Exercise 7.4.

Diesel engines are used, among others, for electrical power generation. The operational time T of a diesel engine until a breakdown, is assumed to follow an Exponential distribution with mean μ_T = 24 months. Normally such an engine is inspected every 6 months and in case that a default is observed this is fully repaired. It is assumed herein that a default is a serious damage that leads to breakdown if the engine is not repaired.

- Exponential distribution with mean μT = 24 months
- engine is inspected every 6 months
- default is observed this is fully repaired

Shifted Exponential $x \geq \varepsilon$

$\lambda > 0$ $\mu = \varepsilon + 1/\lambda$ $\varepsilon = 0$ $\sigma = 1/\lambda$

$f(x) = \lambda*\exp(-\lambda*(x - \varepsilon))$

$F(x) = 1 - \exp(-\lambda*(x - \varepsilon))$

Here:

$\mu = 1/\lambda = 24$ $\lambda = 1/24$

$f(x) = 1/24*\exp(-1/24*x)$

$F(x) = 1 - \exp(-1/24*x)$

7.4. a) Calculate the probability that such an engine will need repair before the first inspection.

$$P(T < 6) = \int 1/24 * \exp(-1/24 * x) * dx = -\exp(-1/24*6) - (-\exp(0))$$

$$= 1 - \exp(-1/24*6) = \underline{\mathbf{0.2212}}$$

7.4. b) Assume that the first inspection has been carried out and no repair was required. Calculate the probability that the diesel engine will operate normally until the next scheduled inspection.

$$P[\ T > 12 \mid T > 6\ ] = P[\ T > 12 \cap T > 6\ ] \,/\, P[\ T > 6]$$
$$= P[\ T > 12\ ] \,/\, P[\ T > 6]$$

$$P[\ T > 12\ ] = 1 - (1 - \exp(-1/24*12)) = \exp(-1/2) = 0.6065$$
$$P[\ T > 6\ ] = 1 - (1 - \exp(-1/24*6)) = \exp(-1/4) = 0.7788$$

$$P[\ T > 12 \mid T > 6\ ] = \exp(-1/2) \,/\, \exp(-1/4) = \exp(-1/4) = \underline{\mathbf{0.7788}}$$

7.4. c) Calculate the probability that the diesel engine will fail between the first and the second inspection.

P[6 ≤ T ≤ 12] = ∫1/24*exp(-1/24*x) *dx = -exp(-1/2) + exp(-1/4)

= **0.1723**

7.4. d) A nuclear power plant owns 6 such diesel engines. The operational lives *t1* , *t2* ,…, *t6* of the diesel engines are assumed statistically independent. What is the probability that at most 1 engine will need repair at the first scheduled inspection?

Binomial dist. for 0 and 1 (at most 1)

P[need repair at the first inspection] = P[T ≤ 6] = 0.2212

(from 7.4. a)

P(y) = nCr(n , y)*p^y * (1-p)^(n-y)   y = 0, 1, 2, …, n

n = 6   p = 0.2212

P(0) = nCr(6 , 0)*p^0 * (1-p)^(6) = 0.2235

P(1) = nCr(6 , 1)*p^0 * (1-p)^(5) = 0.3804

Ptot = P(0) + P(1) = **0.60386**

7.4. e) It is a requirement that the probability of repair at each scheduled inspection is not more than 60%. The operational lives t1, t2, …, t6 of the diesel engines are assumed statistically independent. What should be the inspection i nterval?

0.6 = (1 – (1 - exp(-1/24*x)))^6 => x = **2.04 months**