Application Example:
Performance of Structural Systems

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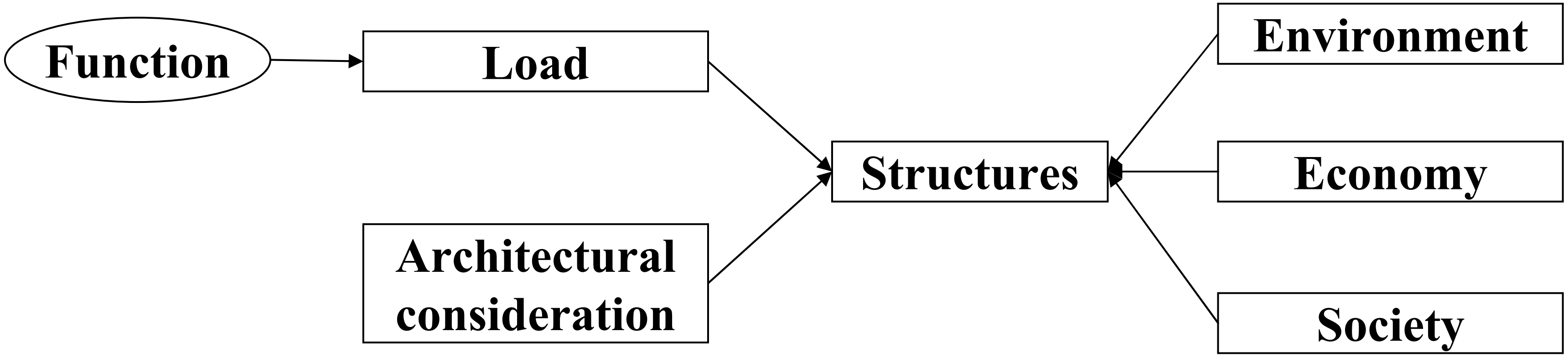
Introduction

Structures come in all shapes and sizes, but their primary function is to support and resist various loads.



Function

Load

Architectural consideration

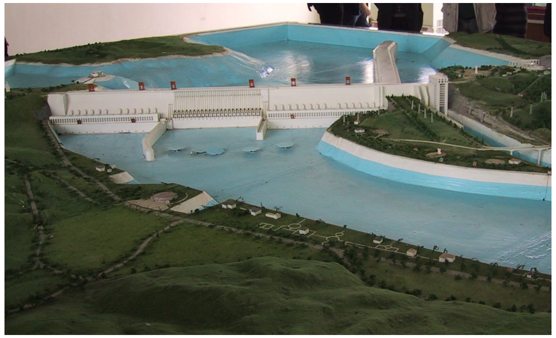
Structures

Environment

Economy

Society

Environment



Economy

Society

Types of structures

- Bending*:
  Beam

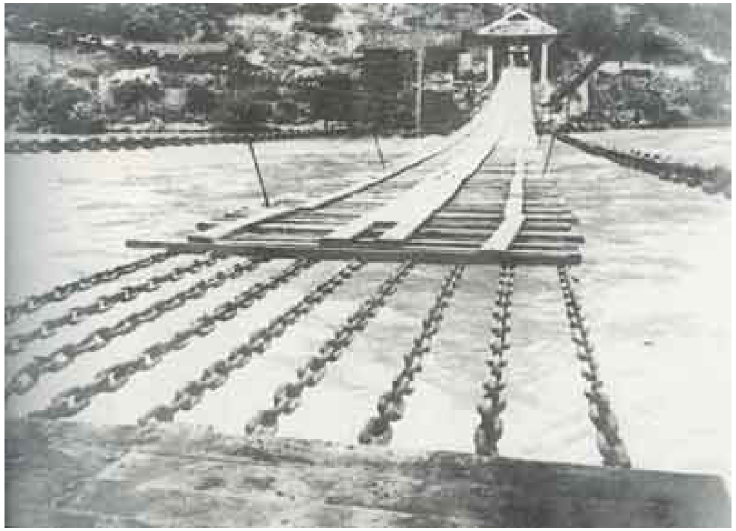
*Reference: Ghali, A., Neville, A. M. and Brown, T. G. (2003). Structural Analysis - A Unified Classical And Matrix Approach. London, United Kingdom, Spon Press.

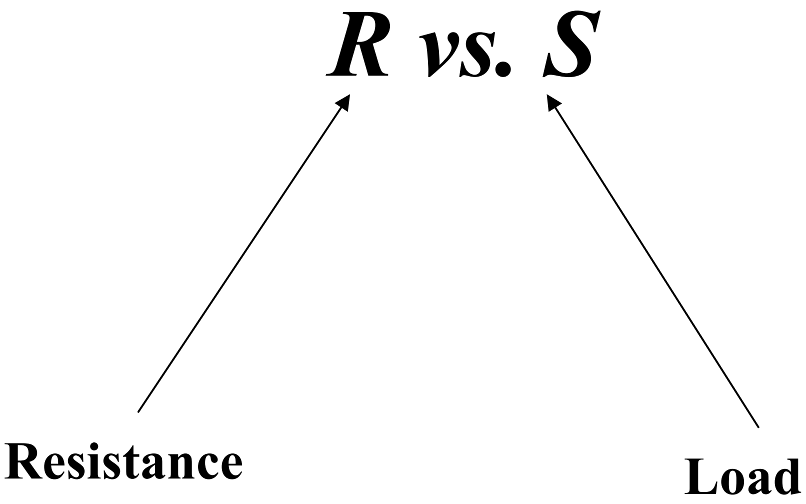
Types of structures

- Compression:
  Arch

Types of structures

- Tension:
  Cable





R vs. S

Resistance

Load

R ↓

S ↑

Mechanical failure modes*

- Buckling
- Corrosion
- Creep
- Fatigue
- Fracture
- Yielding

etc…

Natural hazards and human factors

- Earthquake
- Typhoon
- Fire
- Explosion
- Airplane impacts

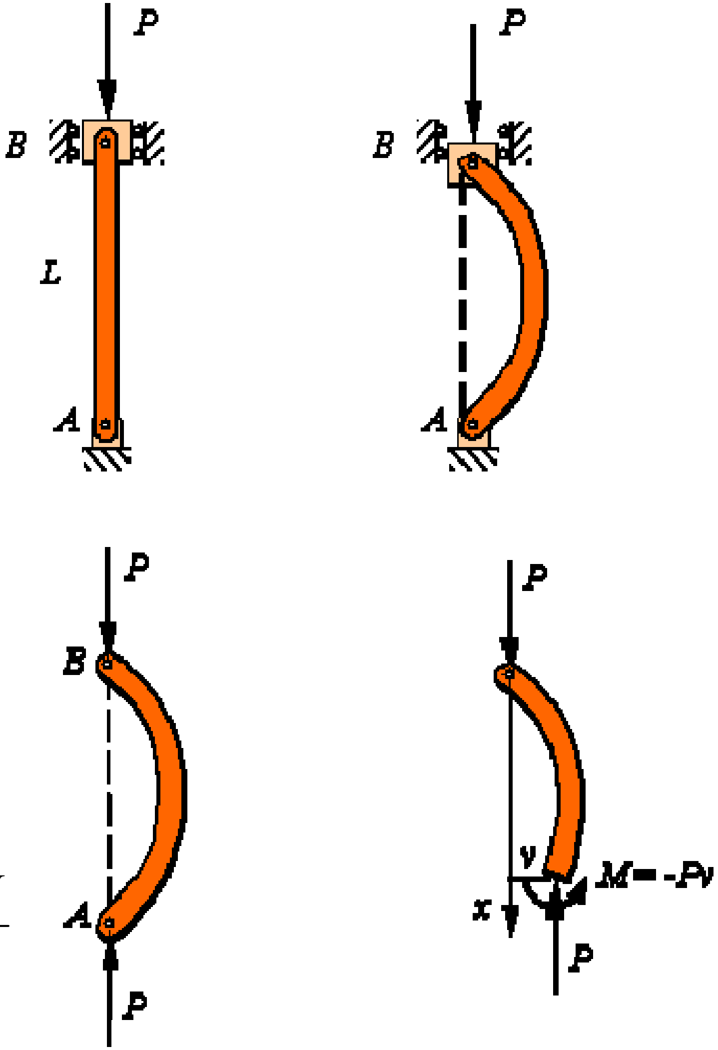
etc…

*Reference: http://en.wikipedia.org/wiki/Structural_failure

Mechanical failure modes

- Buckling

  Buckling is a failure mode characterized by a sudden failure of a structural member subjected to high compressive stresses, where the actual compressive stresses at failure are smaller than the ultimate compressive stresses that the material is capable of withstanding.



$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Mechanical failure modes

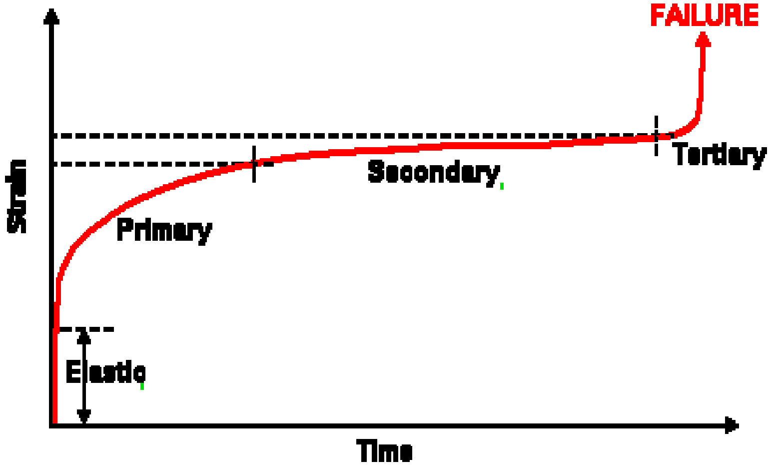
- Corrosion

Corrosion is breaking down of essential properties in a material due to reactions with its surroundings. In the most common use of the word, this means a loss of an electron of metals reacting with water and oxygen.

Mechanical failure modes

- Creep

  Creep is the term used to describe the tendency of a material to move or to deform permanently to relieve stresses.

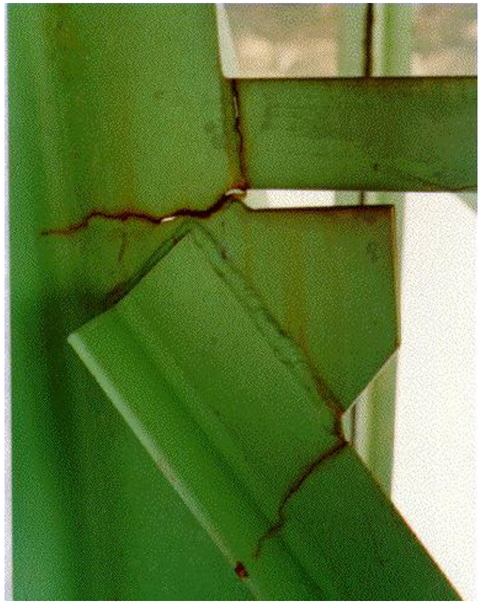


Initially, as the load is applied the elastic strain occurs (virtually instantaneously). As time passes under constant stress, the rate of strain reduces. This period of decelerating strain-rate is called primary creep. The primary creep phase is followed by an extended period of slow (almost steady-state) deformation called secondary creep. At the end of this stage, the strain-rate begins to accelerate and the material rapidly fails. The final stage of accelerating deformation is called tertiary creep.

Mechanical failure modes

- Fatigue

  In materials science, fatigue is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. The maximum stress values are less than the ultimate limit.



Mechanical failure modes

- Fatigue

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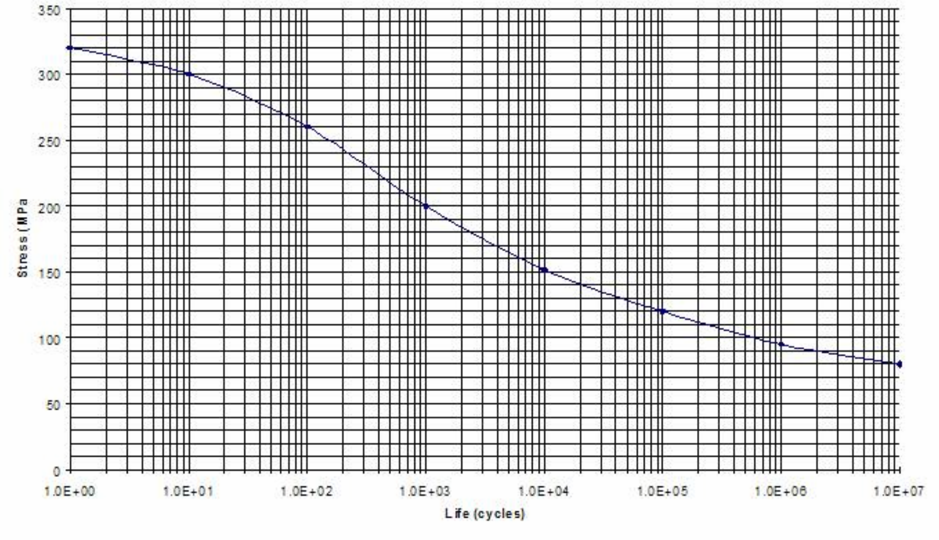
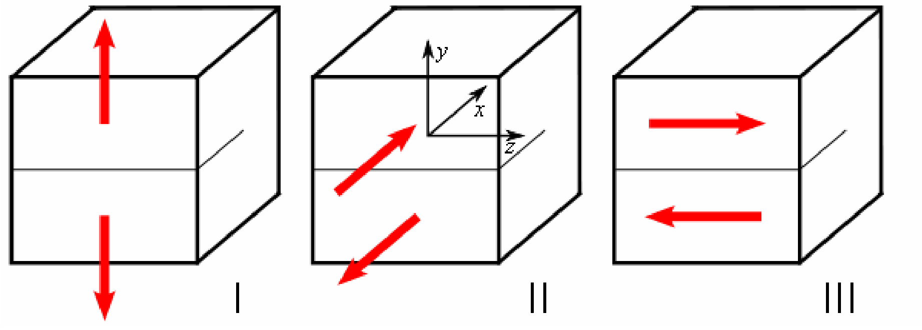


In high-cycle fatigue situations, materials performance is commonly characterized by an S-N curve. This is a graph of the magnitude of a cyclical stress (S) against the logarithmic scale of cycles to failure (N).

Mechanical failure modes

- Fracture

A fracture is the (local) separation of a body into two, or more, pieces under the action of stress.



Mode I crack - Opening mode (a tensile stress normal to the plane of the crack)

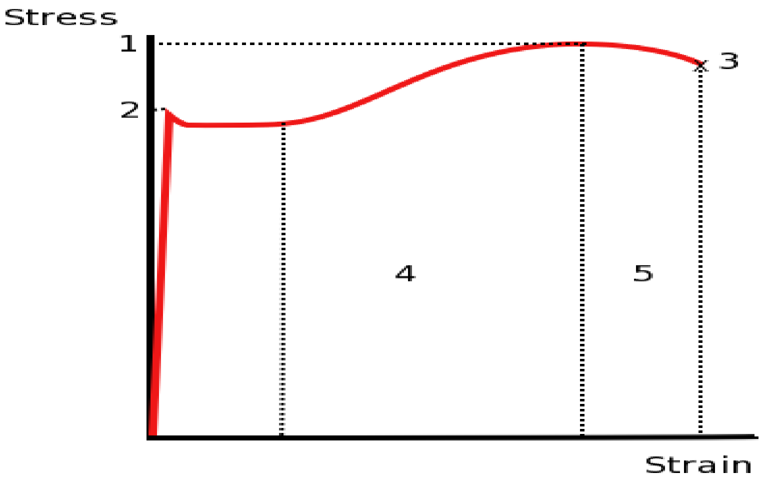
Mode II crack - Sliding mode (a shear stress acting parallel to the plane of the crack and perpendicular to the crack front)

Mode III crack - Tearing mode (a shear stress acting parallel to the plane of the crack and parallel to the crack front)

Mechanical failure modes

- Yielding

  Yielding means that the stress at which the material begins to plastically deform



Typical Stress vs Strain Curve for *low-carbon steel*.

1: Ultimate strength: the maximum stress a material can withstand.

2: Yield strength: the stress at which material strain changes from elastic deformation to plastic deformation, causing it to deform permanently.

3: Rupture (breaking strength)

4: Strain hardening: the strengthening of a material due to plastic deformation.

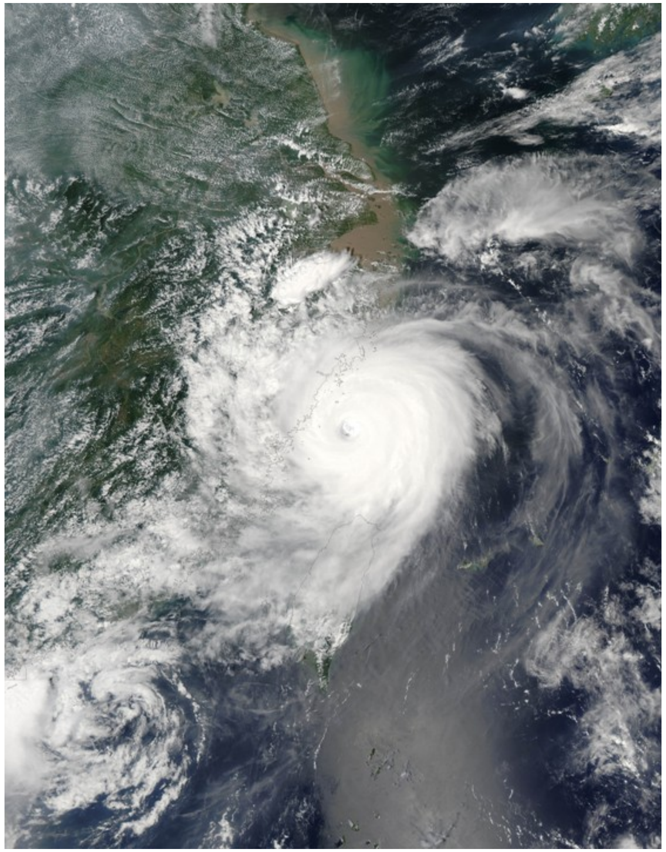
5: Necking: a mode of ductile flow of a material in tension. This is visible when applied stress passes a material’s ultimate strength. The material's cross-sectional area decreases, becoming thinner, and increases in length before it fails completely.

Natural hazards and human factors

- Earthquake

Natural hazards and human factors

- Typhoon



Typhoon Saomai, 2006

Natural hazards and human factors

- Fire

Natural hazards and human factors

- Explosion

Natural hazards and human factors

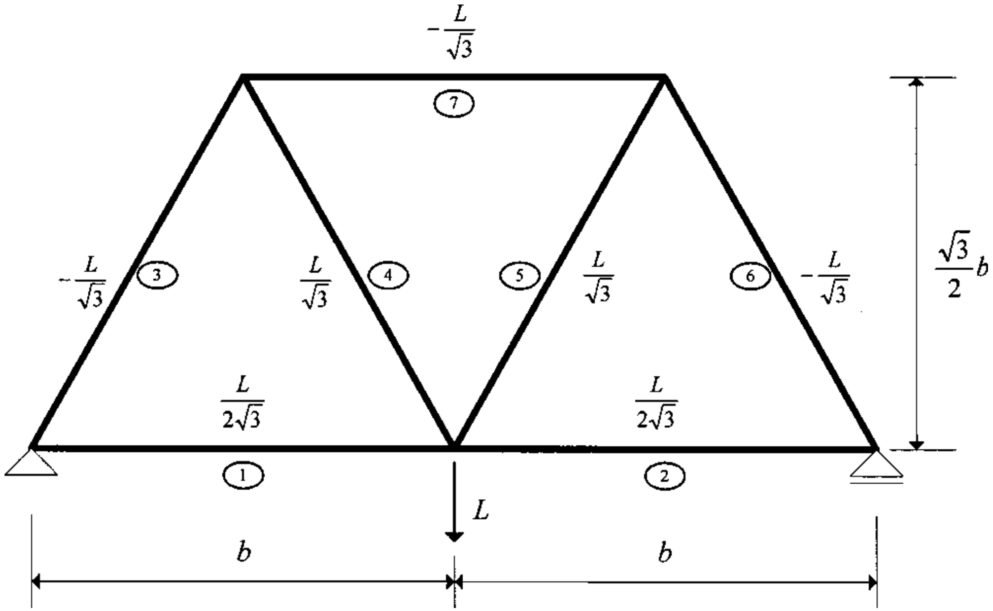
- Airplane impacts:

Jet colliding with World Trade Center simulated by Purdue University:

http://www.youtube.com/watch?v=gH02Eh44yUg

Case 1

Ditlevsen, O. (1979). "Narrow Reliability Bounds for Structural Systems." Journal of Structural Mechanics 7(4): 453-472.
Song, J. and Der Kiureghian, A. (2003). "Bounds on System Reliability by Linear Programming." Journal of Engineering Mechanics 129(6): 627-636.



X_i: the tensile/compressive strength of the i^{th} member

$$L = 100$$

$$X_i \sim N(100, 20)\ i = 1, 2$$

$$X_i \sim N(200, 40)\ i = 3 \sim 7$$

The failure events of the individual members are

$$E_i = \left\{ X_i \leq \frac{L}{2\sqrt{3}} \right\} \ (i = 1, 2)$$

$$E_i = \left\{ X_i \leq \frac{L}{\sqrt{3}} \right\} \ (i = 3 \sim 7)$$

The members have equal probabilities of failure given by

$$P_i = P(E_i) = \Phi\left(\frac{\frac{100}{\sqrt{3}} - 200}{40} \right) = 1.88 \times 10^{-4}$$

If all the information is available, the failure probability could be directly obtained.

$$P\left(\bigcup_{i=1}^{7} E_i\right)=\sum_{i=1}^{7} P\left(E_i\right)-\sum_{i=1}^{6}\sum_{j=i+1}^{7} P\left(E_i \cap E_j\right)+\sum_{i=1}^{5}\sum_{j=i+1}^{6}\sum_{k=j+1}^{7} P\left(E_i \cap E_j \cap E_k\right)$$

$$-\sum_{i=1}^{4}\sum_{j=i+1}^{5}\sum_{k=j+1}^{6}\sum_{l=k+1}^{7} P\left(E_i \cap E_j \cap E_k \cap E_l\right)+\sum_{i=1}^{3}\sum_{j=i+1}^{4}\sum_{k=j+1}^{5}\sum_{l=k+1}^{6}\sum_{m=l+1}^{7} P\left(E_i \cap E_j \cap E_k \cap E_l \cap E_m\right)$$

$$-\sum_{i=1}^{2}\sum_{j=i+1}^{3}\sum_{k=j+1}^{4}\sum_{l=k+1}^{5}\sum_{m=l+1}^{6}\sum_{n=m+1}^{7} P\left(E_i \cap E_j \cap E_k \cap E_l \cap E_m \cap E_n\right)+P\left(\bigcap_{i=1}^{7} E_i\right)$$

The LP model for bounds of failure probability is

$$\min/\max P\left(\bigcup_{i=1}^{7} E_i\right)$$

The LP formulation involves $2^7=128$ design variables, 7 equality constraints for the uni-component probabilities, 21 for bi-component probabilities, and 35 for tri-component probabilities……

Boole

$$\max_i P_i \le P\left(\bigcup_{i=1}^{n} E_i\right) \le \min\left(1, \sum_{i=1}^{n} P_i\right)$$

KHD

$$P_1 + \sum_{i=2}^{n} \max\left(0, P_i - \sum_{j=1}^{i-1} P_{ij}\right) \le P\left(\bigcup_{i=1}^{n} E_i\right) \le P_1 + \sum_{i=2}^{n}\left(P_i - \max_{j<i} P_{ij}\right)$$

Zhang

$$P_1 + P_2 - P_{12} + \sum_{i=3}^{n} \max\left(0, P_i - \sum_{j=1}^{i-1} P_{ij} + \max_{k\in\{1,2,\ldots,i-1\}} \sum_{\substack{j=1\\ j\ne k}}^{i-1} P_{ijk}\right) \le P\left(\bigcup_{i=1}^{n} E_i\right) \le P_1 + P_2 - P_{12} + \sum_{i=3}^{n}\left[P_i - \max_{\substack{k\in\{2,3,\ldots,i-1\}\\ j<k}}\left(P_{ik} + P_{ij} - P_{ijk}\right)\right]$$

Bounds ($x10^{-3}$)		Lower	Upper
Unicomponent	Boole	0.188	1.32
	LP	0.188	1.32
Bicomponent	KHD	0.344	0.961
	LP	0.477	0.912
Tricomponent	Zhang	0.605	0.809
	LP	0.631	0.796

Case 2

Straub, D., Malioka, V. and Faber, M.H. (2007). A framework for the asset integrity management of large deteriorating concrete structures. Structure and Infrastructure Engineering, 2007, ISSN 1573-2479 print / ISSN 1744-8980 online, Taylor & Francis, pp. 1-15.

A model framework for the representation of temporal and spatial variability of deterioration, illustrated by consideration of chloride-induced corrosion of the reinforcement in concrete structures is introduced here.

Deterioration

- **Temporal variability**
- **Spatial variability**

- Decisions in the asset integrity management of large, deteriorating structures

Responsibility of engineers

1. When
2. Where
3. How
4. What

$$\min_{\mathbf{e},d} \mathrm{E}_{\mathbf{Z},\Theta}\left[C_T\left(\mathbf{e},d,T_{SL}\right)\right]$$

$$\text{subject to} \quad \Delta p_F\left(\mathbf{e},d,t\right) \leq \Delta p_F^{\,max}, \qquad t = 0,\ldots,T_{SL}$$

E: the expectation operation

Θ: the (uncertain) condition of the structure

e: the inspection and monitoring actions

Z: the inspection outcomes

d: the decision rule that specifies the repair actions as a function of the inspection outcomes

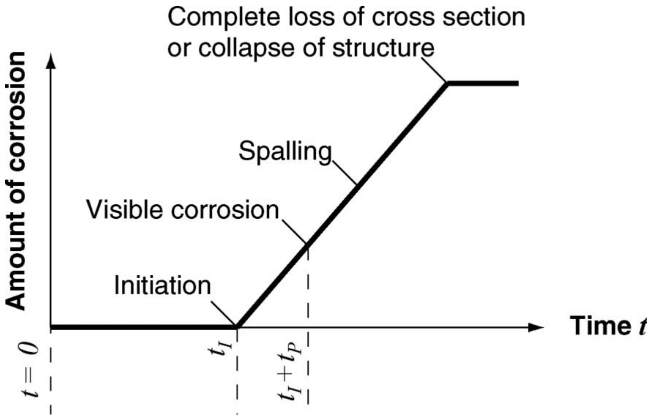
C_T: the total expected life-cycle cost

T_{SL}: the anticipated service life time

Δp_F: the annual probability of failure of the structure or the element

Δp_{Fmax}: the corresponding acceptance criterion

- Temporal modeling



$$g_{CI}(t,\mathbf{X}) = X_I \cdot T_I\left(\mathbf{X}_0\right) - t$$

$$T_I\left(\mathbf{X}_0\right) = \frac{d^2}{4D}\left(erf^{-1}\left(1 - \frac{C_{CR}}{C_S}\right)\right)^{-2}$$

$$g_{CV}(t,\mathbf{X}) = X_I \cdot T_I\left(\mathbf{X}_0\right) + T_V - t$$

X_I: the model uncertainty associated with T_I

T_I: the time till corrosion initiation

d: depth of reinforcement

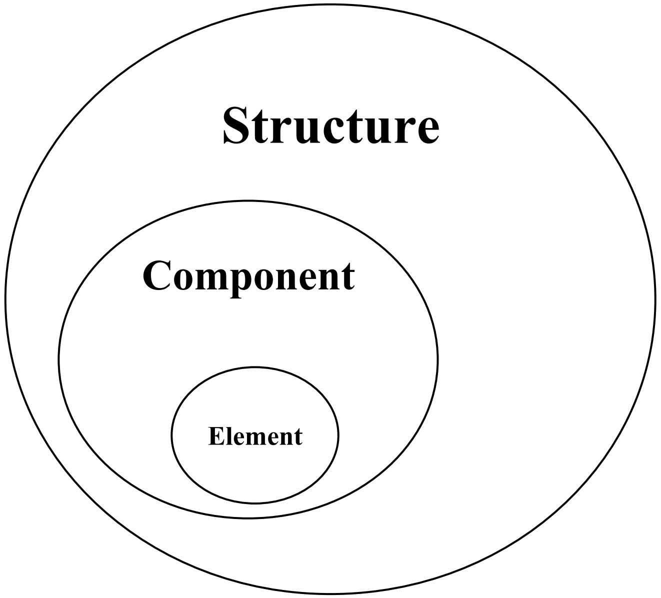
D: diffusion coefficient

C_{CR}: certain critical concentration

C_S: the concentration of chlorides on the surface of the concrete

T_V: the time from corrosion initiation to visible corrosion

- Spatial modeling



The deterioration performance of these elements is, in general, *interdependent*, due to *common influencing parameters*, such as common materials, production processes and environmental influences.

The spatial variability is modeled by identifying zones of the structure with common properties. It is assumed that there is no interdependency between different zones.

- Spatial modeling

- Spatial modeling

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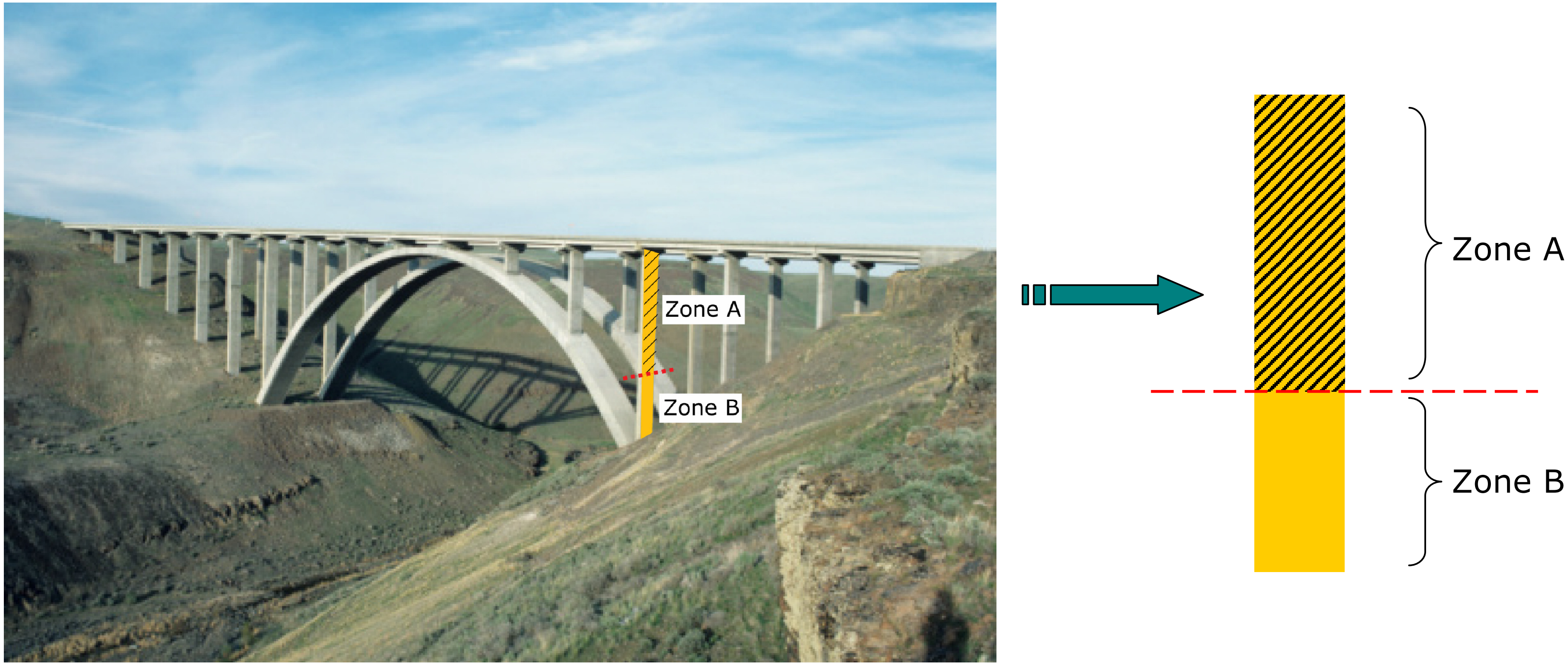
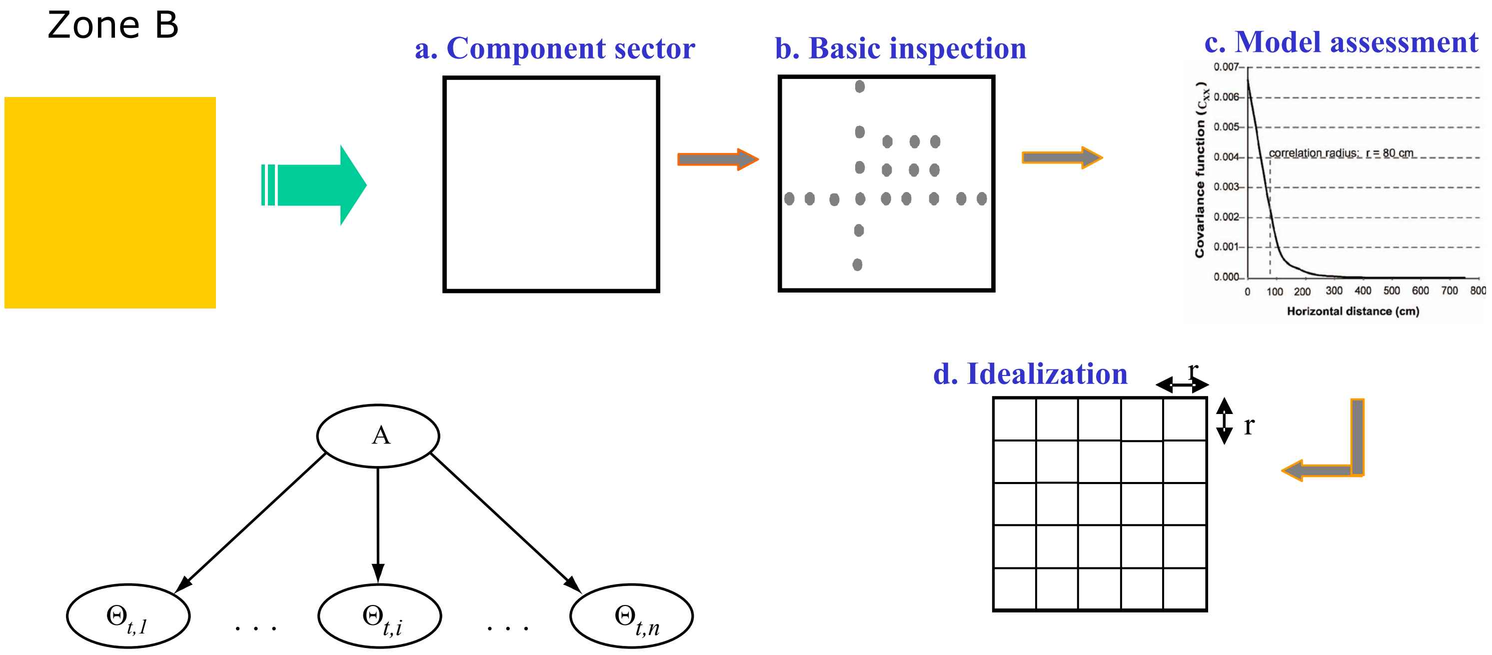


- Spatial modeling



Zone B

a. Component sector

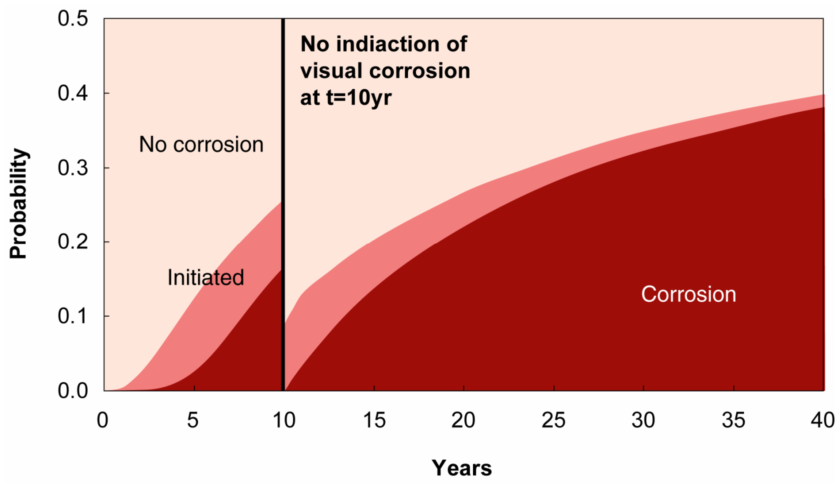
b. Basic inspection

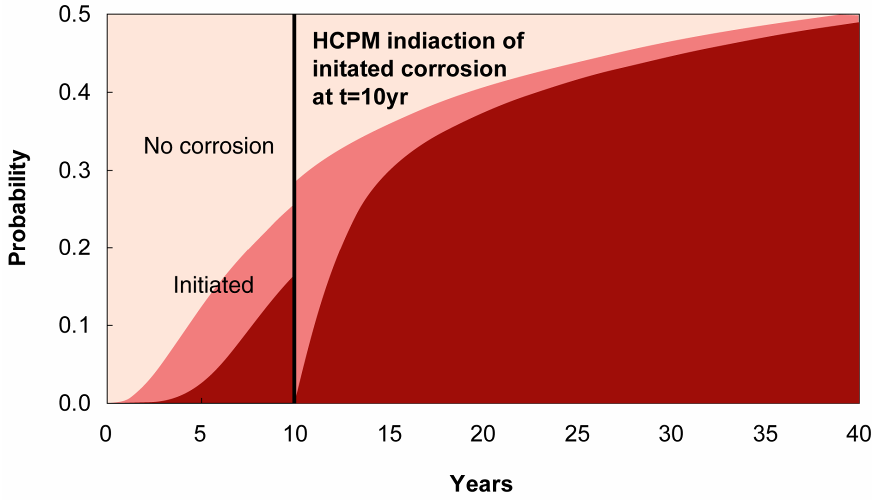
c. Model assessment

d. Idealization

- Effect of inspection for one zone

$$p_{\Theta_{t,i}}\left(\theta_i = \theta^* \middle| z_i, \boldsymbol{\alpha}\right) = \frac{P\left(g_{\theta^*}\left(t, \mathbf{X}\right) \le 0 \cap g_{Z_i}\left(t_{insp}, \mathbf{X}\right) \le 0 \middle| \boldsymbol{\alpha}\right)}{P\left(g_{Z_i}\left(t_{insp}, \mathbf{X}\right) \le 0 \middle| \boldsymbol{\alpha}\right)}$$

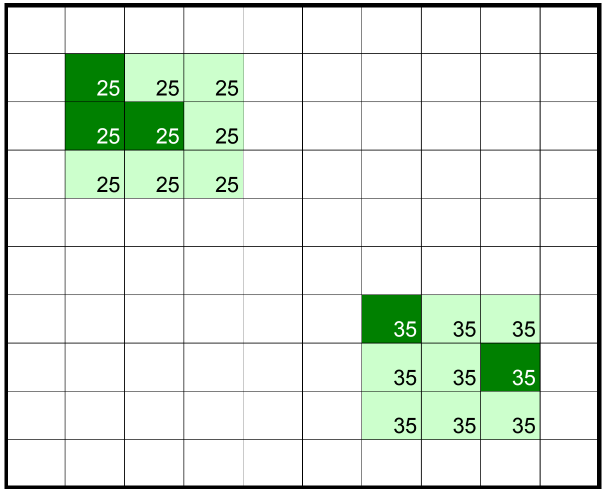




Probabilities of visible corrosion of the entire zone for HCPM at years 25 and 35

🟩 *Resulting in no-indication*
🟩 *Resulting in indication*

Time of HCPM



	25	25	25						
	25	25	25						
	25	25	25						
						35	35	35	
						35	35	35	
						35	35	35	

Probability of visible corrosion at t = 50yr

0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
0.35	0.48	0.27	0.27	0.35	0.35	0.35	0.35	0.35	0.35
0.35	0.48	0.48	0.27	0.35	0.35	0.35	0.35	0.35	0.35
0.35	0.27	0.27	0.27	0.35	0.35	0.35	0.35	0.35	0.35
0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
0.35	0.35	0.35	0.35	0.35	0.35	0.57	0.17	0.17	0.35
0.35	0.35	0.35	0.35	0.35	0.35	0.17	0.17	0.57	0.35
0.35	0.35	0.35	0.35	0.35	0.35	0.17	0.17	0.17	0.35
0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35

Thank you for your attention.