Exercise 9.3 (Group Exercise):

a. The sample first moment is calculated by:

$$m_1 = \frac{1}{10}\sum_{i=1}^{10} x_i = 0.656$$

where x_i (i=1,2,…,10) are the observations. The analytical first moment of the given distribution is obtained as:

$$\lambda_1 = \int_0^1 x f_X(x)\,dx = \int_0^1 x\alpha x^{\alpha-1} dx$$

$$= \left[\frac{\alpha}{\alpha+1}x^{\alpha+1}\right]_0^1 = \frac{\alpha}{\alpha+1}$$

By equating m_1 and λ_1, the estimate of α is obtained:

$$\frac{\alpha}{\alpha+1} = 0.656 \Leftrightarrow \alpha = 1.91$$

b. The likelihood function is written as:

$$L(\alpha) = \prod_{i=1}^{10} \alpha x_i^{\alpha-1}$$

The log likelihood function is written as:

$$l = \ln L(\alpha) = \sum_{i=1}^{10} \log\left(\alpha x_i^{\alpha-1}\right) = 10\log\alpha + (\alpha-1)\sum_{i=1}^{10}\log x_i$$

The maximum likelihood estimator is obtained by maximizing the log-likelihood function as:

$$\frac{dl}{d\alpha} = \frac{10}{\alpha} + \sum_{i=1}^{10}\log x_i = 0$$

$$\hat{\alpha} = -\frac{10}{\sum_{i=1}^{10}\log x_i} = 1.96\,.$$

c. See Figure 9.3.1 where the blue line represents the probability distribution function with the estimated parameter and the red line represents the observed cumulative distribution.

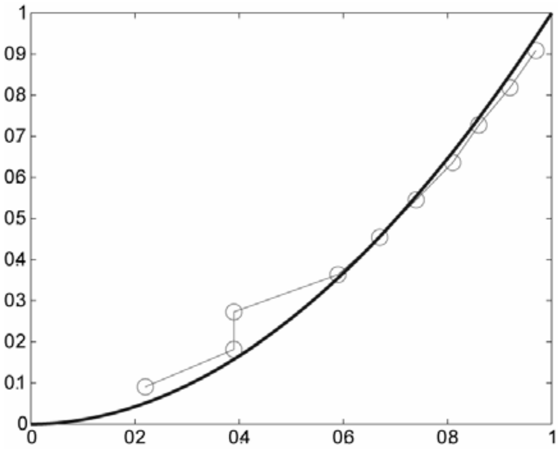

Figure 9.3.1: **Cumulative distribution function and observed cumulative frequency.**