Exercise 11.1 (Group Exercise) - Solution

Using the information provided it is:

$P(SW) = 0.6$

$P(SD) = 0.4$

$P(I_{SD} | SD) = 0.75$

$P(I_{SW} | SW) = 0.75$

$P(I_{SD} | SW) = 1 - P(I_{SD} | SD) = 1 - 0.75 = 0.25$

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Using the Bayes' Theorem it is:

$$P(SD | I_{SD}) = 0.6667 = \frac{P(I_{SD} | SD) \cdot P(SD)}{P(I_{SD} | SD) \cdot P(SD) + P(I_{SD} | SW) \cdot P(SW)} = \frac{0.75 \cdot 0.4}{0.75 \cdot 0.4 + 0.25 \cdot 0.6} = \frac{2}{3} = 0.6667$$

$$P(SD | I_{sw}) = \frac{P(I_{SW} | SD) \cdot P(SD)}{P(I_{SW} | SD) \cdot P(SD) + P(I_{SW} | SW) \cdot P(SW)} = \frac{0.25 \cdot 0.4}{0.25 \cdot 0.4 + 0.75 \cdot 0.6} = \frac{2}{11} = 0.18182$$

$$P(SW | I_{SD}) = \frac{P(I_{SD} | SW) \cdot P(SW)}{P(I_{SD} | SW) \cdot P(SW) + P(I_{SD} | SD) \cdot P(SD)} = \frac{0.25 \cdot 0.6}{0.25 \cdot 0.6 + 0.75 \cdot 0.4} = \frac{1}{3} = 0.3333$$

$$P(SW | I_{SW}) = \frac{P(I_{SW} | SW) \cdot P(SW)}{P(I_{SW} | SW) \cdot P(SW) + P(I_{SW} | SD) \cdot P(SD)} = \frac{0.75 \cdot 0.6}{0.75 \cdot 0.6 + 0.25 \cdot 0.4} = \frac{9}{11} = 0.8181$$

And:

$$P(I_{SW}) = P(I_{SW} | SW) \cdot P(SW) + P(I_{SW} | SW) \cdot P(SW) = 0.75 \cdot 0.6 + 0.25 \cdot 0.4 = 0.55$$

$$P(I_{SD}) = P(I_{SD} | SW) \cdot P(SW) + P(I_{SD} | SD) \cdot P(SD) = 0.25 \cdot 0.6 + 0.75 \cdot 0.4 = 0.45$$

The event tree can now be filled in. An example of calculation is provided in the following.

Consider the branch associated with the activity "clean up the roof". If the roof is cleaned up there are two events that may occur according to our problem:

    a. the roof may collapse (due to various reasons)
    b. the roof will not collapse (survival of the roof)

These events are associated with a probability as shown in the event tree branches:

a. $P_f(SN) = 5 \cdot 10^{-4}$ and b. $P_s(SN) = 1 - 5 \cdot 10^{-4} = 0.9995$. Hence the expected cost of this action is:

$$E[C_{clean\;up}] = P_f(SN) \cdot 1000004 + P_s(SN) \cdot 4000 = 5 \cdot 10^{-4} \cdot 1004000 + 0.9995 \cdot 4000$$
$$= 4500\ CHF$$

In a similar way the rest of the event tree may be completed.
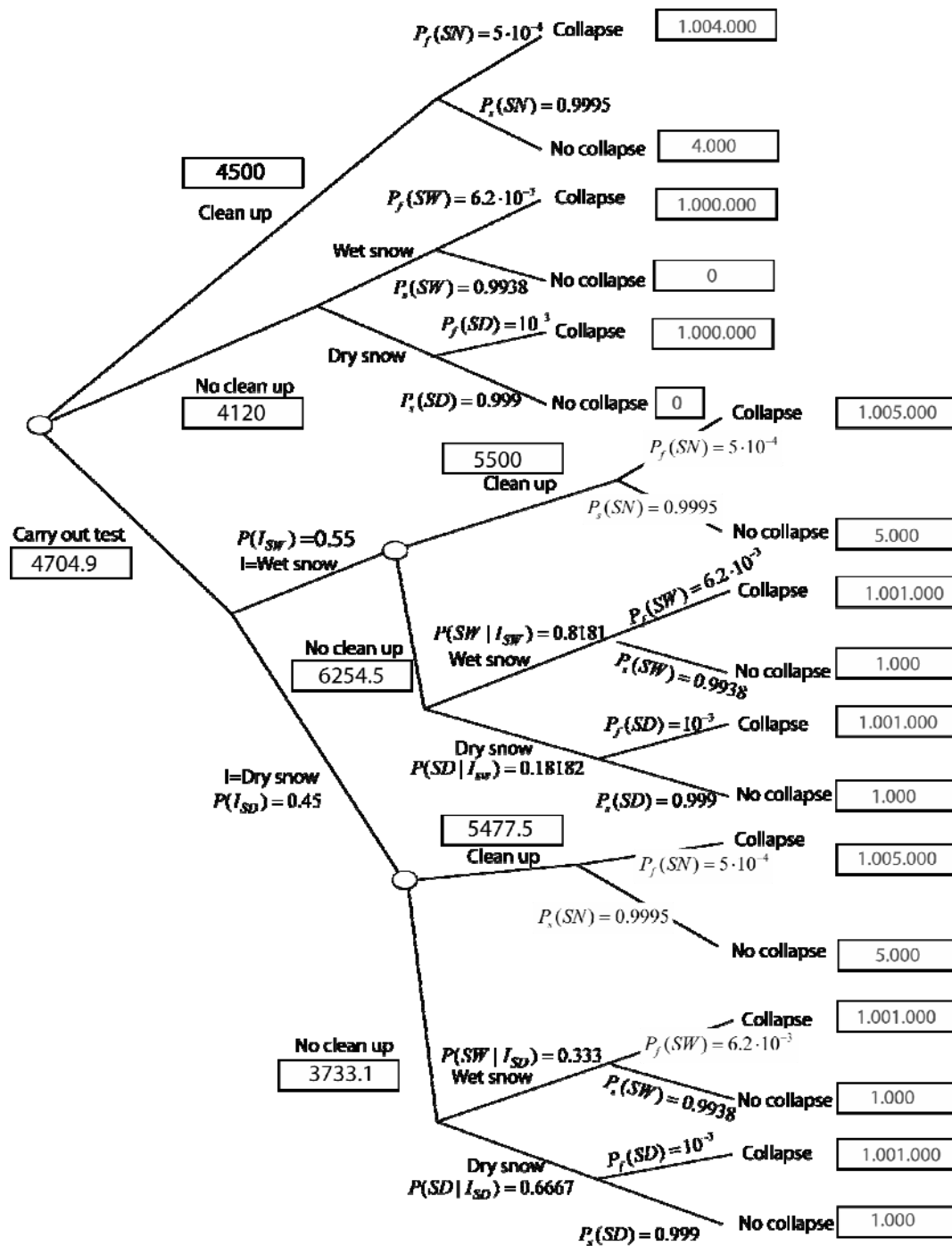

Figure 11.1.2. Completed event tree.

It can be seen that the action associated with the smaller cost is not to clean up the roof.