Assessment of System Reliability and Robustness for Structural Systems

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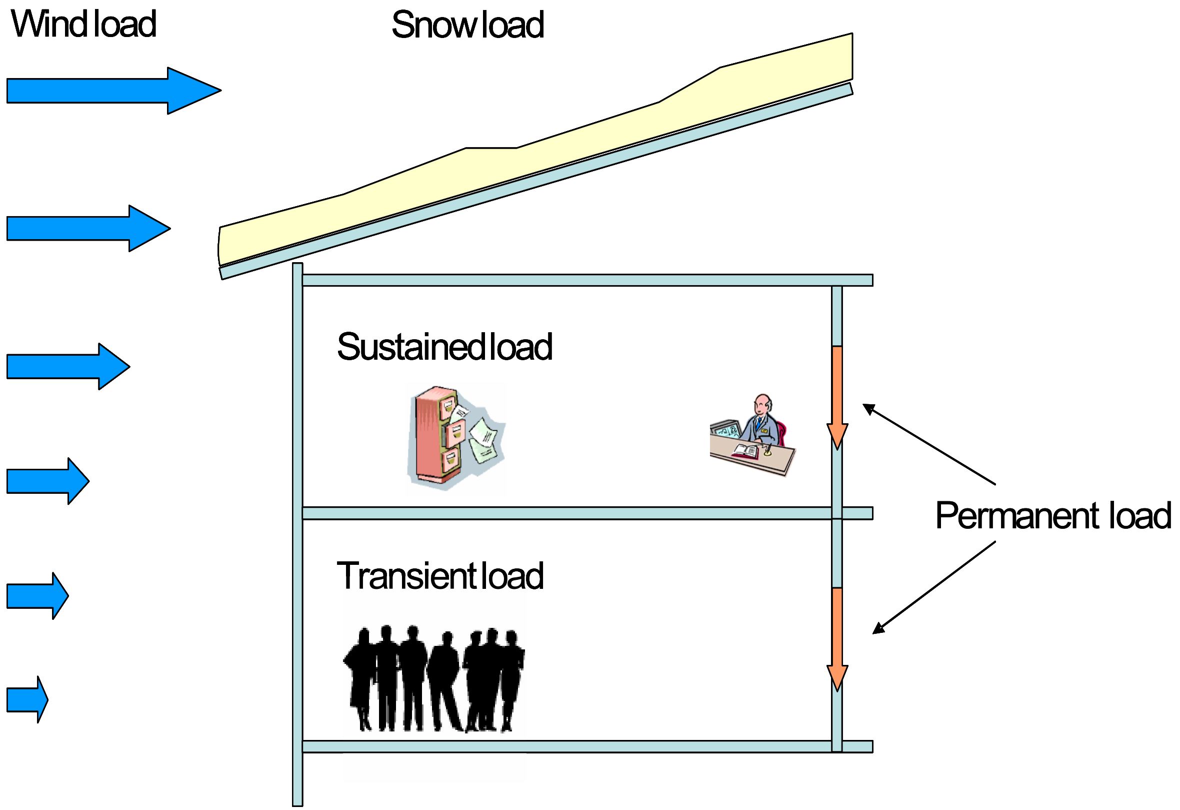
Introduction

Structural engineering design involves the consideration and appropriate quantification of **loads** and **resistances**.

Loads refer to forces acting on structural components/systems or influences on the structure from the ambient environment.

Resistances refer to characteristic properties or abilities of structural components/systems to withstand loads.

Loads


Wind load
Snow load
Sustained load
Transient load
Permanent load

Loads

Loads on structures are uncertain due to:

- Random variations in space (spatial) and time (temporal)
- Model uncertainties
- Statistical uncertainties

The probabilistic modelling of loads includes the following steps:

- specifying the definition of the random variables used to represent the uncertainties in the loading

- selecting a suitable distribution type to represent the random variable

- assigning the distribution parameters of the selected distribution.

Resistances

Resistances of structural components/systems are typically functions of material strength, section geometry and dimensions.

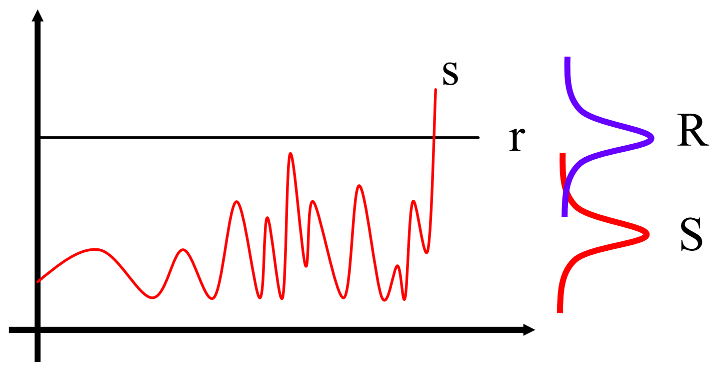
They are commonly expressed in the form of compressive strength, yield strength, moment capacity, shear capacity, etc.

Resistances are associated with the following uncertainties:
- Geometrical uncertainties
- Material characteristics
- Model uncertainties

The steps in the probabilistic modelling of resistances include:
- defining the random variables used to represent the uncertainties in the resistances
- selecting a suitable distribution type to represent the random variable
- assigning the distribution parameters of the selected distribution.

Reliability Analysis of Structural Components

Structural reliability analysis is concerned with the rational treatment of uncertainties in structural engineering design and the associated problems of optimal decision making.



Structural failures normally take place due to extreme loads exceeding the residual resistance

Therefore in structural reliability, models are established for resistances **R** and loads **S** individually and the structural reliability is assessed through establishing the **probability of failure** given by

$$P_f = P(R - S \leq 0)$$

Reliability Analysis of Structural Components

If only the resistance is uncertain, the failure probability can be assessed by

$$P_f = P(R \leq s) = F_R(s) = P(R/s \leq 1)$$

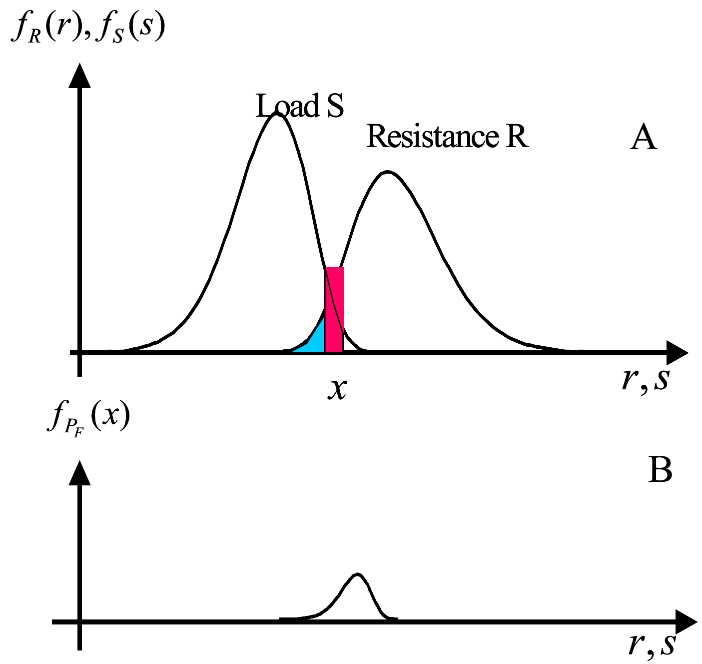
If also the load is uncertain, then

$$P_f = P(R \leq S) = P(R - S \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx$$

where it is assumed that the load and the resistance are independent.

This is called the

"**Fundamental Case**"



Reliability Analysis of Structural Components

The **safety margin** M is defined as

$$M = R - S$$

Then the failure probability is

$$P_F = P(R - S \le 0) = P(M \le 0)$$

In case R and S are Normal distributed, the safety margin M is also Normal distributed.

with a mean value of

$$\mu_M = \mu_R - \mu_S$$

and standard deviation of

$$\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2}$$

The **failure probability** is then

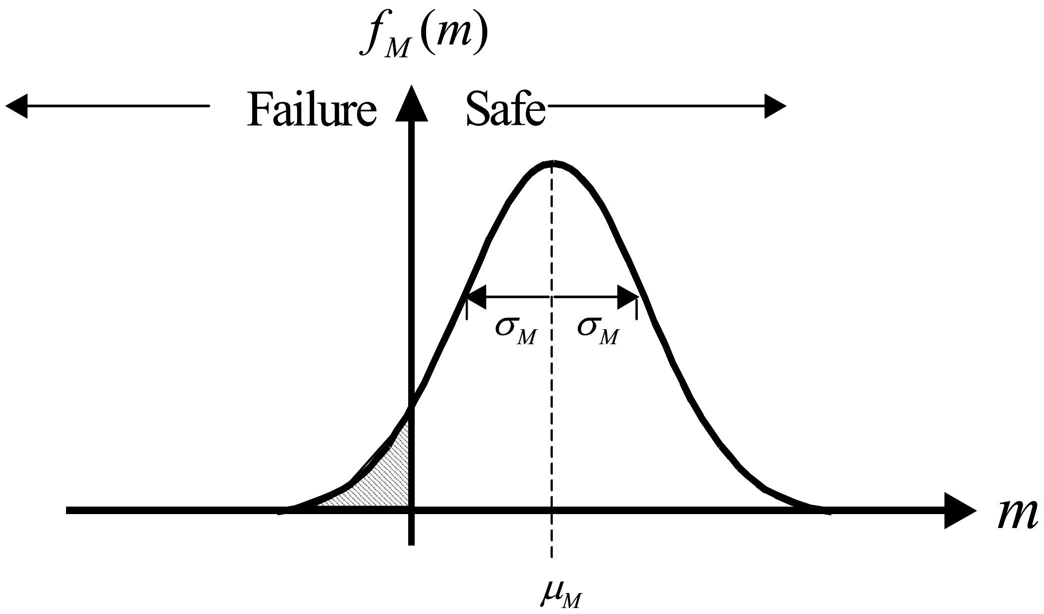
$$P_F = \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right) = \Phi(-\beta)$$

where the **reliability index** is

$$\beta = \mu_M / \sigma_M$$

Reliability Analysis of Structural Components

The Normal distributed safety margin M



Reliability Analysis of Structural Components

In the general case, the resistance and the load may be defined in terms of functions where ***X*** are the basic random variables

$$R = f_1(\mathbf{X})$$
$$S = f_2(\mathbf{X})$$

The safety margin is then

$$M = R - S = f_1(\mathbf{X}) - f_2(\mathbf{X}) = g(\mathbf{X})$$

Here g(x) is called the **limit state function**

Failure occurs when

$$g(\mathbf{x}) \leq 0$$

Reliability Analysis of Structural Components

Failure occurs when $g(\mathbf{x}) \le 0$

The **probability of failure** is then written as:

$$P_f = \int_{g(\mathbf{x}) \le 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function for the basic random variables ***X***

The probability of failure can be calculated by:

- Numerical integration (cumbersome for larger dimensions)
- FORM (First Order Reliability Method)
- SORM (Second Order Reliability Method)
- Simulation (Monte Carlo)

System reliability analysis

Systems normally consist of a large number of interconnected components.

The failure of a single component may or may not mean failure of the system.

It is hence important to distinguish between the reliability of an individual component and the reliability of the system

System reliability analysis provides the means to determine (or at least estimate or bound) the reliability or probability of failure of an entire system, building upon the reliability analysis of components

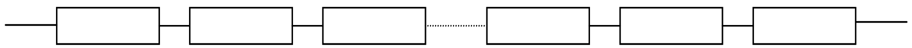
System reliability analysis

Block diagrams are normally used in the representation of systems in structural systems reliability analysis

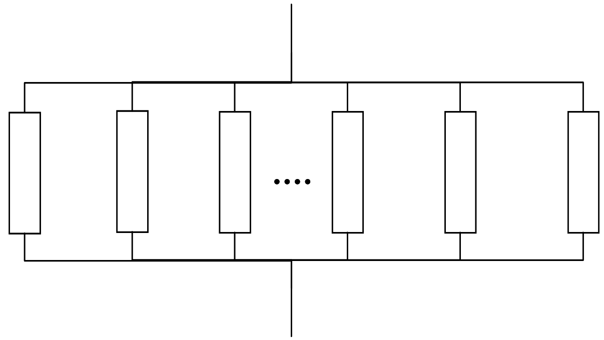
Each component in the block diagrams represent one failure mode for the structure

a) series system
b) parallel system
c) mixed system

a)



b)



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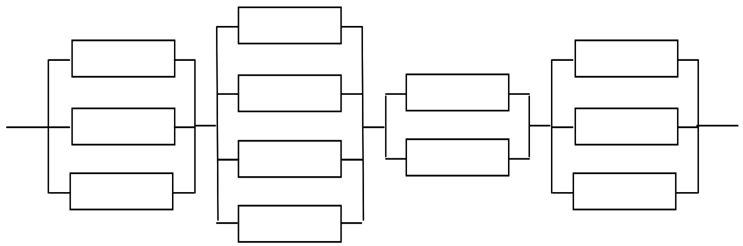


System reliability analysis

Uncorrelated components

The failure probability of a **series** system may be determined by

$$P_F = 1 - P_S = 1 - \prod_{i=1}^{n} (1 - P(F_i))$$

The failure probability of a **parallel** system may be determined by

$$P_F = \prod_{i=1}^{n} P(F_i)$$

System reliability analysis

Uncorrelated components

If the individual components of the systems have **linear** and **Normal** distributed safety margins

The failure probability of a **series** system may be determined by

$$P_F = 1 - P_S = 1 - \Phi_n(\boldsymbol{\beta}, \boldsymbol{\rho})$$

The failure probability of a **parallel** system may be determined by

$$P_F = \Phi_n(-\boldsymbol{\beta}, \boldsymbol{\rho})$$

System reliability analysis

Simple bounds on systems reliability

The failure probability of a **series** system may be bounded by

$$\max_{i=1}^{n}\{P(F_i)\} \leq P_F \leq 1-\prod_{i=1}^{n}(1-P(F_i))$$

Full correlation Uncorrelated

The failure probability of a **parallel** system may be bounded by

$$\prod_{i=1}^{n} P(F_i) \leq P_F \leq \max_{i=1}^{n}\{P(F_i)\}$$

Uncorrelated Full correlation

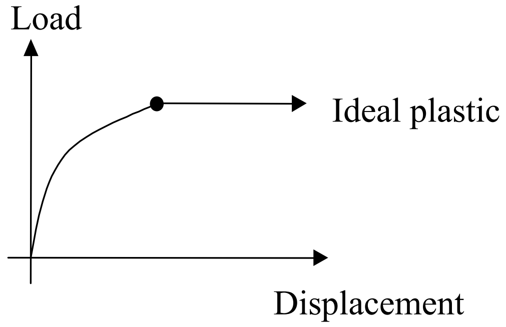
System reliability analysis

Mechanical modelling of structural systems

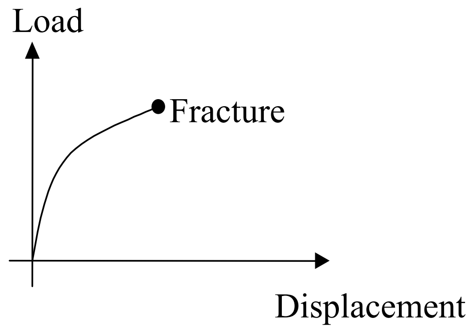
The behaviour of structural failure modes after failure is important for the assessing the reliability and safety of the system

Two extreme cases are

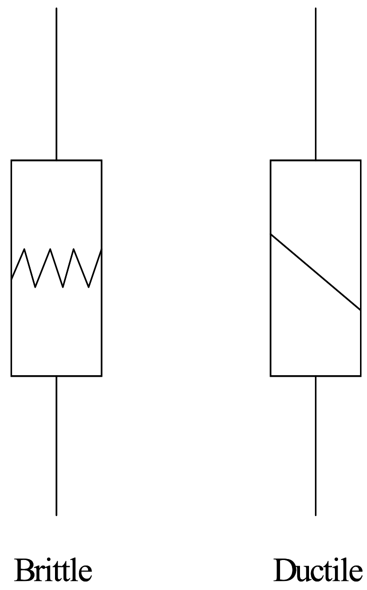
- ductile components
- brittle components



Ductile behaviour



Brittle behaviour



Representation

System reliability analysis

Series systems

Distinction between brittle and ductile failures is irrelevant.

Failure of the system occurs when the weakest element fails.

System reliability analysis

Parallel systems with ductile components

Assume a parallel system with n ductile components.

Such a system fails when all of its components fail

The strength of this system is given by

$$R_S = \sum_{i=1}^{n} R_i$$

The mean value and variance of the strength are then given by

$$\mu_{R_S} = \sum_{i=1}^{n} \mu_{R_i} \qquad \sigma_{R_S}^2 = \sum_{i=1}^{n} \sigma_{R_i}^2$$

System reliability analysis

Parallel systems with ductile components

From the central limit theorem, the strength is Normal distributed, independent of the distribution of individual component strengths.

If $\mu_{R_1} = \mu_{R_2} = \ldots = \mu_{R_n} = \mu$ and

$\sigma_{R_1} = \sigma_{R_2} = \ldots = \sigma_{R_n} = \sigma$

then the coefficient of variation (CoV) is:

$$CoV = \frac{\sigma}{\sqrt{n} \cdot \mu}$$

The uncertainty of the strength of parallel systems with ductile components approaches zero for large *n*

System reliability analysis

Parallel systems with brittle components

Assume a parallel system with n brittle components.

When one component fails, it loses its capacity to carry load and this leads to load redistribution among remaining elements.

If, after the load is redistributed, the system does not fail, the load can be increased until the next element fails.

Repeating this process of component failure and load redistribution, the strength of the system can be obtained as

$$R_S = \max [\, nR_1 , (n-1) R_2 , (n-2) R_3 , \ldots\ldots\ldots\ldots , 2R_{n-1} , R_n \,]$$

System reliability analysis

Methods of structural systems reliability analysis

Different methods have been developed for reliability analysis of structural systems. Two of these are:

- β-unzipping method

- fundamental mechanism method

System reliability analysis

β-unzipping method

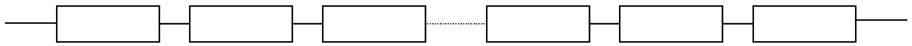
The failure of a structural system may be defined at different **levels** - the levels correspond to the number of failure modes assumed to be associated with the failure of the structural system.

A failure element is defined as an element or point where failure can take place.

Systems reliability at level 0 is an estimate of the system reliability on the basis of the failure of a single element (the element with the lowest reliability index).

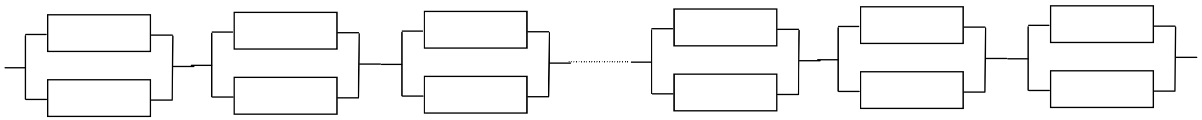
System reliability analysis

β-unzipping method



Systems modelling at level 1

Systems reliability at level 1 is an estimate of the system reliability by modelling a series system with all the failure elements.



Systems modelling at level 2

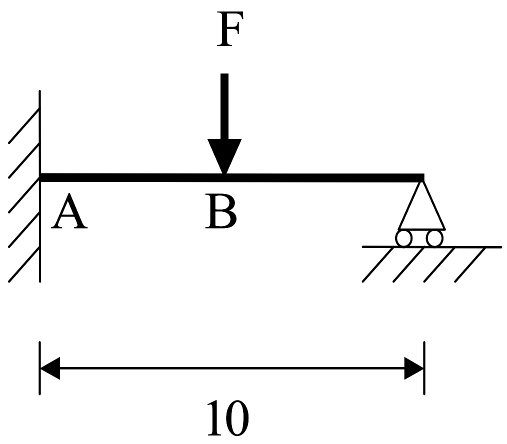
Systems reliability at level 2 is an estimate of the system reliability by modelling a series system where all elements are parallel systems with 2 failure elements.

Critical failure elements (within a defined interval starting from the element with the lowest reliability index) are selected for analysis at each level.

System reliability analysis

Example

The bending moment capacity *R* and the loading *F* on the beam structure are assumed to be Normal distributed



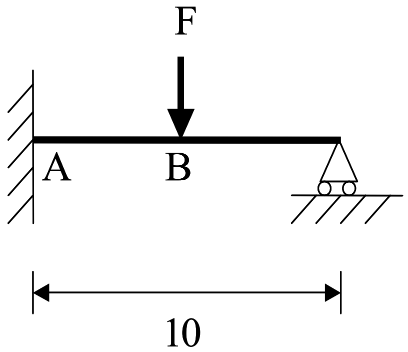
$$\mu_R = 300, \sigma_R = 30$$

$$\mu_F = 100, \ \sigma_F = 20$$

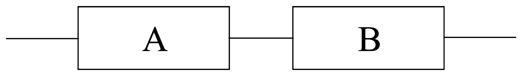
System reliability analysis

Example

It is assumed that that bending failures will occur at location A or location B.



For a **level 1** analysis, the system to be considered is a series system



The limit state functions for moment failure at locations A and B are easily established as

$$g_A(\mathbf{x}) = r + m_A = r - 1.875 \cdot f$$

$$g_B(\mathbf{x}) = r - m_B = r - 1.563 \cdot f$$

A FORM analysis yields

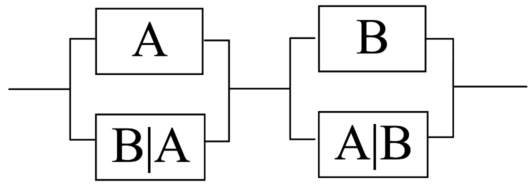
$$P_{f,A} = 9.58 \cdot 10^{-3} \qquad P_{f,B} = 4.56 \cdot 10^{-4}$$

The simple bounds are obtained as

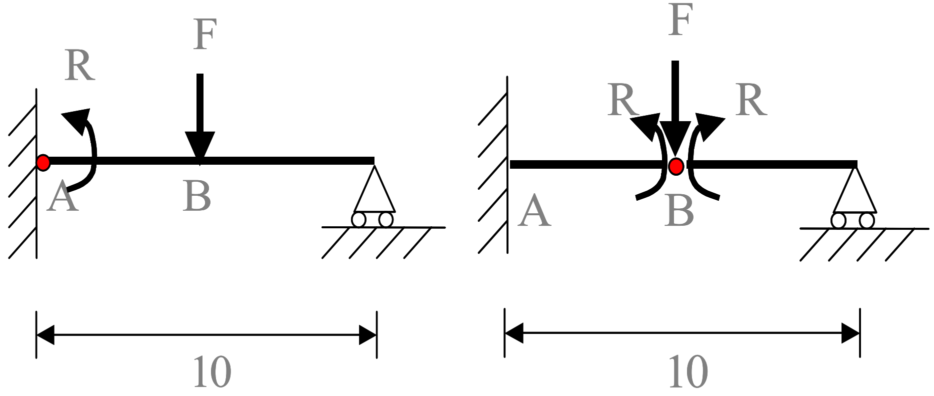
$$9.58 \cdot 10^{-3} \leq P_f \leq 1 \cdot 10^{-2}$$

System reliability analysis

Example



System failure is now defined by the event that two failure modes have failed. (equivalent to the formation of a plastic/collapse mechanism for the beam)



For a **level 2** analysis, the system to be considered is a mixed system

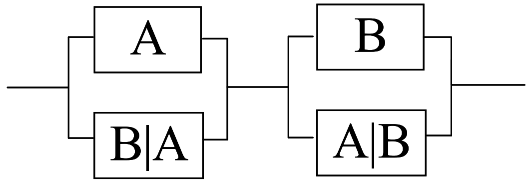
At the location of failures, fictitious forces are introduced corresponding to the moment capacity

The limit state equations are found as:

$$g_{B|A}(\mathbf{x}) = r - m_{B|A} + 0.5 \cdot r = r - 2.5 \cdot f + 0.5 \cdot r$$

$$g_{A|B}(\mathbf{x}) = r - m_{A|B} + 2 \cdot r = 3 \cdot r - 5 \cdot f$$

System reliability analysis



Example

A FORM analysis yields:

$$P_{f,B|A} = 1.47 \cdot 10^{-3} \qquad P_{f,A|B} = 1.47 \cdot 10^{-3}$$

The simple bounds for the parallel system can be calculated as:

$$1.41 \cdot 10^{-5} \le P(A \cap B|A) \le 9.58 \cdot 10^{-3}$$

$$6.71 \cdot 10^{-7} \le P(B \cap A|B) \le 1.47 \cdot 10^{-3}$$

and finally the simple bounds for the series system as:

$$1.48 \cdot 10^{-5} \le P_f \le 9.58 \cdot 10^{-3}$$

The lower bound on the system failure probability at level 1 is seen to be equal to the upper bound in the level 2 analysis.

Accepting a more developed failure in the beam before the beam is considered to be in a state of failure reduces the failure probability.

Robustness

Robustness means different things to different people.

The Oxford online dictionary defines "robust" as:

1 sturdy or resilient. **2** strong and healthy**.** **3** uncompromising and forceful; not subtle: *a robust defence.* **4** (of wine or food) strong and rich in flavour or smell.

The Cambridge online dictionary defines "robust" as:

(of a person or animal) strong and healthy, or **(of an object or system) strong and unlikely to break or fail**.

Different fields of science, engineering and technology provide essentially contextual definitions and understanding of the concept of robustness.

Broadly speaking, robusntess can refer to the manner in which certain performance objectives or system properties are affected by **extreme/unexpected/hazardous/ambiguous/abnormal** conditions.

Robustness of Structural Systems – Background

Robustness has now come to be recognized as a property of great significance in structural engineering and is generally accepted as a principle of good system design.

Robustness is commonly understood by the following statement as stipulated in the structural design codes EN1990 and EN1991-1-7:

> A structure shall be designed and executed in such a way that it will not be damaged by events such as :
>
> explosion,
> impact, and
> the consequences of human errors,
>
> to an extent **disproportionate** to the original cause.

Structural design codes are seen to be consistently ambiguous in their treatment of robustness, primarily owing to their greater focus on individual structural member and member failure modes.

Robustness of Structural Systems – Background

Events of malevolence and terrorism have served to emphasize the need for a clear and quantifiable understanding of robustness in order to minimise the resulting risks to society

Several attempts have been made to evolve suitable measures for robustness.

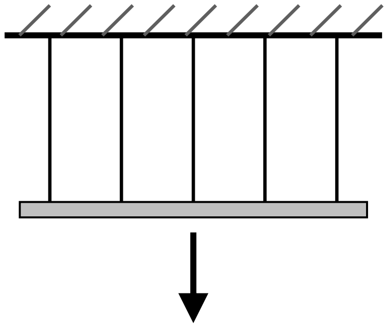
However there is presently no consensual agreement in the structural engineering community on an unequivocal interpretation of robustness that readily facilitates its quantification and easy use

Robustness of Structural Systems – Developing a Framework

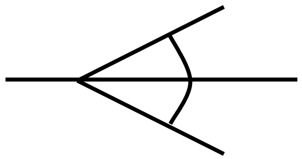
- An objective quantification of robustness for use in structural design is required.
- Some desirable properties for such a measure include:
  - General applicability to systems
  - Provision of a ranking system for alternative systems
  - Establishment of a criterion for identifying acceptable robustness
- The accepted definition of robustness in structural engineering requires the identification and consideration of all possible events, the resulting damages and the ensuing consequences.
- It is hence important to establish a consistent definition and representation of a system to determine the computational platform.
- In structural engineering, the Joint Committee on Structural Safety (JCSS) has developed guidelines for risk assessment of engineered systems.

  "A system can be considered as a spatial and temporal representation of all constituents required to describe the interrelations between all relevant exposures and their consequences."

A Risk-based Framework for Assessment of Robustness

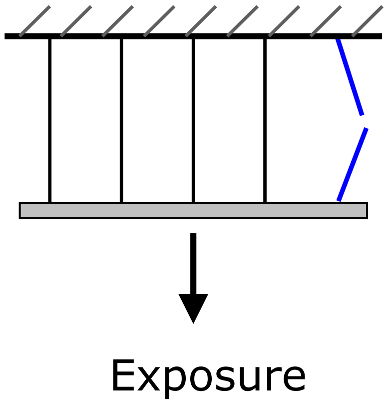


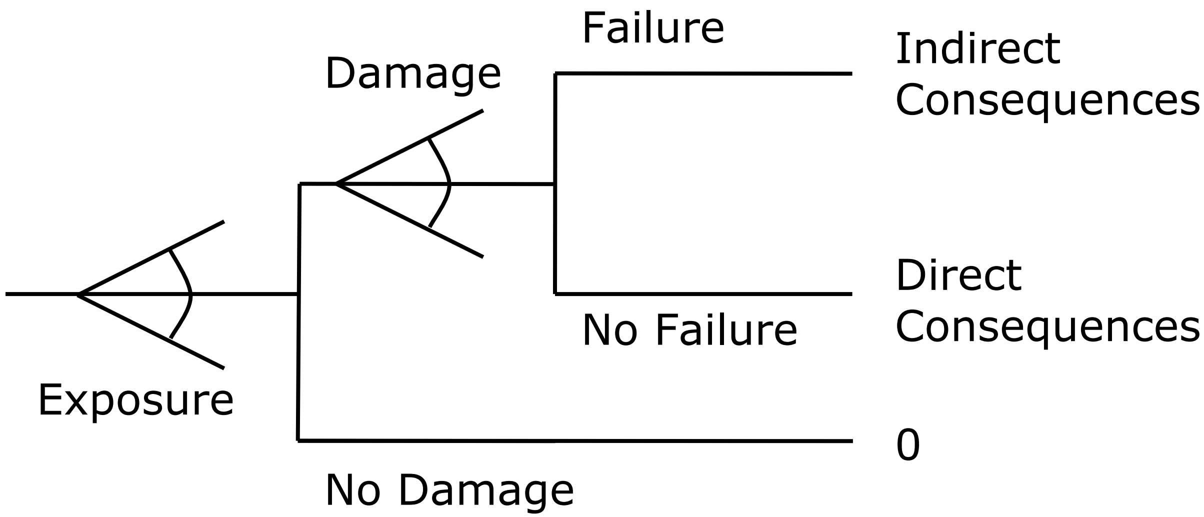
Exposure



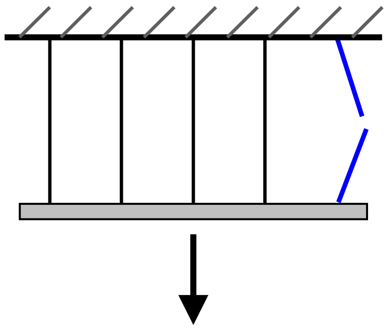
Exposure

A Risk-based Framework for Assessment of Robustness



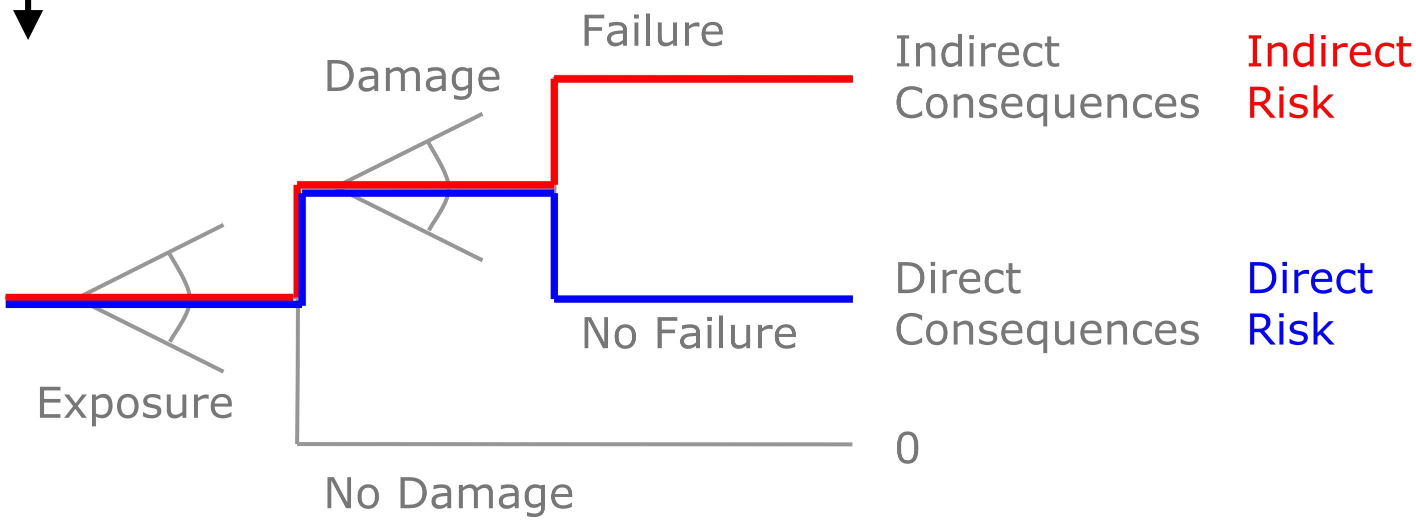


A Risk-based Framework for Assessment of Robustness



Calculation of Risk

Risk = Probability of Occurrence * Consequences



An index of robustness: $$I_{Rob} = \frac{\text{Direct Risk}}{\text{Direct Risk} + \text{Indirect Risk}}$$

A Risk-based Framework for Assessment of Robustness

Features of the robustness index

$$I_{Rob} = \frac{\text{Direct Risk}}{\text{Direct Risk} + \text{Indirect Risk}}$$

- Assumes values between zero and one
- Measures relative risk only
- Dependent upon the probability of damage occurrence
- Dependent upon consequences
- Can be used for assessing multiple exposure events

A Risk-based Framework for Assessment of Robustness

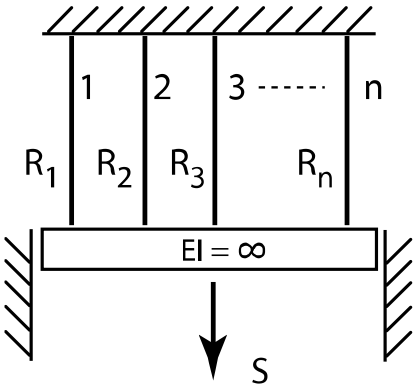
The integration of **system reliability methods** in the assessment procedure for robustness provides a means to overcome the inherent complexity in a structural system on account of the existence of multiple failure modes.

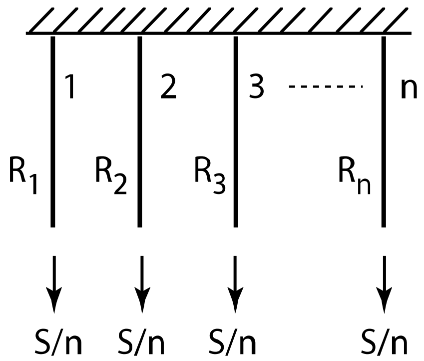
Basic approach is to consider finite number of failure modes as dominating or significant and combining them in complex reliability systems.

Identification of such significant failure methods is possible by use of suitable methods such as the β-unzipping method.

This provides a computational platform for evaluation of system probability of failure and robustness for complex structures with numerous failure modes

A Risk-based Framework for Assessment of Robustness





Example - Structural Systems

Parallel system with *n* elements

Perfect ductile / brittle

Load distribution after component failure

Element damage / system failure

Consequences of system failure set equal to 100 times the consequences of component failure

A Risk-based Framework for Assessment of Robustness

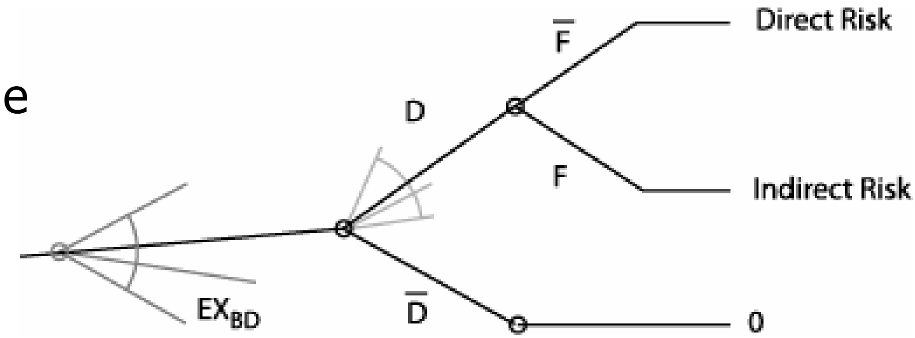
Modelling of System

Loads (Dead load and live load)
Weibull distribution with mean value of one and different CoV values for analysis

Resistances
Lognormal distribution with a CoV of 7%

Component failure
Each component has the same annual probability of failure of 0.001

Event/decision tree



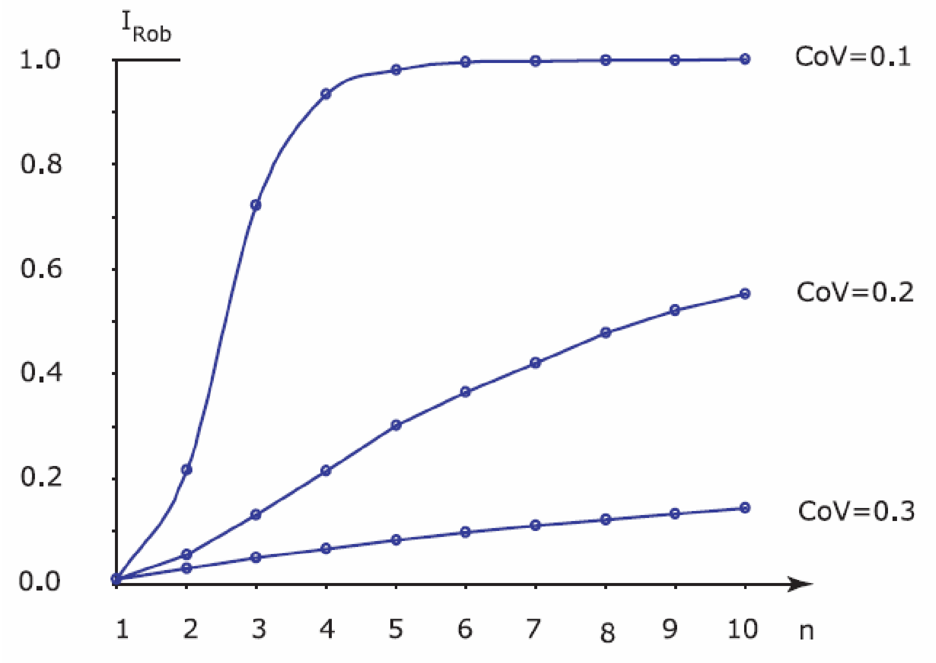
A Risk-based Framework for Assessment of Robustness

Number of components – ductile material

The greater the number of components, the more robust the system

A one component has almost zero robustness

The one element case can be considered to represent series systems



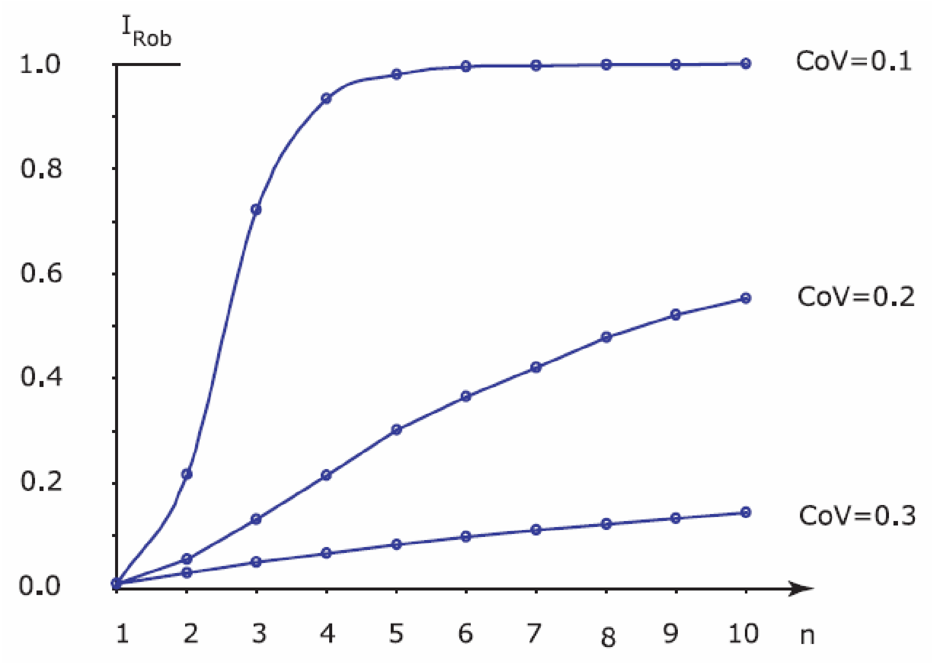
A Risk-based Framework for Assessment of Robustness

Load variability – ductile material

Higher CoV leads to less robustness

Higher Cov increases the probability that the system fails if one component is damaged

Here uncorrelated resistances are considered – correlation has the same effect as reducing the number of components



A Risk-based Framework for Assessment of Robustness

Load variability – brittle material

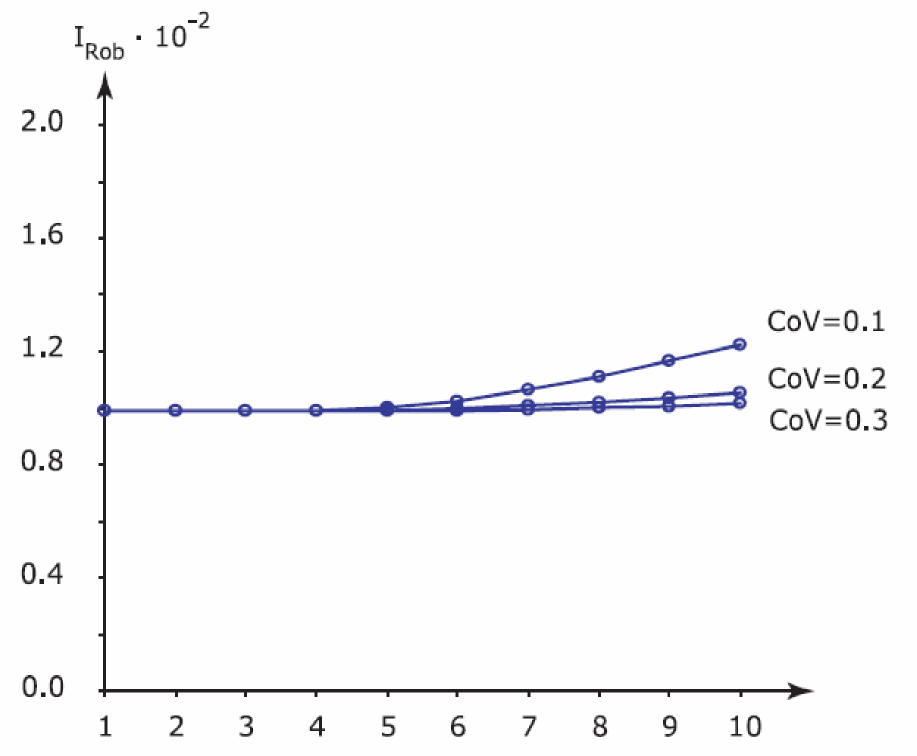
There is no residual carrying capacity.

Brittle failure is likely to trigger cascading system failure when load redistribution takes place.

The robustness values are close to zero

Indirect risks are dominating

In general, low values probably indicate that high system failure consequences outweigh their probabilities of occurrence.



A Risk-based Framework for Assessment of Robustness

Failure Consequences

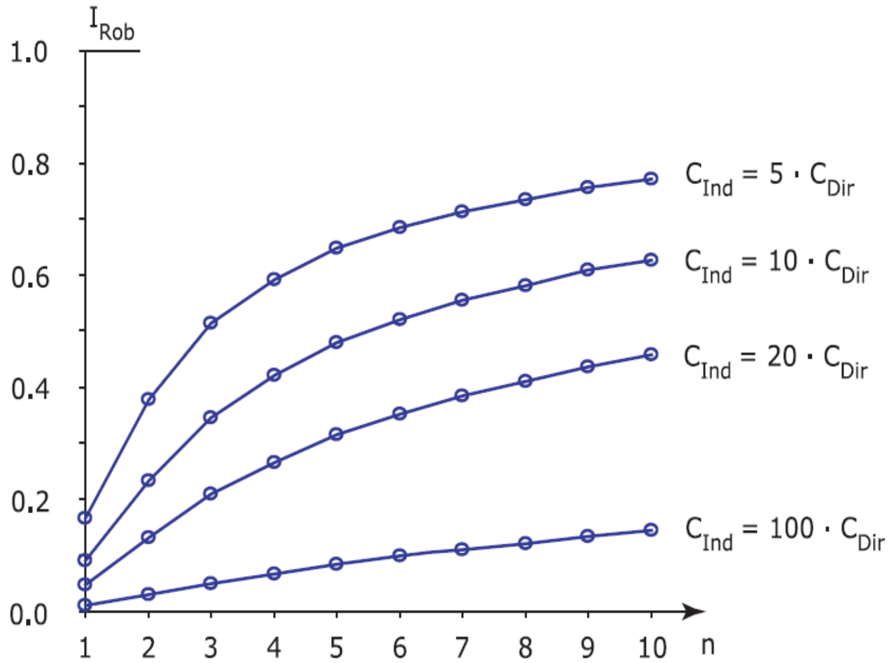
The higher the indirect consequences, the lower the robustness

Increasing robustness through:

- effective egress routes
- decisions in rescue action
- effective warning systems

For fixed indirect consequences, robustness increases if the direct consequences increase

Hence robustness cannot be considered in isolation



A Risk-based Framework for Assessment of Robustness

Load redistribution

An important consideration is how will loads be carried by the structure in the event of damage.

Decision between "tying together" or compartmentalize damage.

Depending on exposure intensity, load redistribution might increase system failure probability

For no load redistribution, robustness is constant, however indirect consequences can occur in the event of local failure

An assessment of robustness can help to identify a suitable strategy