

Assignment 3

1. Finite element code for a cantilever beam

Create a finite element code to calculate the deformation of a shear flexible cantilever beam (Timoshenko beam) using Matlab[®]. The beam and the characteristics of the cross section are given in Figure 1. The beam element and the corresponding degrees of freedom are given in Figure 2.

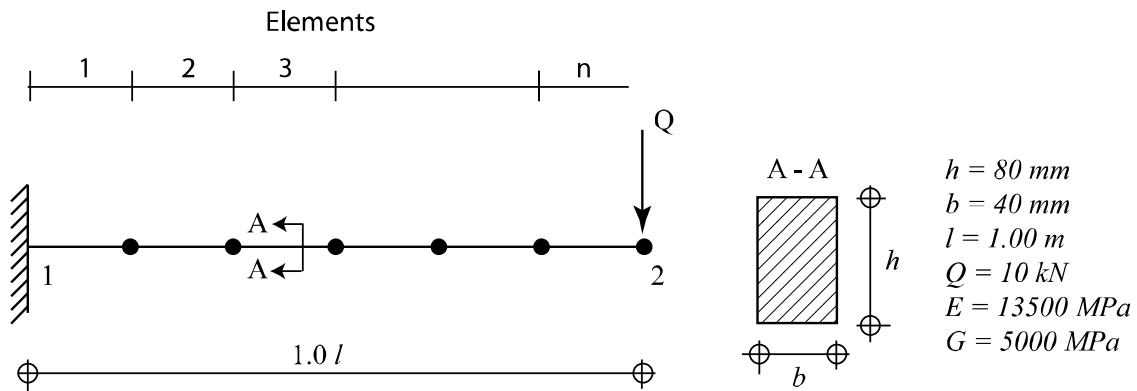


Figure 1: Cantilever beam

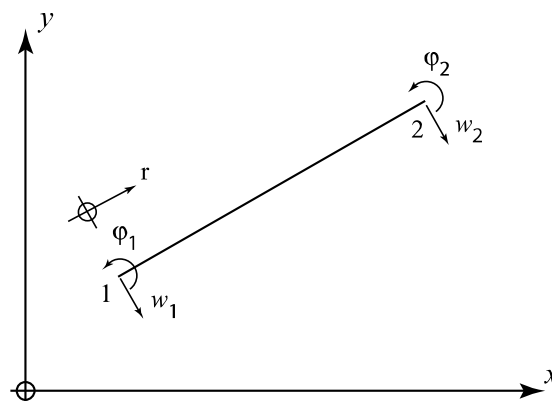


Figure 2: Beam element

The local stiffness matrix for the shear flexible beam element given in Figure 2 is given as:



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Lecture "Methods of Finite Elements I"
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$$\mathbf{k}_{local} = \begin{bmatrix} \frac{GA_Q}{L} & -\frac{1}{2}GA_Q & -\frac{GA_Q}{L} & -\frac{1}{2}GA_Q \\ -\frac{1}{2}GA_Q & \frac{1}{3}GA_Q L + \frac{EI}{L} & \frac{1}{2}GA_Q & \frac{1}{6}GA_Q L - \frac{EI}{L} \\ -\frac{GA_Q}{L} & \frac{1}{2}GA_Q & \frac{GA_Q}{L} & \frac{1}{2}GA_Q \\ -\frac{1}{2}GA_Q & \frac{1}{6}GA_Q L - \frac{EI}{L} & \frac{1}{2}GA_Q & \frac{1}{3}GA_Q L + \frac{EI}{L} \end{bmatrix} \quad (1)$$

where L denotes the length of one element and A_Q denotes the effective shear area of the element, which is given as:

$$A_Q = \frac{5}{6}A \quad (2)$$

- A) Calculate the displacement at the point 2 with the created code and plot the calculated displacement over different numbers of elements. Compare the results with the exact solution given in Equation (3) and check the convergence of the results by increasing the number of elements.

$$w(x=l) = \frac{Ql^3}{3EI} + \frac{Ql}{GA_Q} \quad (3)$$

- B) Calculate the displacement at the point 2 for different shear modules and different number of elements given in Table 1. Compare the results with the result obtained in Assignment 2 for a shear stiff beam element (corresponding to Bernoulli Beam).

The Matlab[®] code should be submitted as an .m file in an electronic format.

Table 1: Displacement at point 2

	No. of elements	Shear modul [MPa]					
		4500	5000	5500	6000	6500	6750
Displacement [mm] (Timoshenko beam element)	10						
	100						
Displacement [mm] (Bernoulli beam element)	1						