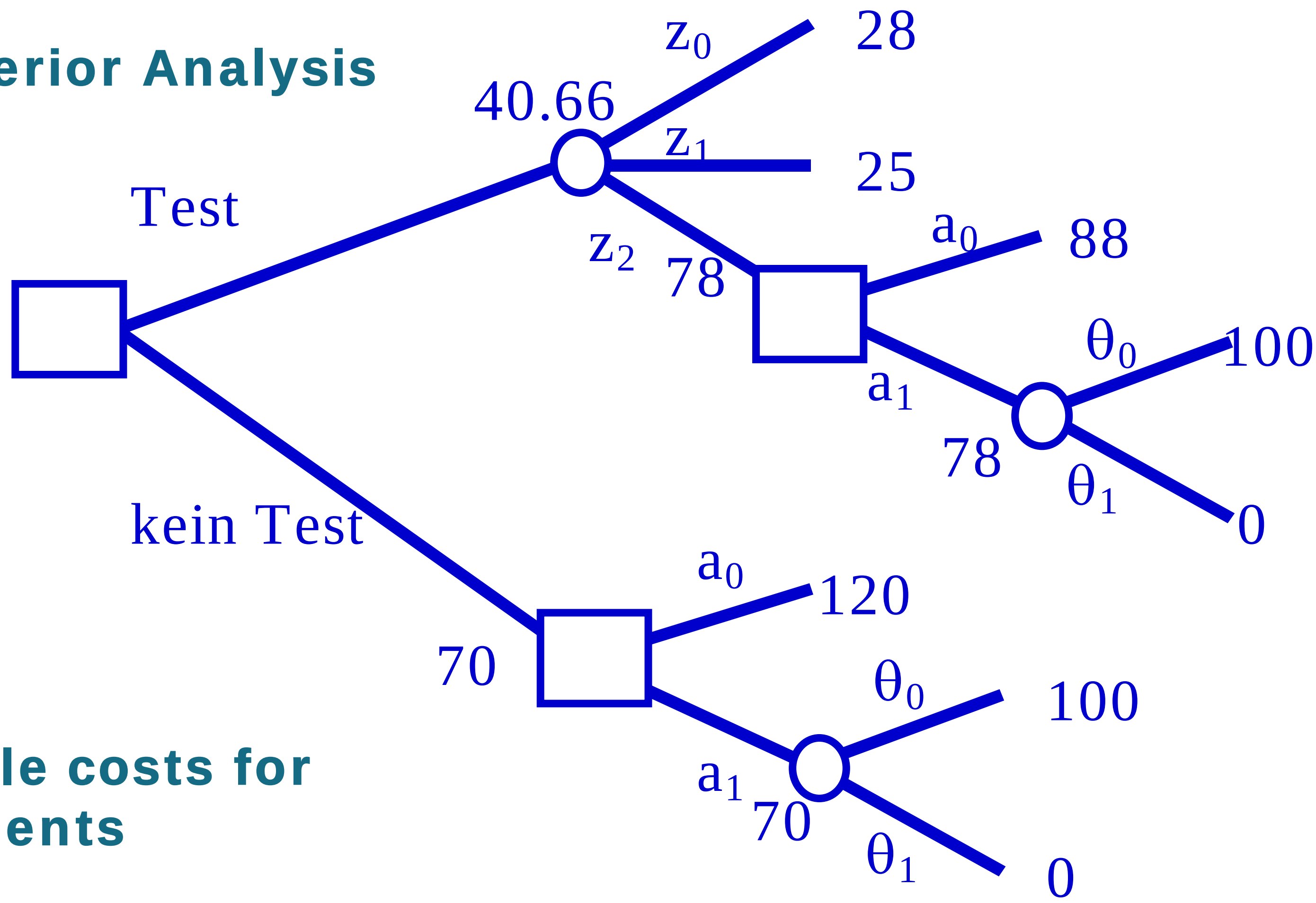
Pre-posterior Analysis



Allowable costs for experiments

$$E'[u] - E[u] = 70.00 - 40.66 = 29.34$$