Basic Statistics and Probability Theory

in

Civil, Surveying and Environmental Engineering

Prof. Dr. Michael Havbro Faber

Swiss Federal Institute of Technology

ETH Zurich, Switzerland

Rooms information

Before…

Group	Tutorial 1	Tutorials 2-9 and 11	Tutorial 10
E	HIL D 10.2	HCI D 2	To be announced
H	HIL B 21	HCI H 2.1	
K	HIL F 10.3	HCI D 8	
V	HIL E 1	HPH G 3	

Now…

Group	Tutorial 1	Tutorials 2-9 and11	Tutorial 10
E	HCI D 2	HCI D 2	To be announced
H	HPT C103	HCI H 2.1	
K	HIL F 10.3	HCI D 8	
V	HIL E 1	HPH G 3	

Time starting (Lecture/Tutorials):

HIL: 8 **Physics/Chemistry Buildings: 7.45**

Contents of Todays Lecture

- Risk and Motivation for Risk Assessment
- Overview of Probability Theory
- Interpretation of Probability
- Sample Space and Events
- The three Axioms of Probability Theory
- Conditional Probability and Bayes's Rule

Why Statistics and Probability in Engineering?

Risk is a characteristic of an activity relating to all possible events n_E which may follow as a result of the activity

The risk contribution R_{E_i} from the event E_i is defined through the product between

the Event probability P_{E_i}

and

the Consequences of the event C_{E_i}

The Risk associated with a given activity R_A may then be written as

$$R_A = \sum_{i=1}^{n_E} R_{E_i} = \sum_{i=1}^{n_E} P_{E_i} \cdot C_{E_i}$$

Decision Problems in Engineering

Uncertainties must be considered in the decision making throughout all phases of the life of an engineering facility

Planning and feasibility study ↔ Investigations and tests

Idea & Concept

Design

Manufacturing

Execution

Operation & maintenance

Decommissioning

Uncertainties

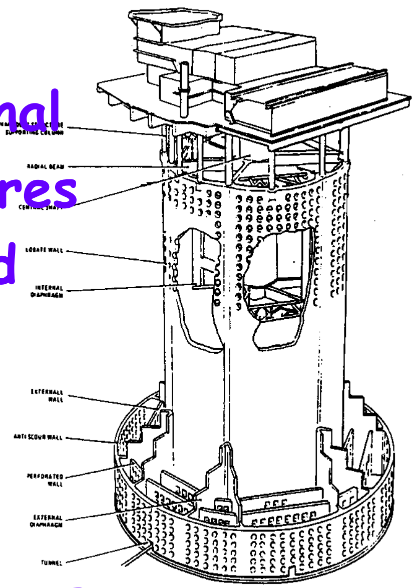
- **Traffic volume**
- **Loads**
- **Resistances (material, soil,..)**
- **Degradation processes**
- **Service life**
- **Manufacturing costs**
- **Execution costs**
- **Decommissioning costs**

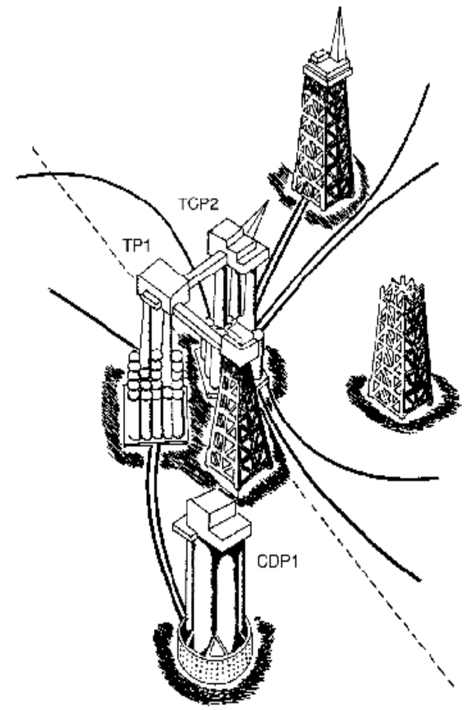
Example – Decommissioning of the Frigg Field

- The Frigg Field – built 1972-1978

  - TCP2
  - TP1
  - CDP1

According to international conventions the structures must be decommissioned





Each structure :

Weight : 250000 t

Costs : 200 – 600 Mio. SFr

- None of the platforms were designed for decommissioning !

Example – Decommissioning of the Frigg Field

- The decision problem

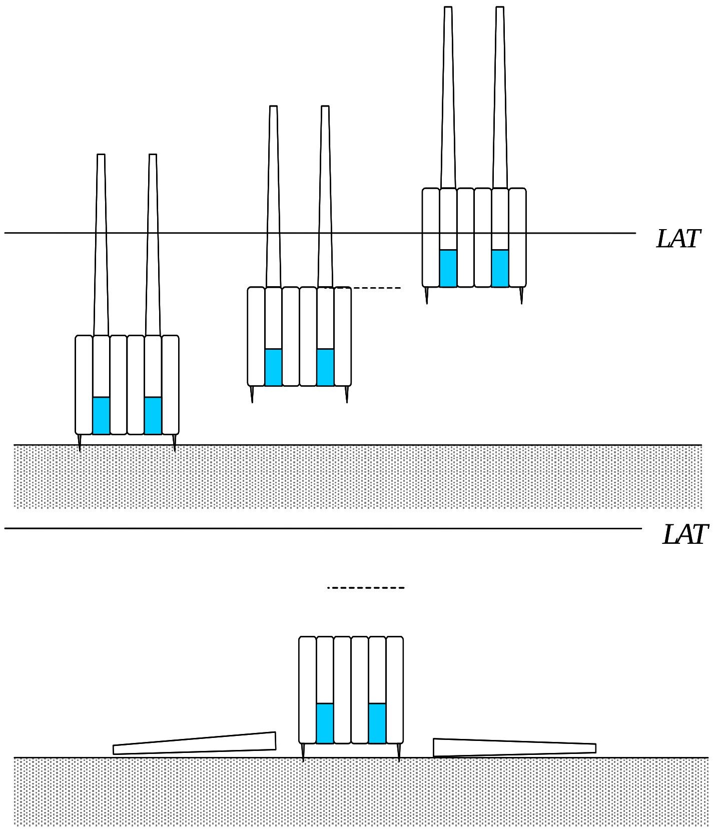
  Decommissioning/removal taking into account

  - Safety of personnel
  - Safety of the environment
  - Costs
  - Interest groups

  Greenpeace
  Fishers
  IMO
  ....

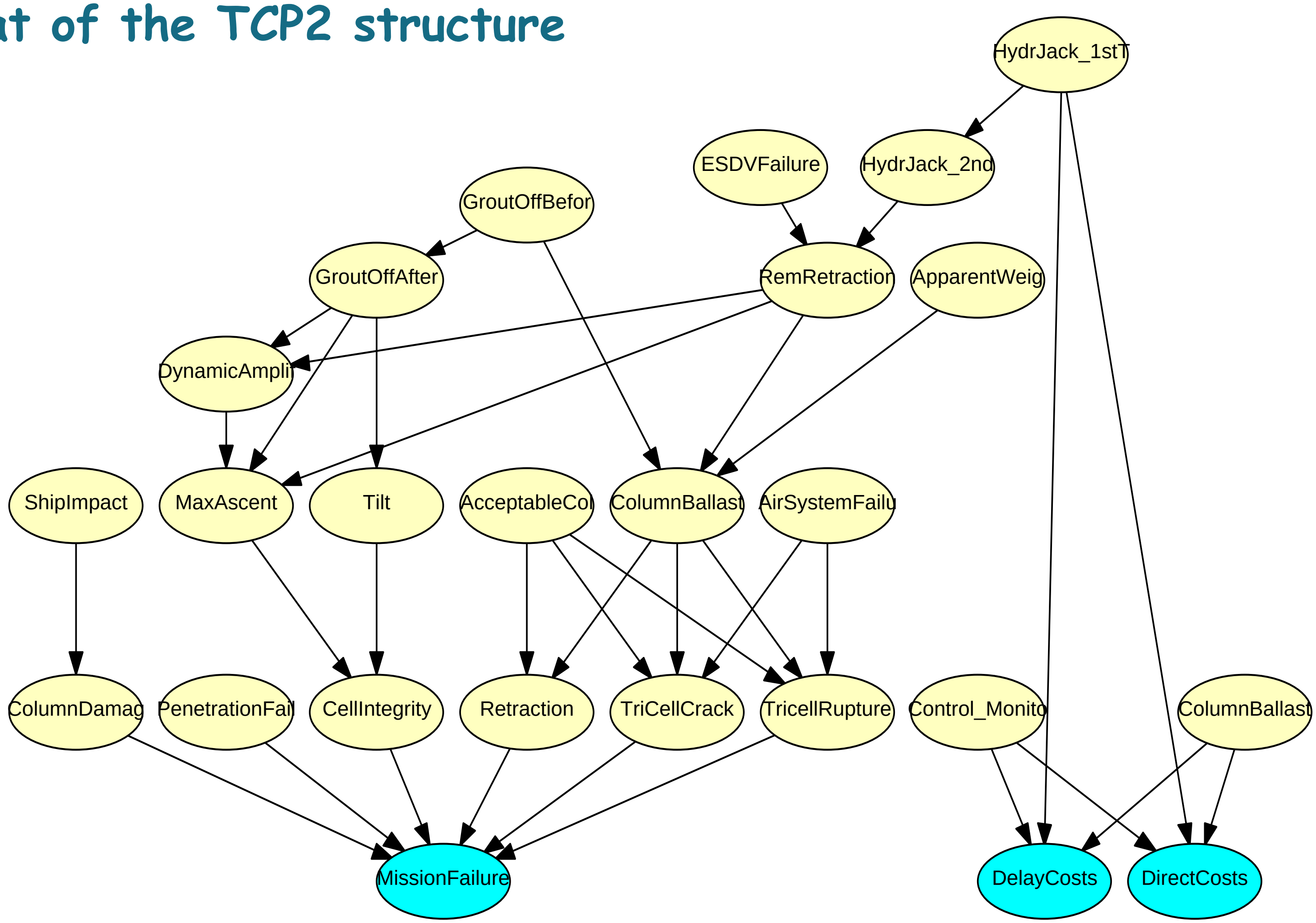
Example – Decommissioning of the Frigg Field

- Three options are considered
  - „Refloat“ and demolition Onshore
  - „Refloat“ and demolition Offshore
  - Removal to a free passage of 55 m depth
- The approach
  - Identification of hazard scenarios chronologically
  - Quantification of occurrence probabilities
  - Quantification of consequences
- Applied approach – Bayesian Nets



Example – Decommissioning of the Frigg Field

- Re-float of the TCP2 structure

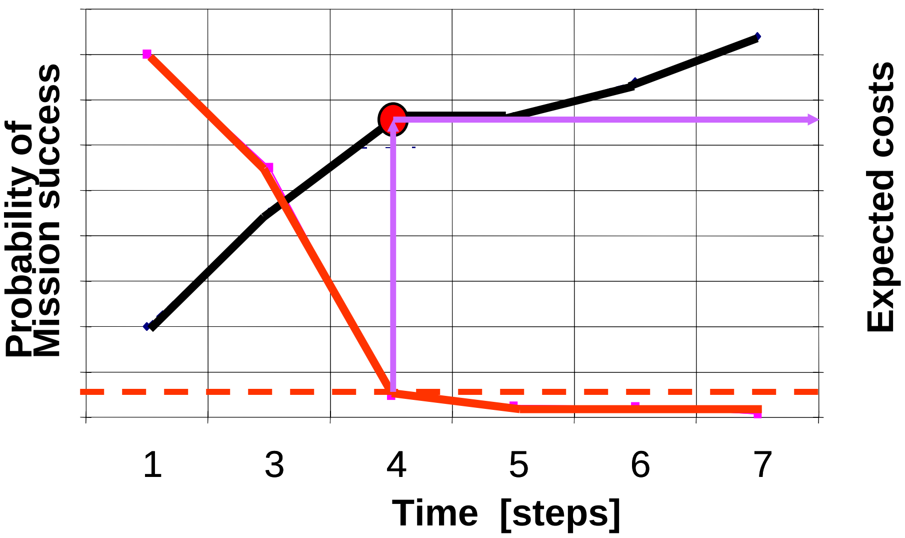


Example – Decommissioning of the Frigg Field

- Results of the decision analysis

Time variation of

- Expected costs
- Probability of mission success



Decision support

- How much to invest before a satisfactorily level of probability of mission success has been reached

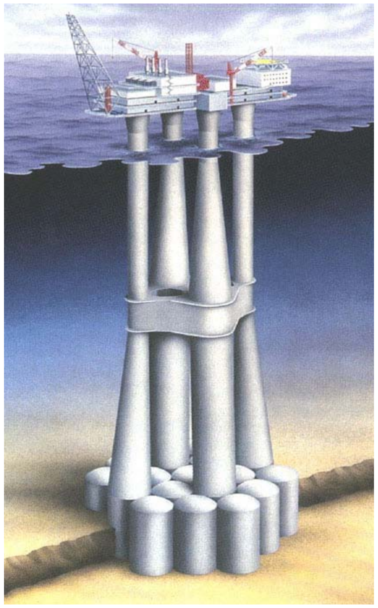
Decision Problems in Engineering

- Structural Design

Exceptional structures are often associated with structures of

„Extreme Dimensions“

Great Belt Bridge
under Construction



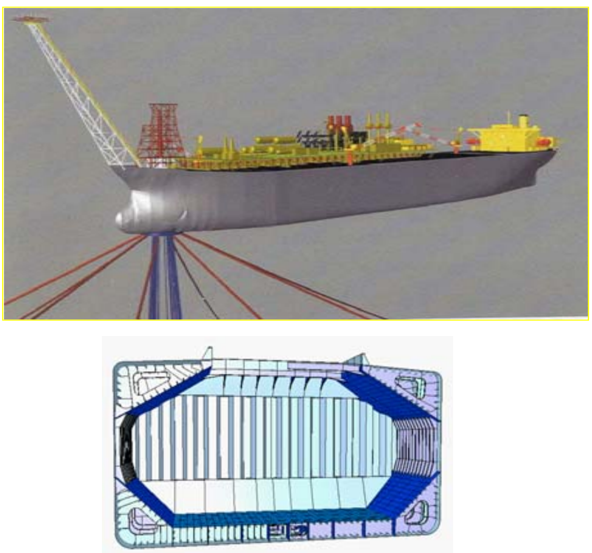
Concept drawing
of the Troll platform

Decision Problems in Engineering

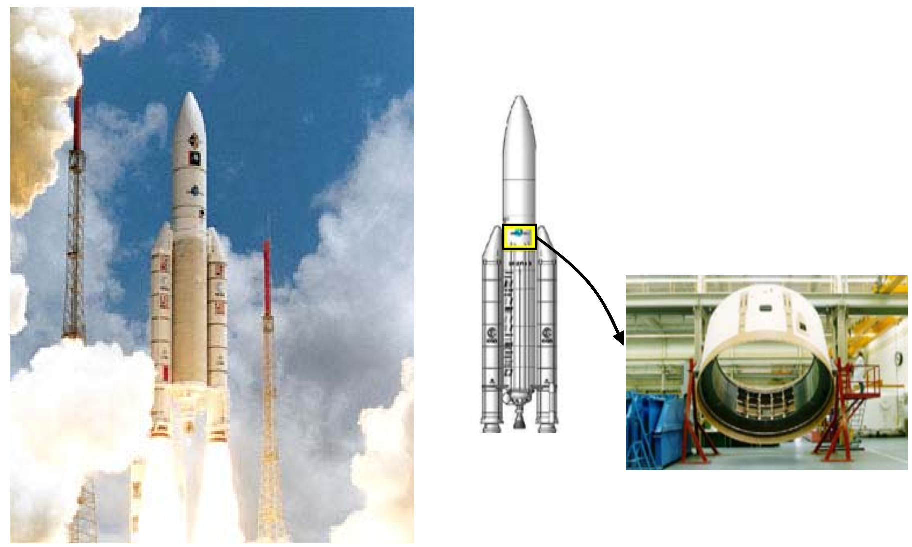
- **Structural Design**

  or associated with structures fulfilling

  „New and Innovative Purposes"

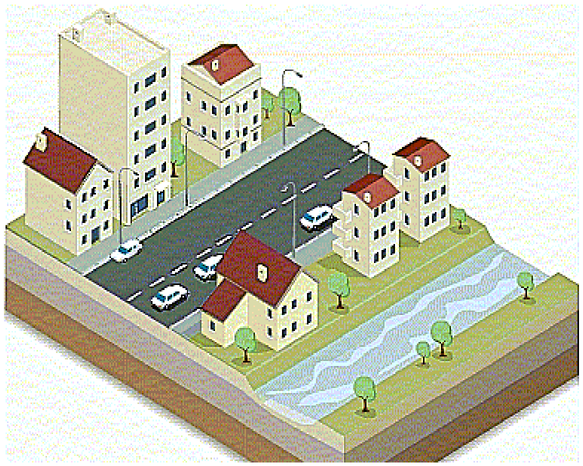


Concept drawing of
Floating Production, Storage and Offloading unit

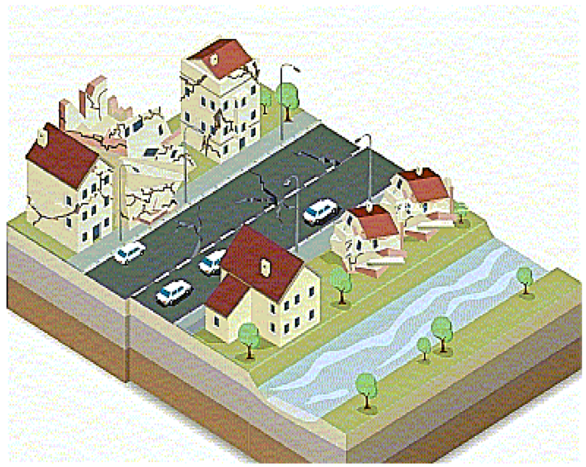


Illustrations of the ARIANE 5 rocket

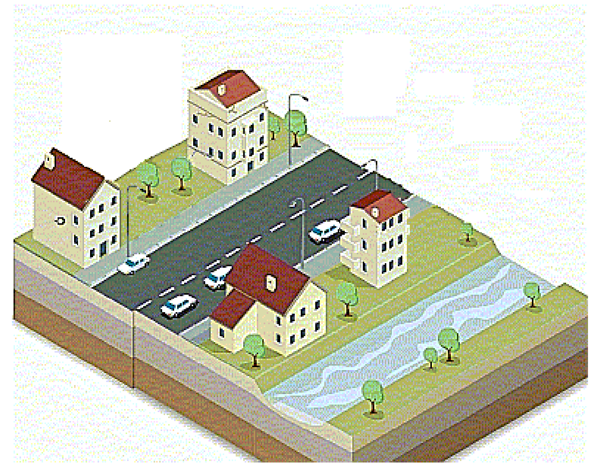
Decision Problems in Engineering



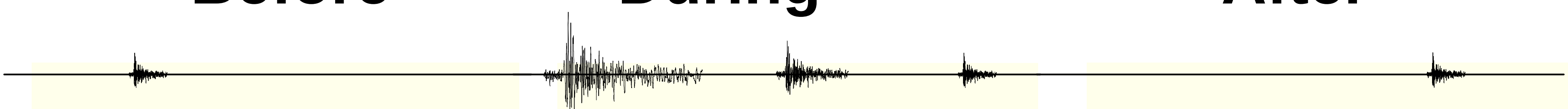
Before



During



After



Before

Optimal allocation of available resources for risk reduction

- strengthening
- rebuilding

in regard to possible earth-quakes

During

Damage reduction/Control

Emergency help and rescue

After quake hazards

After

Rehabilitation of infrastructure functionality

Condition assessment and updating of reliability and risks

Optimal allocation of resources for rebuilding and strengthening

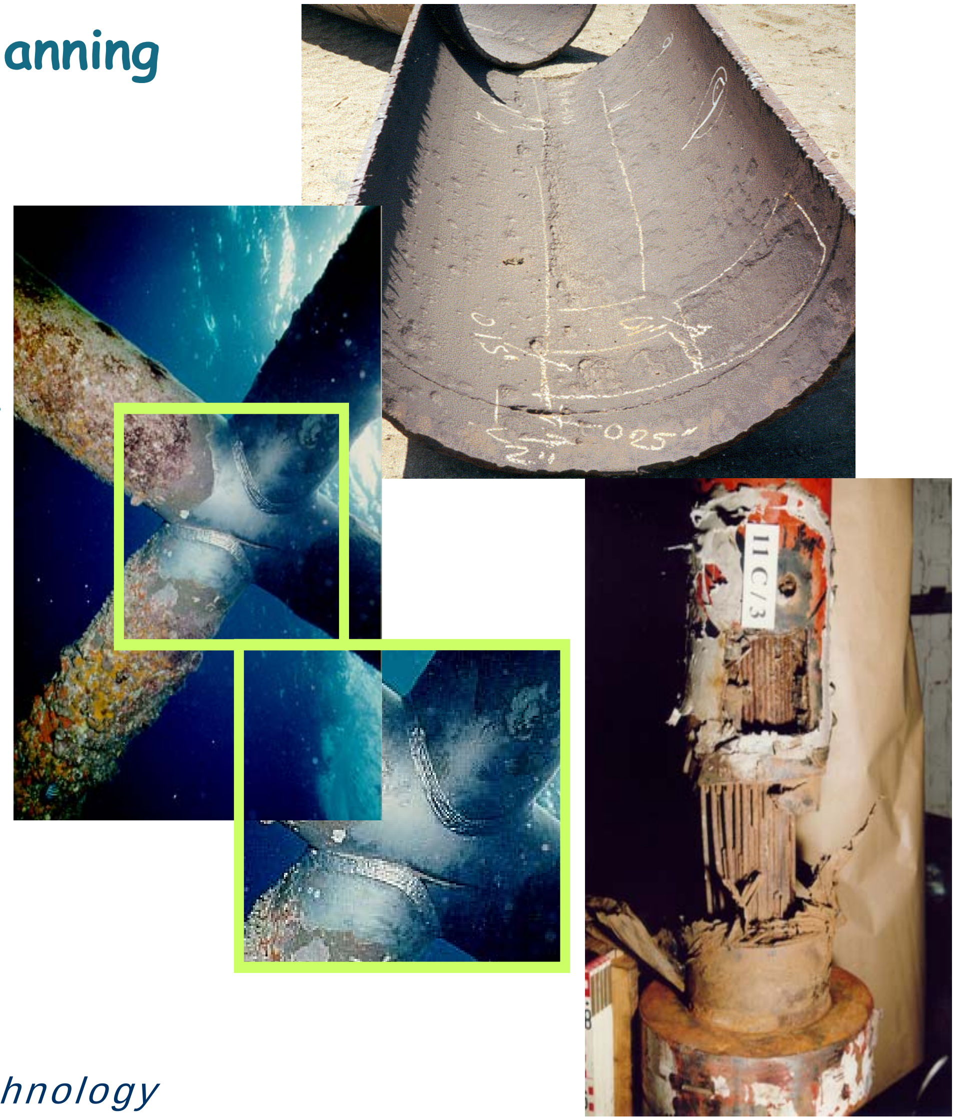
Decision Problems in Engineering

- Inspection and Maintenance Planning

  Due to
  - operational loading
  - environmental exposure

  structures will always to some degree be exposed to degradation processes such as

  - fatigue
  - corrosion
  - scour
  - wear



Why Statistics and Probability in Engineering?

In summary

statistics and probability theory is needed in engineering to

- quantify the uncertainty associated with engineering models
- evaluate the results of experiments
- assess importance of measurement uncertainties
- safe guard

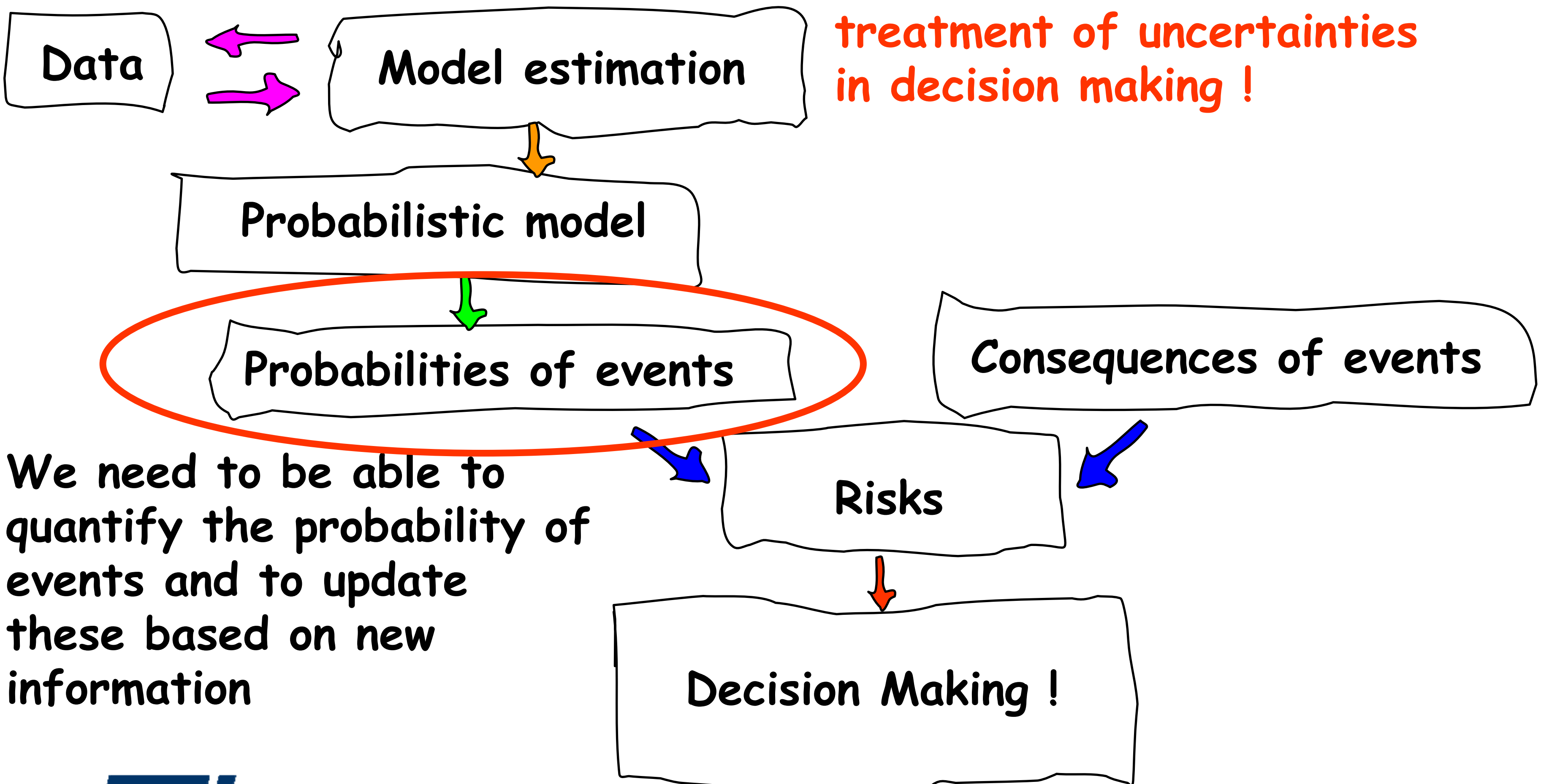
  safety for persons
  qualities of environment
  assets

ENHANCE DECISION MAKING

Overview of Probability Theory

- What are we aiming for ?

The probability theory provides the basis for the consistent treatment of uncertainties in decision making !



We need to be able to quantify the probability of events and to update these based on new information

Interpretation of Probability

- What is Probability ?

  We all have some notion of probability !

  and frequently use words like

  - Chance
  - Likelihood
  - Frequency
  - Probability

Nuclear accidents

Stock exchange

Smoking

Passing the exam

Crossing the street

Hunting

Succeeding difficult tasks

Interpretation of Probability

States of nature of which we have interest such as:

- a bridge failing due to excessive traffic loads
- a water reservoir being over-filled
- an electricity distribution system „falling out“
- a project being delayed

are in the following denoted „events“

we are generally interested in quantifying the probability that such events take place within a given „time frame“

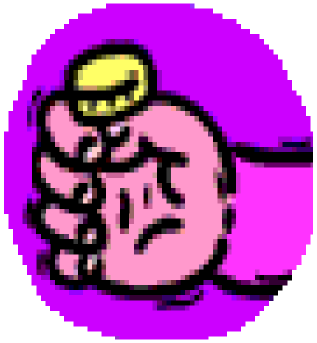
Interpretation of Probability

- There are in principle three different interpretations of probability

  - Frequentistic $P(A) = \lim \frac{N_A}{n_{\exp}}$ **for** $n_{\exp} \to \infty$
  - Classical $P(A) = \frac{n_A}{n_{tot}}$
  - Bayesian $P(A) = $ degree of belief that A will occur

Interpretation of Probability

Consider the probability of getting a „head“ when flipping a coin



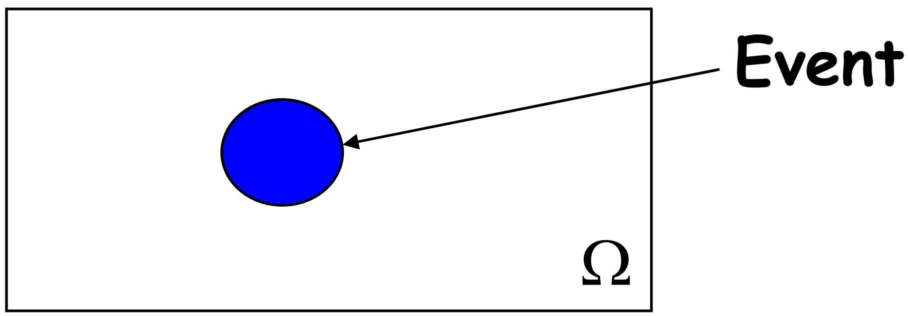
- **Frequentistic** $$P(A) = \frac{510}{1000} = 0.51$$
- **Classical** $$P(A) = \frac{1}{2}$$
- **Bayesian** $$P(A) = 0.5$$

Sample Space and Events

The set of all possible outcomes of the state of nature e.g. concrete compressive strength test results is called the sample space Ω. For concrete compressive strength test results the sample space can be written as $\Omega =]0;\infty[$

A sample space can be continuous or discrete.

Typically we illustrate the sample space and events using Venn diagrams

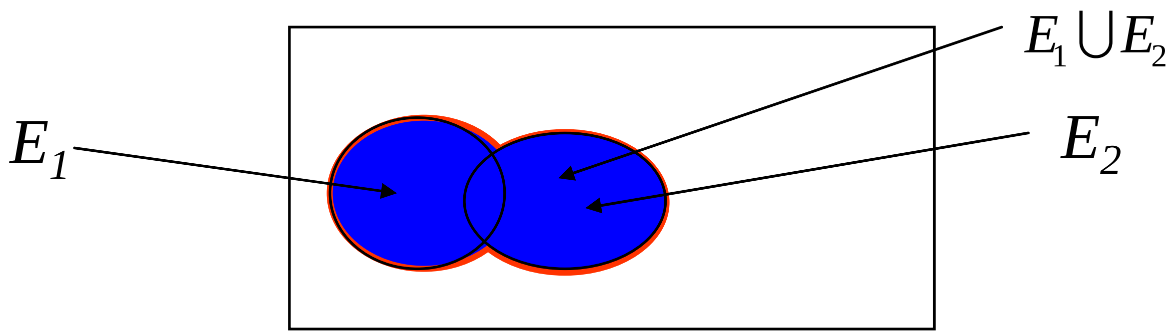


Sample Space and Events

An event is a sub-set of the sample space

- if the sub-set is empty the event is impossible
- if the sub-set contains all of the sample space the event is certain

Consider the two events E_1 and E_2:
The sub-set of sample points belonging to the event E_1 and/or the event E_2 is called the union of E_1 and E_2 and is written as : $E_1 \bigcup E_2$

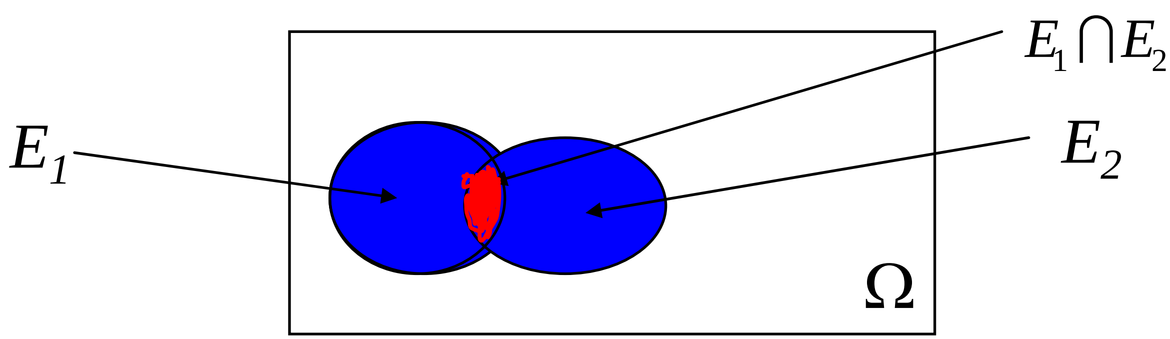


Sample Space and Events

An event is a sub-set of the sample space

- if the sub-set is empty the event is impossible
- if the sub-set contains all of the sample space the event is certain

Consider the two events E_1 and E_2 :
The sub-set of sample points belonging to the event E_1 and the event E_2 is called the **intersection** of E_1 and E_2 and is written as: $E_1 \cap E_2$

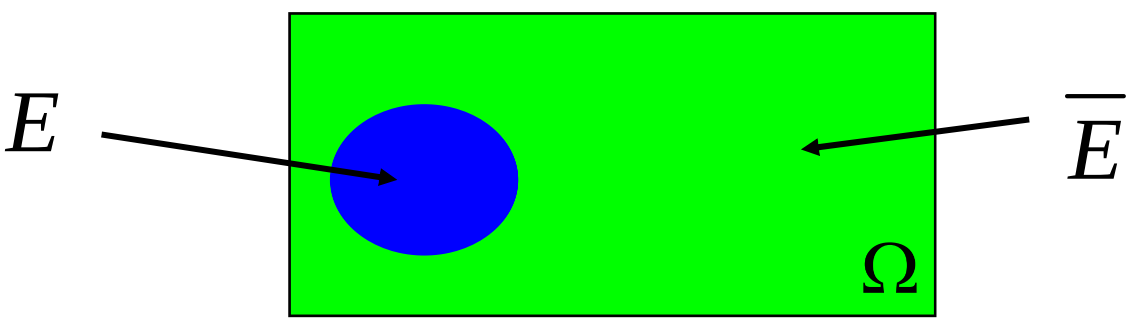


Sample Space and Events

The event containing all sample points in Ω not included in the event E is called the complementary event to E and written as : $\overline{E}$

It follows that $E \cup \overline{E} = \Omega$

and $E \cap \overline{E} = \varnothing$



Sample Space and Events

It can be show that the intersection and union operations obey the following commutative, associative and distributive laws:

$$E_1 \cap E_2 = E_2 \cap E_1$$

Commutative law

$$E_1 \cap (E_2 \cap E_3) = (E_1 \cap E_2) \cap E_3$$

$$E_1 \cup (E_2 \cup E_3) = (E_1 \cup E_2) \cup E_3$$

Associative law

$$E_1 \cap (E_2 \cup E_3) = (E_1 \cap E_2) \cup (E_1 \cap E_3)$$

$$E_1 \cup (E_2 \cap E_3) = (E_1 \cup E_2) \cap (E_1 \cup E_3)$$

Distributive law

Sample Space and Events

From the commutative, associative and distributive laws the so-called De Morgan's laws may be derived:

$$E_1 \cap E_2 = E_2 \cap E_1$$

$$E_1 \cap (E_2 \cap E_3) = (E_1 \cap E_2) \cap E_3$$

$$E_1 \cup (E_2 \cup E_3) = (E_1 \cup E_2) \cup E_3$$

$$E_1 \cap (E_2 \cup E_3) = (E_1 \cap E_2) \cup (E_1 \cap E_3)$$

$$E_1 \cup (E_2 \cap E_3) = (E_1 \cup E_2) \cap (E_1 \cup E_3)$$

$\Rightarrow$

$$E_1 \cap E_2 = \overline{\overline{E}_1 \cup \overline{E}_2}$$

$$E_1 \cup E_2 = \overline{\overline{E}_1 \cap \overline{E}_2}$$

The Three Axioms of Probability Theory

The probability theory is built up on - only - three axioms due to Kolmogorov:

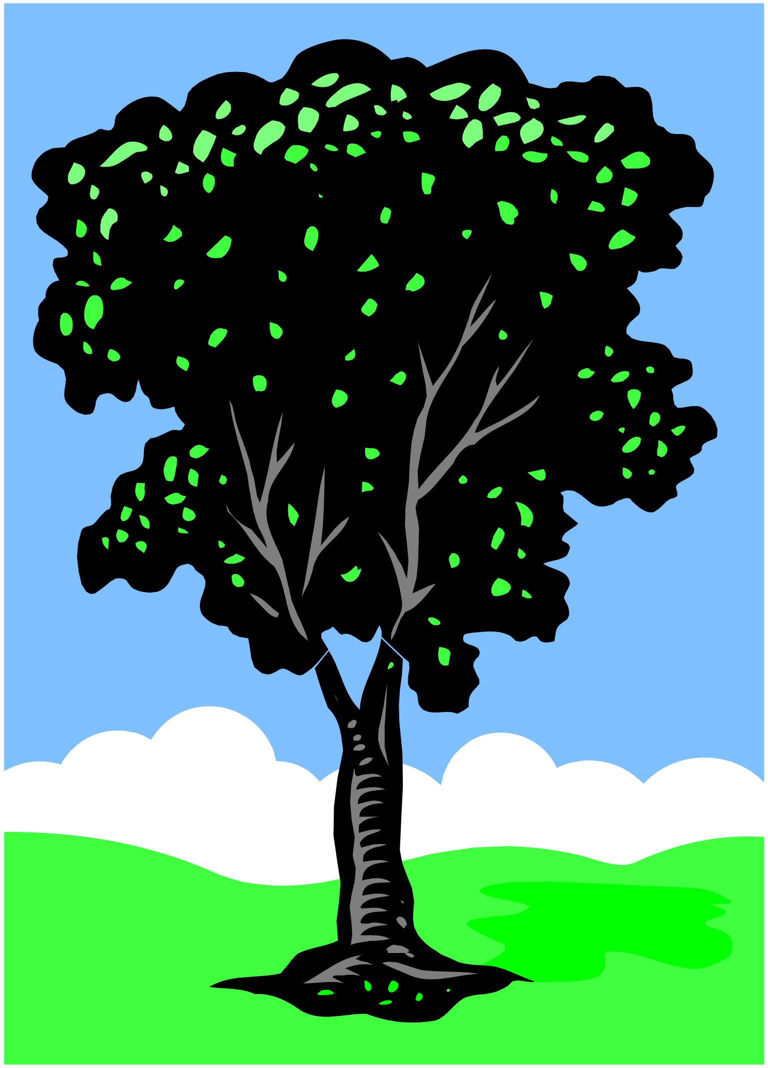
Axiom 1: $$0 \leq P(E) \leq 1$$

Axiom 2: $$P(\Omega) = 1$$

Axiom 3: $$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i)$$

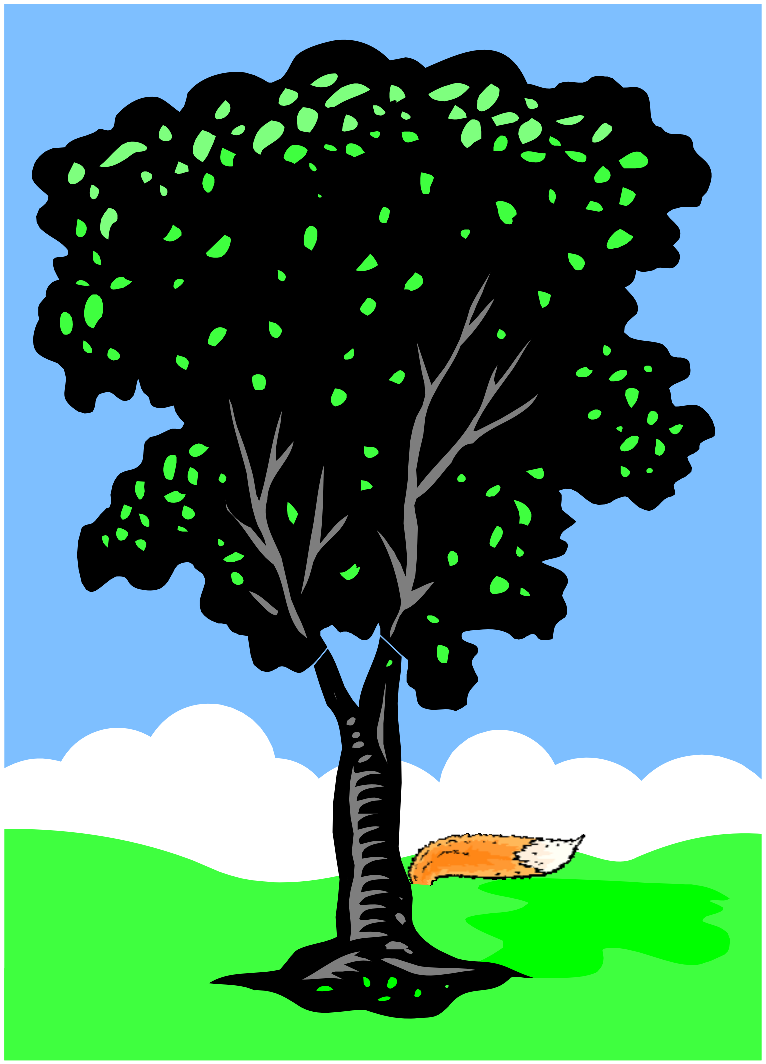
When E_1, E_2, ... are mutually exclusive

Conditional Probability and Bayes's Rule



ETH Swiss Federal Institute of Technology

Conditional Probability and Bayes's Rule



ETH Swiss Federal Institute of Technology

Conditional Probability and Bayes's Rule

Swiss Federal Institute of Technology

Conditional Probability and Bayes's Rule

Formulate hypothesis about the world

Utilize existing knowledge

Combine with data

Learn how to develop knowledge !

Conditional Probability and Bayes's Rule

Conditional probabilities are of special interest as they provide the basis for utilizing new information in decision making.

The conditional probability of an event E_1 given that event E_2 has occured is written as:

$$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

Not defined if $P(E_2) = 0$

The events E_1 and E_2 are said to be statistically independent if:

$$P(E_1 | E_2) = P(E_1)$$

Conditional Probability and Bayes's Rule

From $P(E_1|E_2) = \dfrac{P(E_1 \cap E_2)}{P(E_2)}$

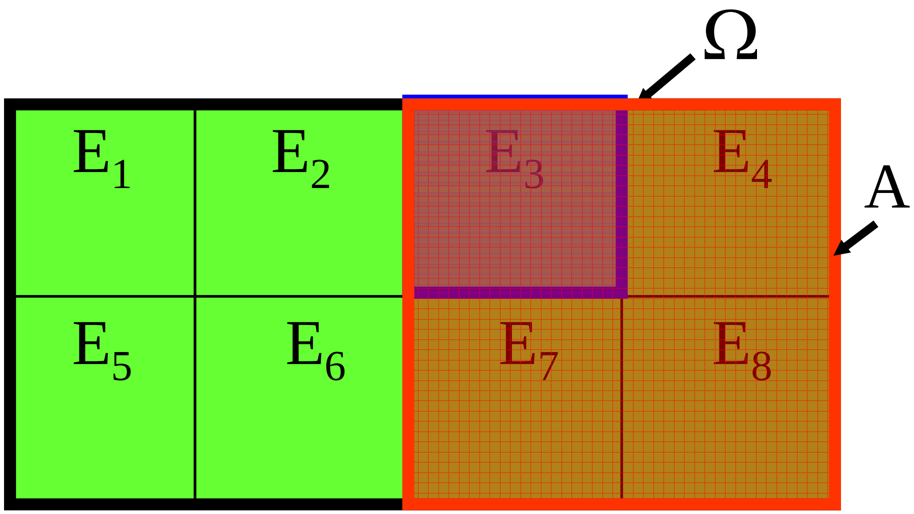
it follows that $P(E_1 \cap E_2) = P(E_2)P(E_1|E_2)$

and when E_1 and E_2 are statistically independent there is

$$P(E_1 \cap E_2) = P(E_2)P(E_1)$$

Conditional Probability and Bayes's Rule

Consider the sample space Ω divided up into n mutually exclusive events $E_1,\ E_2,\ \dots,\ E_n$



$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \dots + P(A \cap E_n)$$

$$P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + \dots + P(A|E_n)P(E_n) =$$

$$\sum_{i=1}^{n} P(A|E_i)P(E_i)$$

Conditional Probability and Bayes's Rule

as there is $P(A \cap E_i) = P(A|E_i)P(E_i) = P(E_i|A)P(A)$

we have

Likelihood

Prior

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)} = \frac{P(A|E_i)P(E_i)}{\sum_{i=1}^{n} P(A|E_i)P(E_i)}$$

Posterior

Bayes Rule

Reverend Thomas Bayes (1702-1764)