Correction!!

EXERCISE TUTORIAL 8 - SOLUTION

Exercise 8.1 - Solution

a. → From the Pythagorean Theorem it follows that:

→ $f^2 + a^2 + b^2 = d^2$

Therefore, the error in d propagates according to $\varepsilon_d = \sqrt{\varepsilon_f^2 + \varepsilon_a^2 + \varepsilon_b^2}$.

→ Then, $\frac{\varepsilon_d}{\sigma_\varepsilon} = \sqrt{(\frac{\varepsilon_f}{\sigma_\varepsilon})^2 + (\frac{\varepsilon_a}{\sigma_\varepsilon})^2 + (\frac{\varepsilon_b}{\sigma_\varepsilon})^2}$ is Chi-distributed with three degrees of freedom.

→ The probability density function of $Z = \frac{\varepsilon_d}{\sigma_\varepsilon}$ is:

→ $f_Z(z) = \frac{z^{(3-1)}}{2^{3/2-1}\Gamma(3/2)} e^{(-z^2/2)}$ $\quad f_{\varepsilon_d}(\varepsilon_d) = \frac{1}{\sqrt{2}}\left(\frac{\varepsilon_d}{\sigma_\varepsilon}\right)^2 \frac{1}{\sqrt{\pi}/2} e^{\left(-\left(\frac{\varepsilon_d}{\sigma_\varepsilon}\right)^2/2\right)} \left|\frac{dz}{d\varepsilon_d}\right| = \sqrt{\frac{2}{\pi}}\left(\frac{\varepsilon_d}{\sigma_\varepsilon}\right)^2 e^{\left(-\left(\frac{\varepsilon_d}{\sigma_\varepsilon}\right)^2/2\right)} \frac{1}{\sigma_\varepsilon}$

→ Therefore, the probability density function of ε_d is obtained as:

→ $f_{\varepsilon_d}(\varepsilon_d) = \frac{1}{2\sqrt{2}}(\frac{\varepsilon_d}{\sigma_\varepsilon})^2 \frac{1}{\sqrt{\pi}/2} e^{(-(\frac{\varepsilon_d}{\sigma_\varepsilon})^2/2)} \left|\frac{dz}{d\varepsilon_d}\right| = \frac{1}{\sqrt{2\pi}}(\frac{\varepsilon_d}{\sigma_\varepsilon})^2 e^{(-(\frac{\varepsilon_d}{\sigma_\varepsilon})^2/2)} \frac{1}{\sigma_\varepsilon}$