

Assignment 1

1. Cantilever Beam

Using the variational approach calculate the vertical displacement w at point A and the bending moment distribution $M(x)$ for a cantilever beam (Figure 1) subjected to

- a uniform distributed load with q
- a concentrated load Q at point A .

Here, EI is assumed to be a constant. Approximate the displacement $w(x)$ by a third-order polynomial.

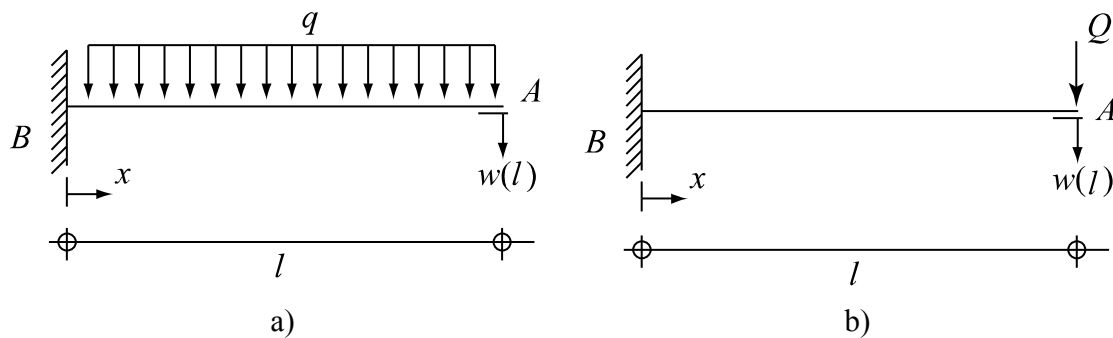


Figure 1. Cantilever beam

Hint: The potential energy of the system is given as:

$$\Pi = \int_0^l \frac{EI}{2} \left(\frac{d^2 w}{dx^2} \right)^2 dx - \int_0^l q(x) w(x) dx - Q w(l)$$

And the relationship between the vertical displacement and internal forces of the beam is:

$$\frac{dw}{dx} = -\Psi(x)$$

$$EI \frac{d^2 w}{dx^2} = -M(x)$$

$$EI \frac{d^3 w}{dx^3} = -Q(x)$$

$$EI \frac{d^4 w}{dx^4} = q(x)$$