Exercise 2.7 (Group Exercise) - Solution:

a. It is: $P(I = C_1|C_1) + P(I = C_2|C_1) + P(I = C_3|C_1) = 1$

    Hence:

    $0.84 + P(I = C_2|C_1) + 0.03 = 1$

    $P(I = C_2|C_1) = 0.13$

    Similarly the rest of the table can be completed.

Category of the thickness of the clay layer C_i	Indication of the thickness of the clay layer		
	$I = C_1$	$I = C_2$	$I = C_3$
C_1	0.84	**0.13**	0.03
C_2	0	0.77	**0.23**
C_3	**0.09**	0.02	0.89

Table 2.7.2: Probability of indication on each ground category

b. The probability that the true state is C_1 given the indication of C_3 is obtained as:

$$P(C_1 | I = C_3) = \frac{P(I = C_3 | C_1)P(C_1)}{P(I = C_3 | C_1)P(C_1) + P(I = C_3 | C_2)P(C_2) + P(I = C_3 | C_3)P(C_3)} = 0.015$$

In the same way, the posterior probability of the other state is obtained as:

$$P(C_2 | I = C_3) = \frac{P(I = C_3 | C_2)P(C_2)}{P(I = C_3 | C_1)P(C_1) + P(I = C_3 | C_2)P(C_2) + P(I = C_3 | C_3)P(C_3)} = 0.265$$

$$P(C_3 | I = C_3) = \frac{P(I = C_3 | C_3)P(C_3)}{P(I = C_3 | C_1)P(C_1) + P(I = C_3 | C_2)P(C_2) + P(I = C_3 | C_3)P(C_3)} = 0.720$$