

# Bayesian Probabilistic Networks

hierarchical modelling

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# outline

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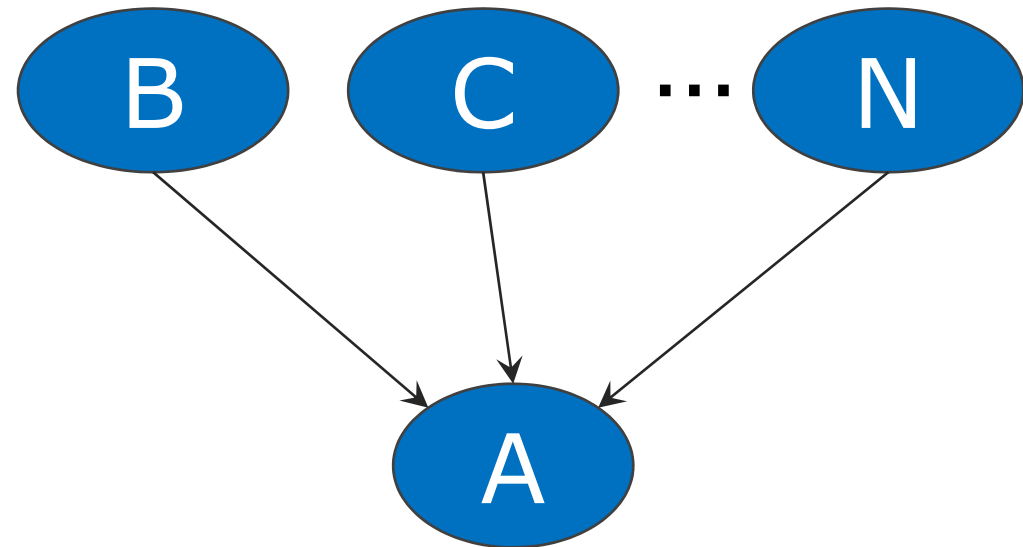
- Bayesian Probability Network (BPN) – basic theories
- Hierarchical BPN
- examples for Hierarchical BPNs in engineering

# Bayesian Probabilistic Networks

# Bayesian Probabilistic Networks (BPN)

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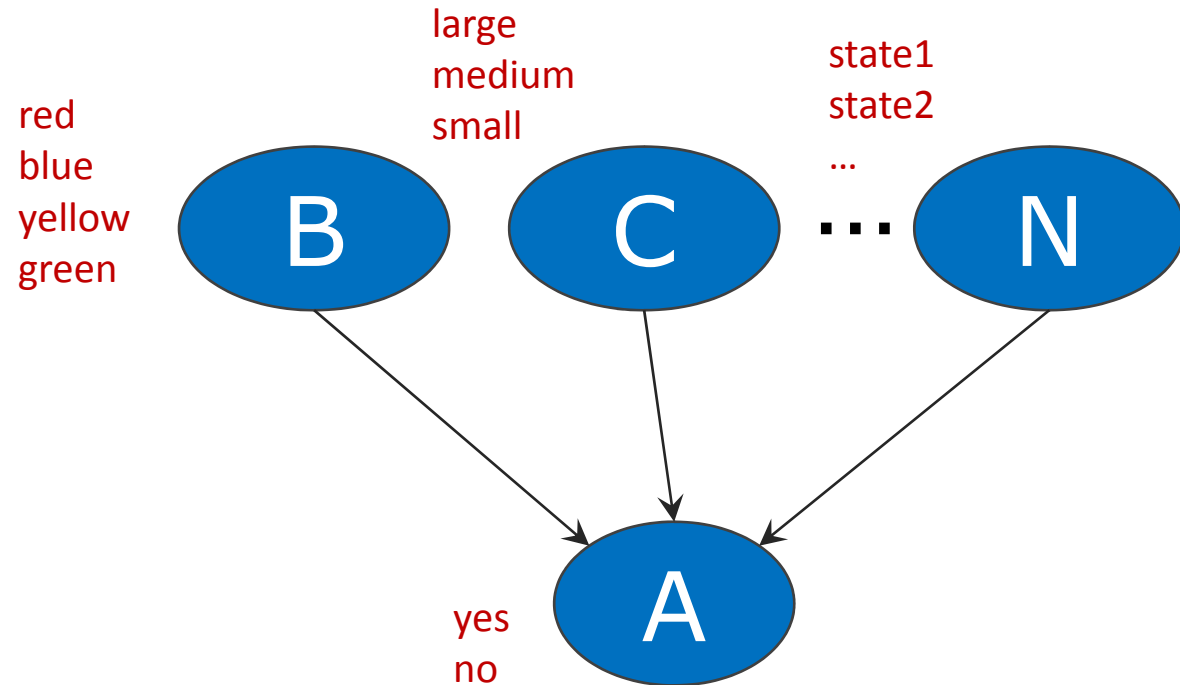
consists of:



1. A set of *variables* so-called *nodes* and a set of *directed arrows* between these variables.

# Bayesian Probabilistic Networks (BPN)

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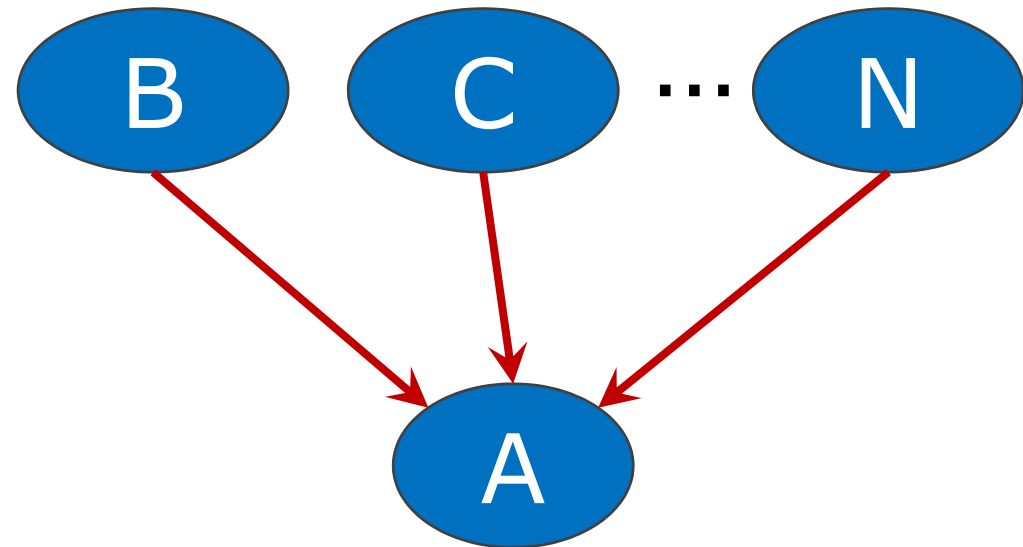


2. Each discrete node has a finite set of mutually exclusive *states*.

# Bayesian Probabilistic Networks (BPN)

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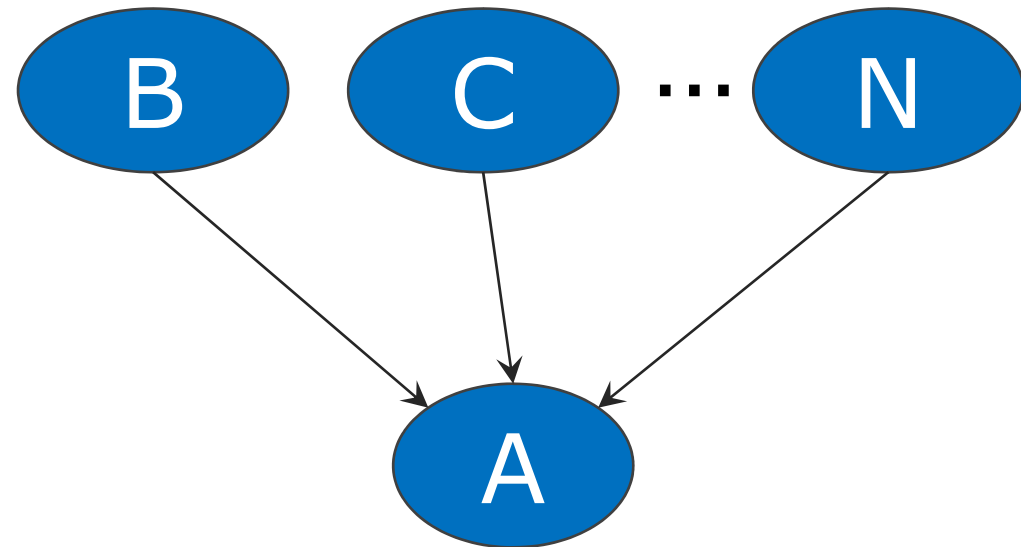


3. The variables together with the directed arrows form a *directed acyclic graph* (DAG).

# Bayesian Probabilistic Networks (BPN)

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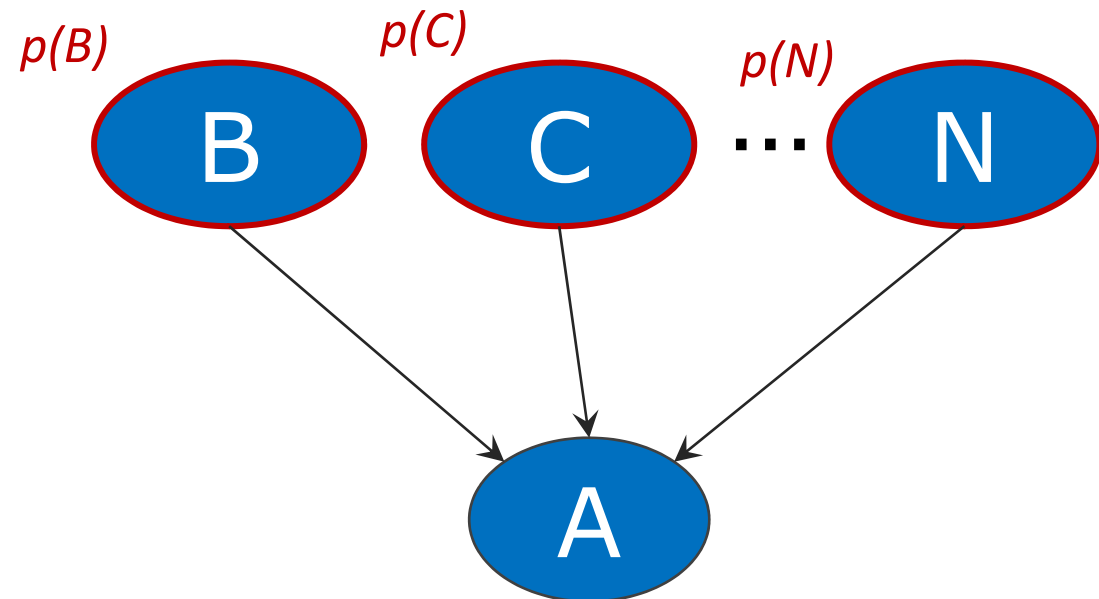
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4. To each variable  $A$  with parents  $B, C, \dots, N$  there is attached a *conditional probabilistic table* (CPT), containing  $P(A|B, C, \dots, N)$ .

# Bayesian Probabilistic Networks (BPN)

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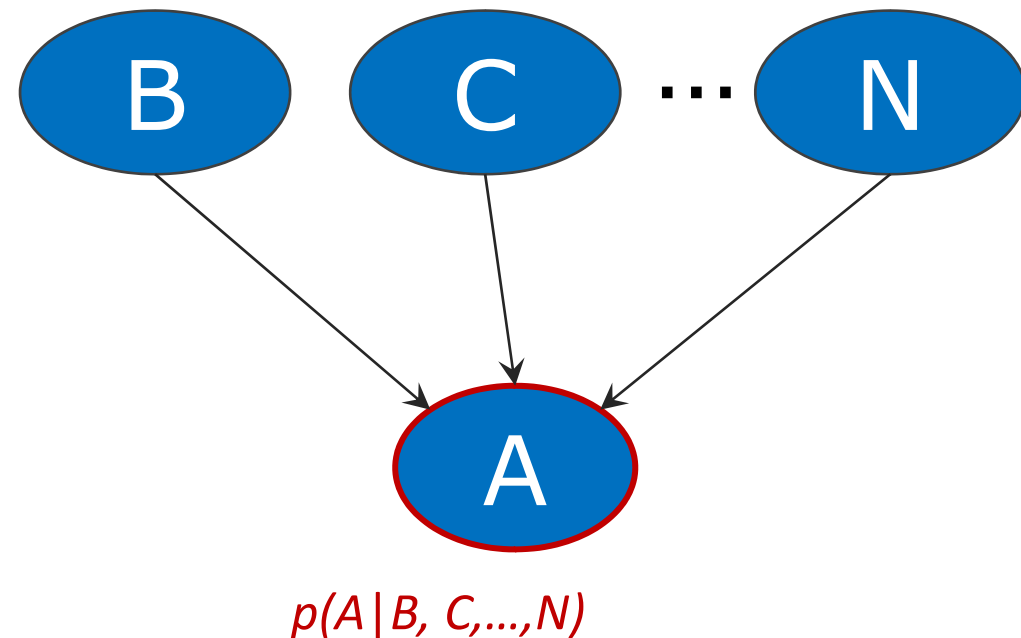


5. For the variables B, C, ..., N *'input' probabilities* must be specified  $P(B), P(C), \dots, P(N)$ .



# Bayesian Probabilistic Networks (BPN)

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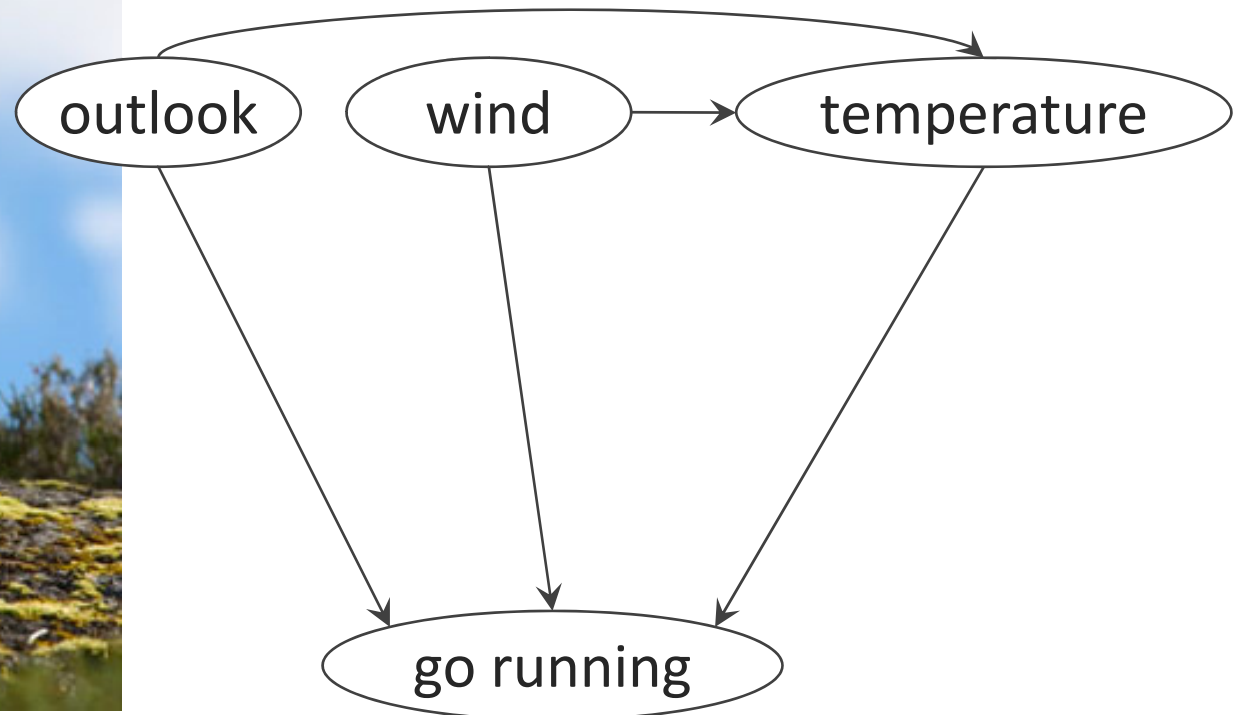
6. For the variable A a joint *'output' probability* can be assessed based on the algorithms of the conditional probability tables .

# Bayesian Probabilistic Networks (BPN)

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example



# Bayesian Probabilistic Networks (BPN)

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## conclusion

A Bayesian Probabilistic Network is a compact representation of the full joint probability of the random variables in the graph.

# Hierarchical Bayesian Probability Networks

# Hierarchical BPN

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## definition

- Hierarchical Bayesian Networks can be considered as an extension of Bayesian Networks.
- Probabilistic inference mechanisms of standard Bayesian Networks can generally be extended for Hierarchical Bayesian Networks as well.
- They are a representation formalism for probabilistic independencies between variables that belong to structured domains.

# Hierarchical BPN

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## definition

- Hierarchical BPNs are a generalization of standard Bayesian Networks, where a node in the network may be an aggregate data type.
- Within a single node, there may also be links between components, representing probabilistic dependencies among parts of the structure.
- Hierarchical BPNs can express further knowledge about variable structure and use that knowledge to build more realistic probabilistic models.

# Hierarchical BPN

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## What is similar to standard BPNs?

- Probabilistic inference mechanisms.
- Representation of probabilistic dependencies between variables as a directed acyclic graph (DAG).
- Each node of the graph corresponds to a random variable and is quantified by the conditional probability of that variable given the values of its parents in the graph by means of conditional probabilistic tables (CPT).

# Hierarchical BPN

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## What is the extension to BPNs?

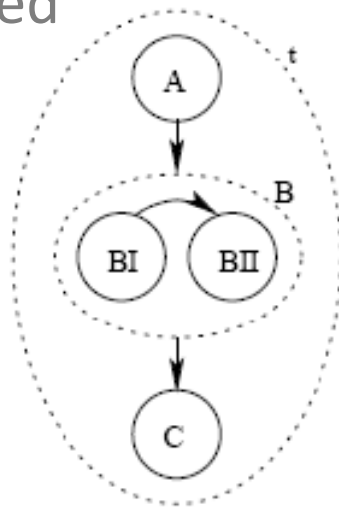
- Expressive power of HBN due to the possibility that one node may correspond to an aggregation of simpler types of variables.
- Allows the representation of complex systems, since causal and mind mapping representations of the system characteristics and functionalities are provided.
- Probabilistic dependencies can be expressed at any level, between nodes that are contained in the same structure.



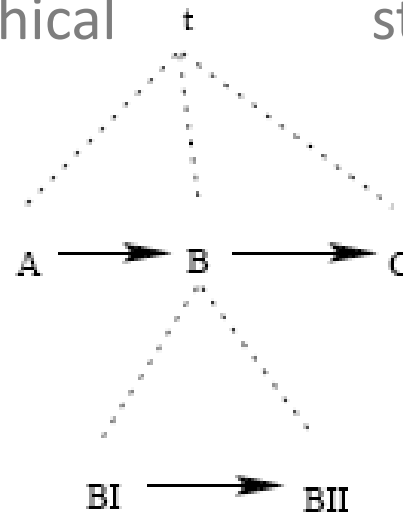
# Hierarchical BPN

## possible structural illustrations

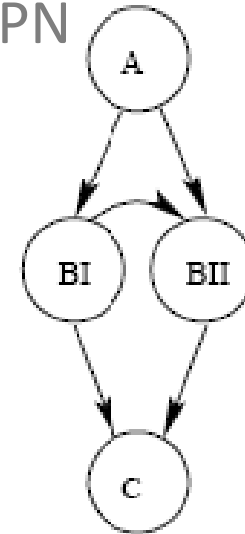
nested



hierarchical



standard BPN



- Contains the variables of the network and describes the part-of relationships and the probabilistic dependencies between them.
- The part-of relationships in a structural part may be illustrated either as nested nodes or as tree hierarchy.

# Hierarchical BPN

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example

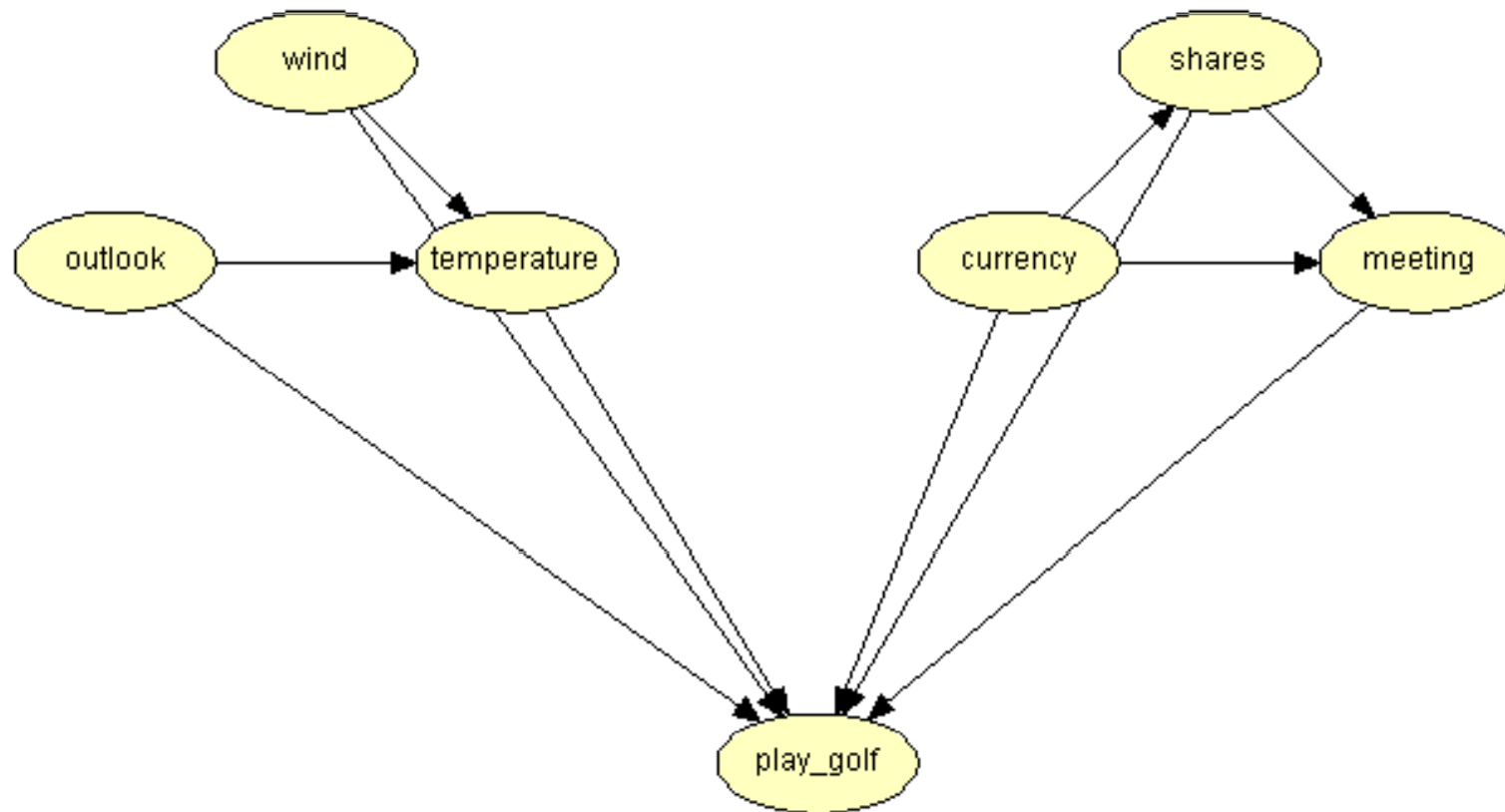
shall I play or  
shall I stay?



# Hierarchical BPN

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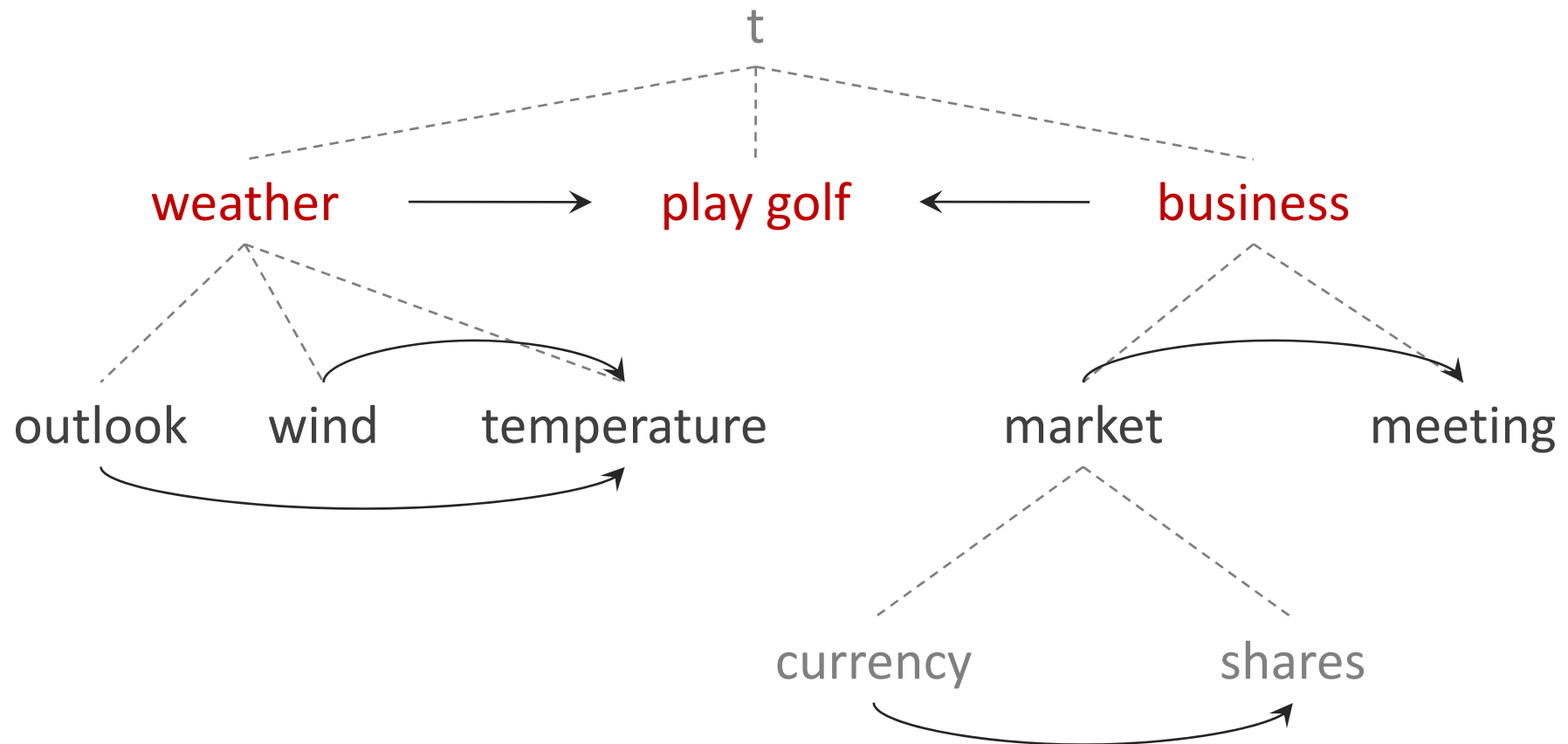
## example – standard BPN



# Hierarchical BPN

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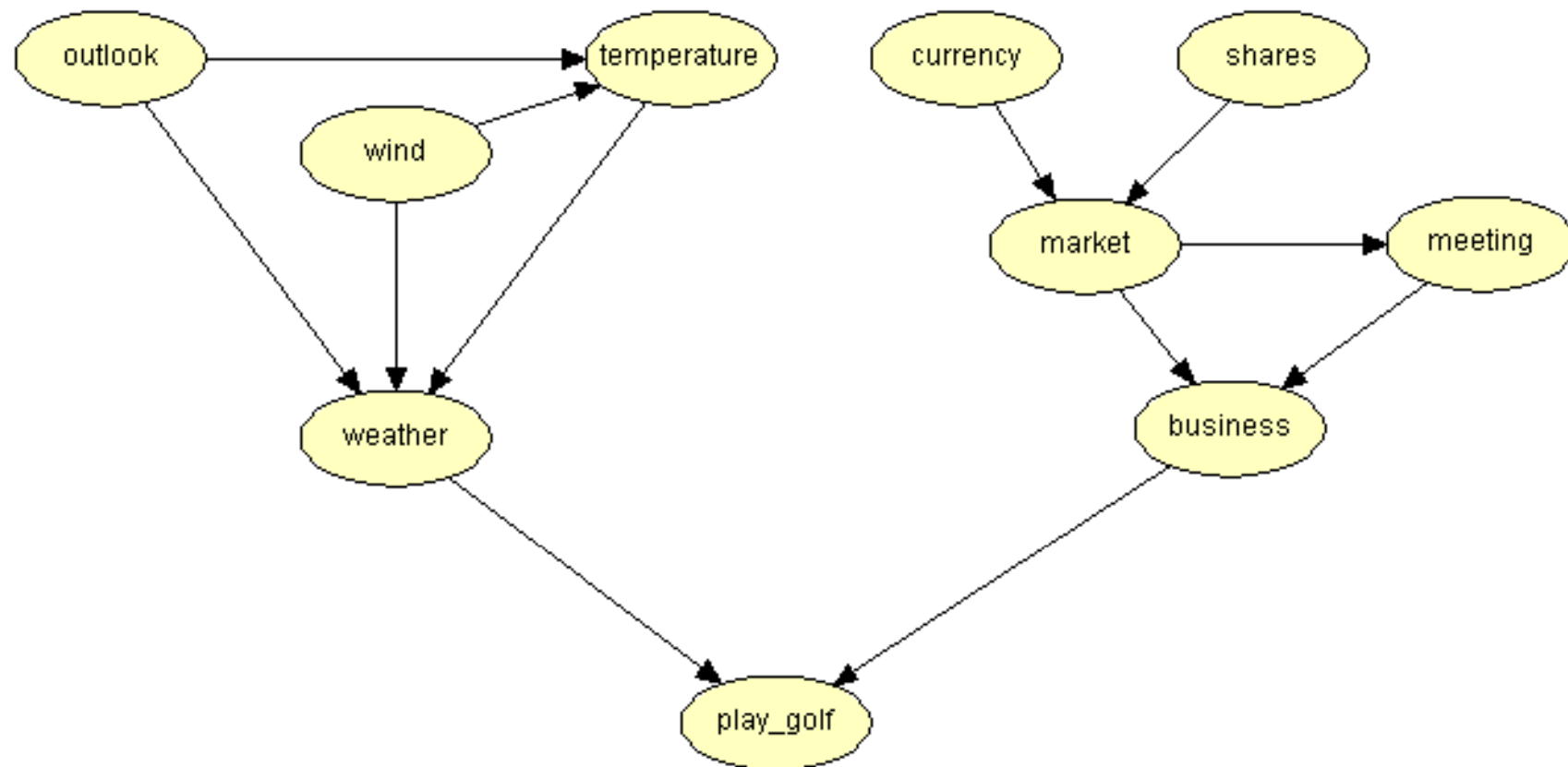
## example – Hierarchical BPN



# Hierarchical BPN

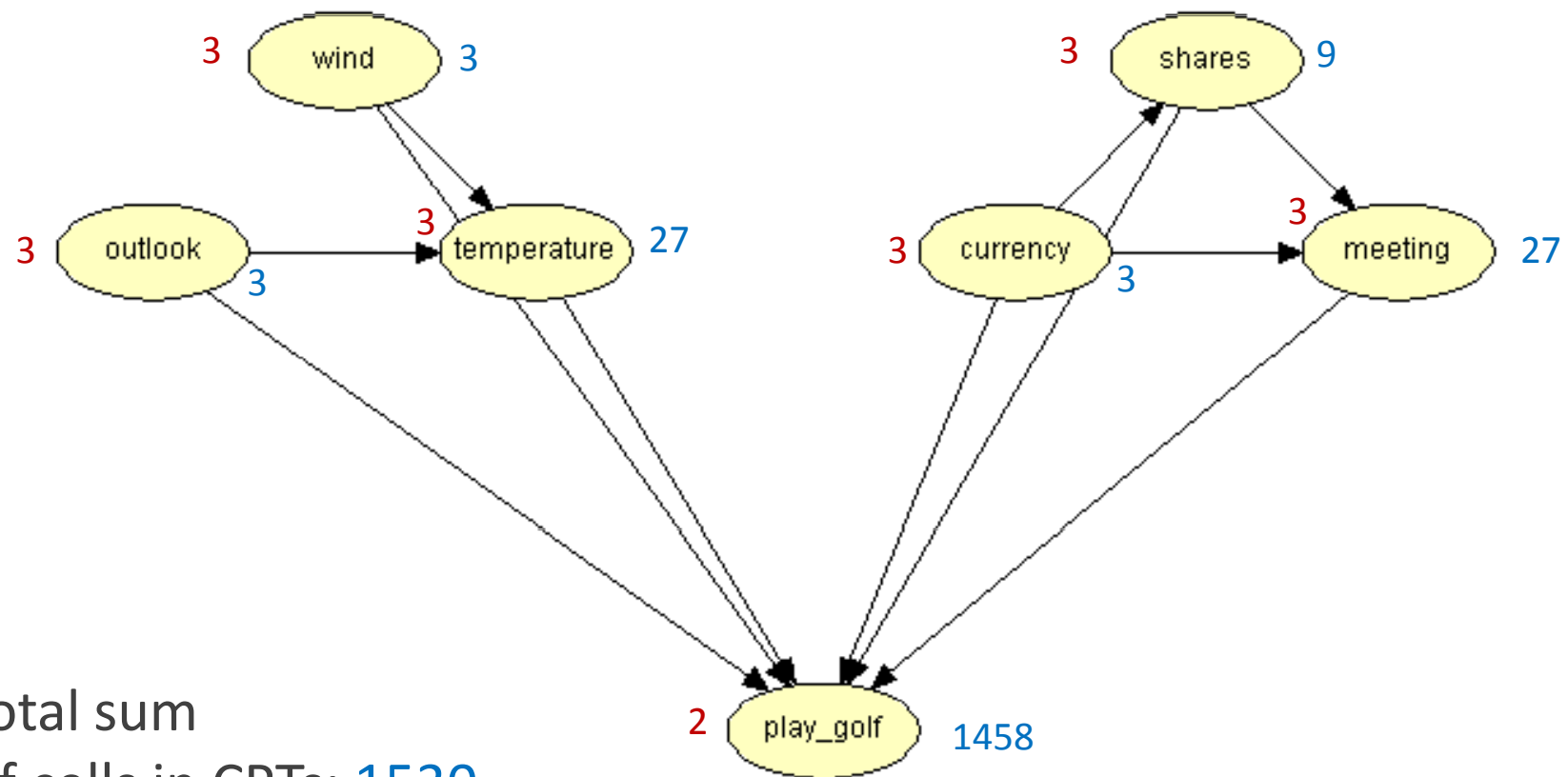
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## example – Hierarchical BPN



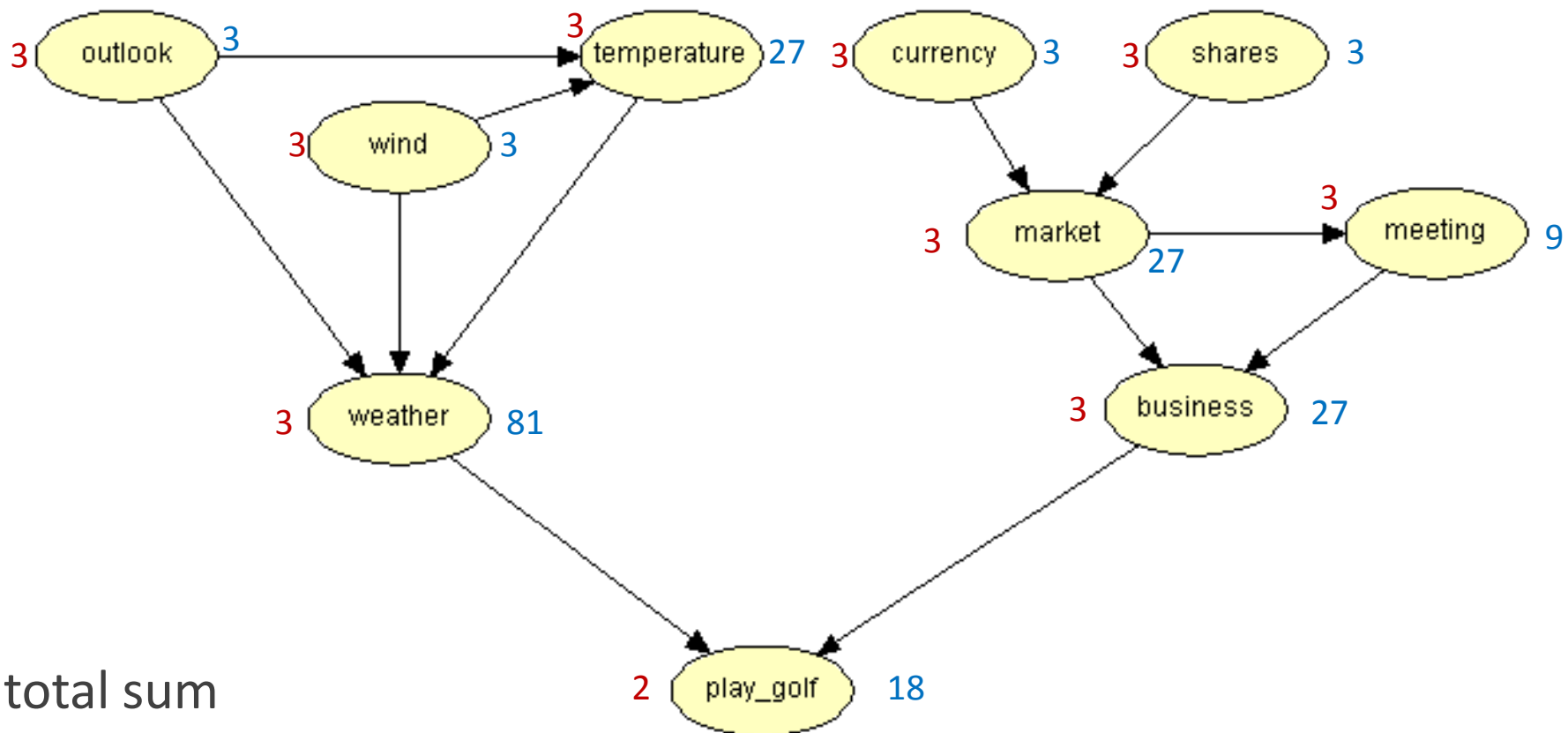
# Hierarchical BPN

## example – states of standard BPN



# Hierarchical BPN

## example – states of Hierarchical BPN



total sum  
of cells in CPTs: **201**

# Hierarchical BPN

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## computing advantages

- reduced number of states and therefore reduced size of conditional probability tables (CPT)
- consequences:
  - less computation time
  - less space needed
  - less complexity



# examples for HBN in engineering

# Hierarchical BPN

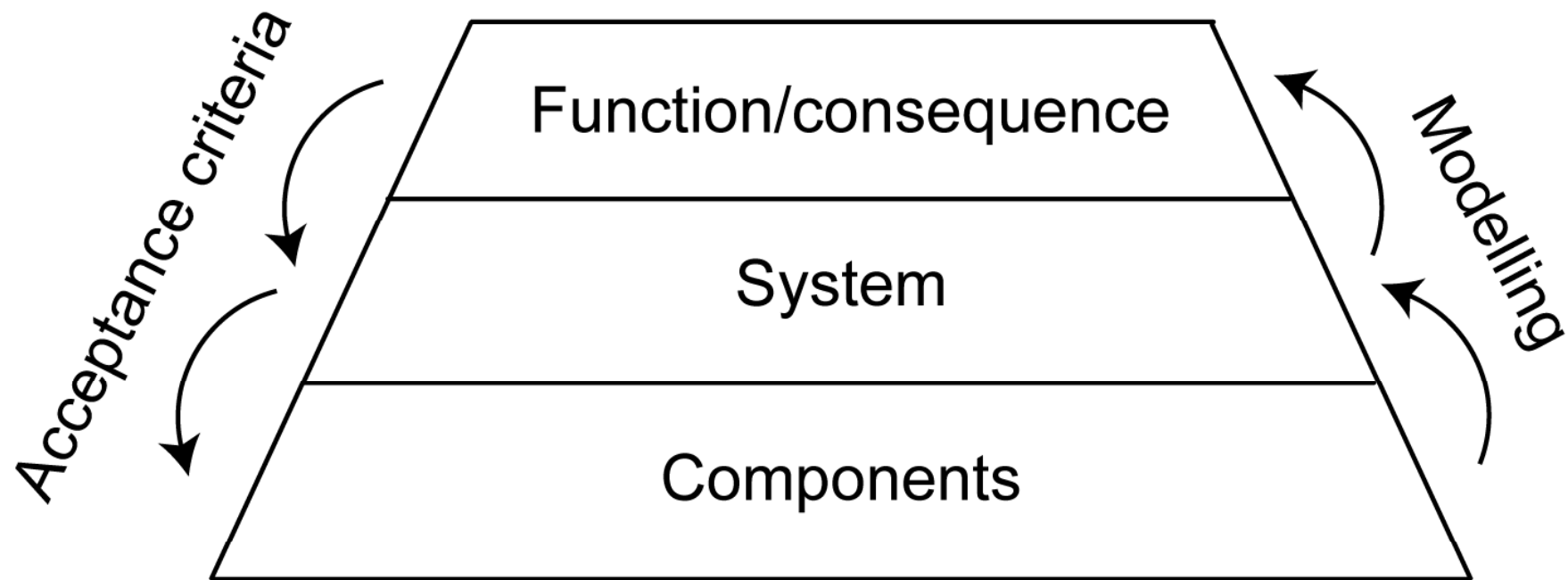
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For modelling e.g. engineered complex systems the main task is the representation of the physical understanding, the relevant experience and the data available at different hierarchical levels in terms of probabilities of states of variables or in terms of decision nodes or utility nodes.

# Hierarchical BPN

hierarchical aggregation of components and their interrelations



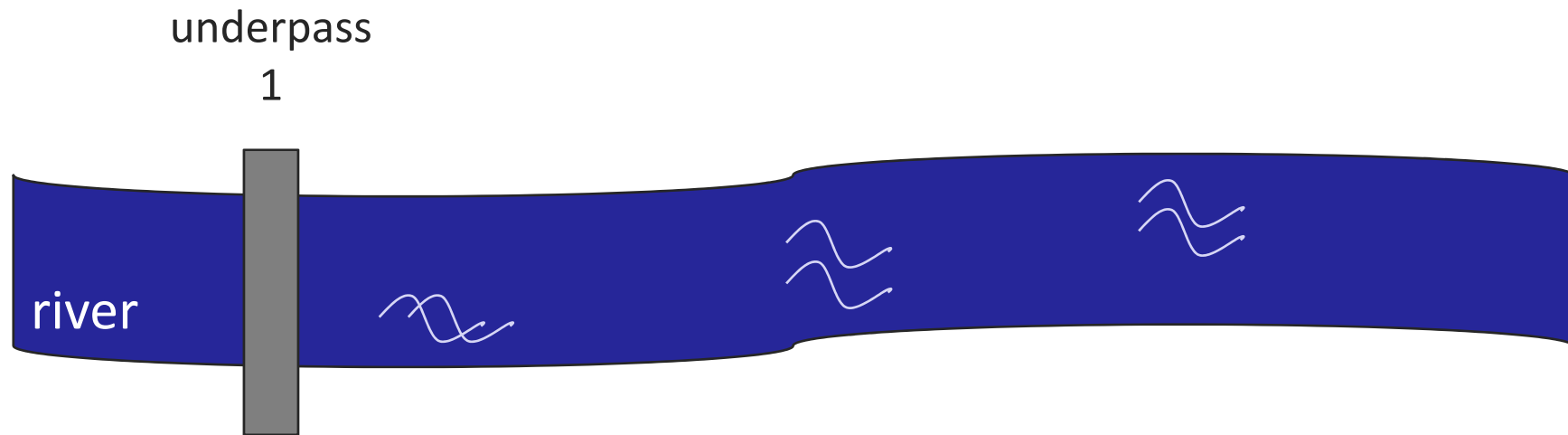
Nishijima et al. 2007

# example for Hierarchical BPN (Schubert 2007)

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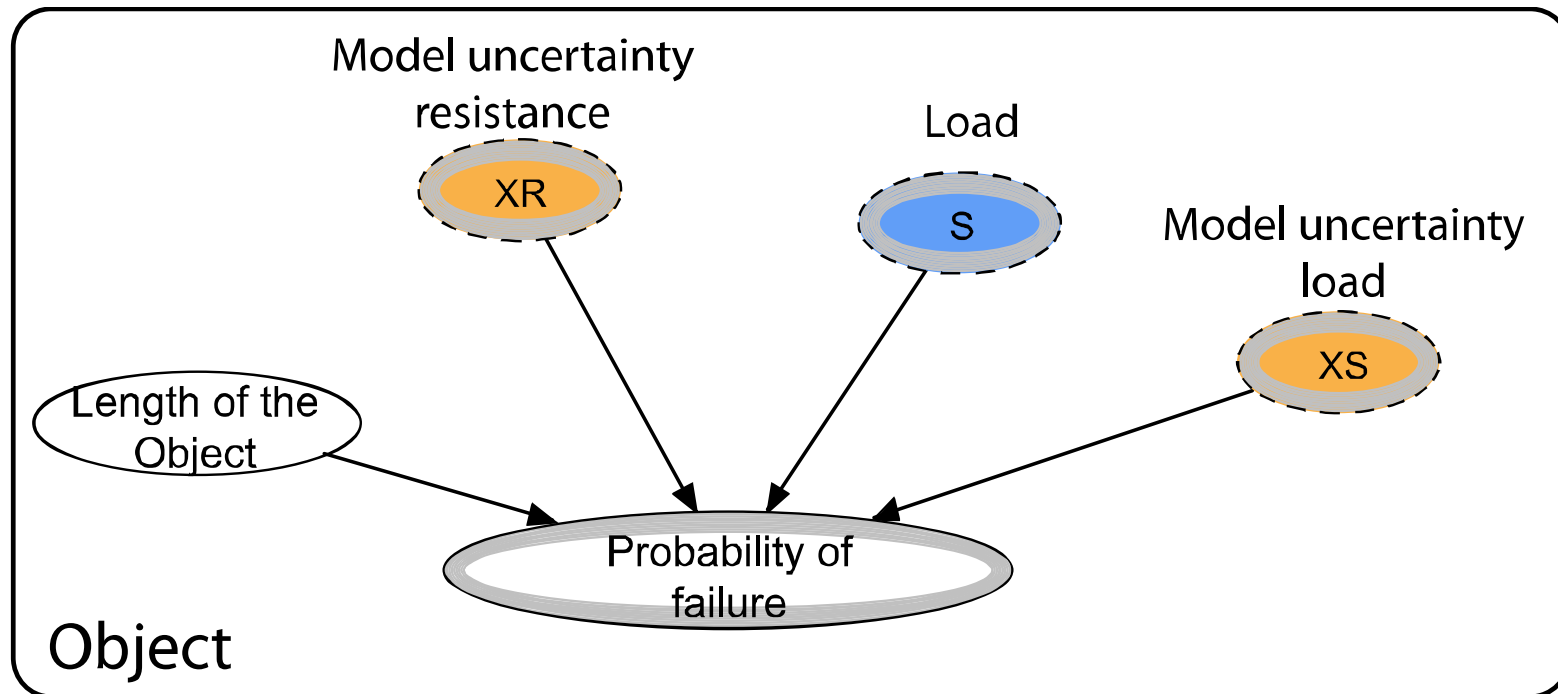
## example I: river underpass system



# example for Hierarchical BPN (Schubert 2007)

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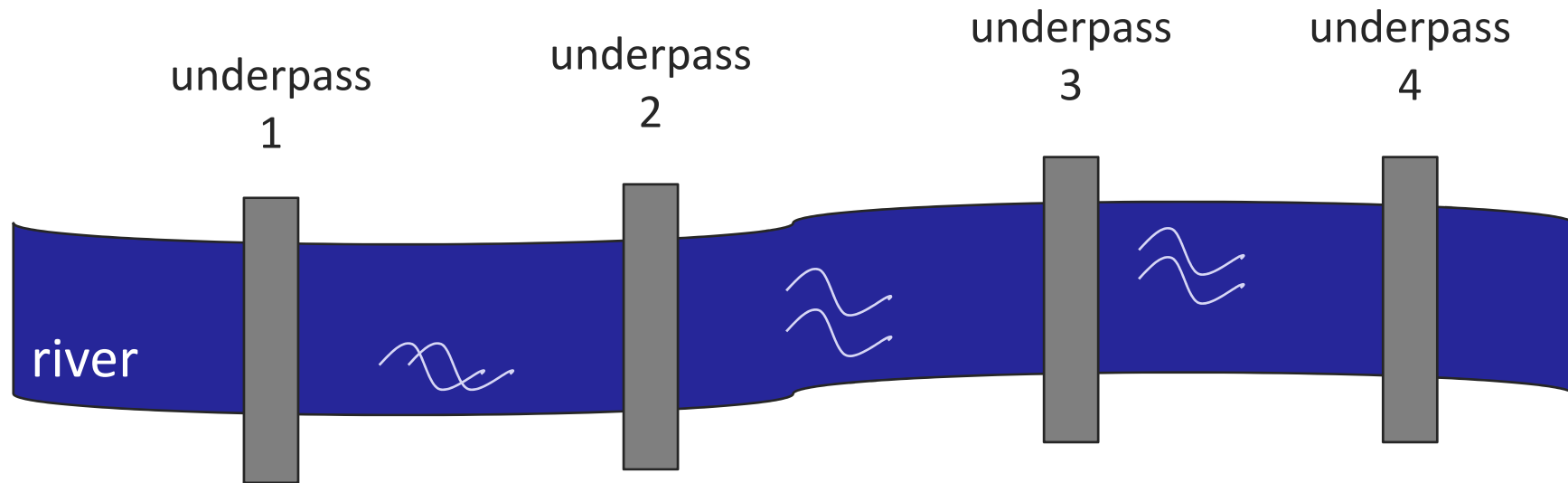
## subnet for one single underpass (object)



# example for Hierarchical BPN (Schubert 2007)

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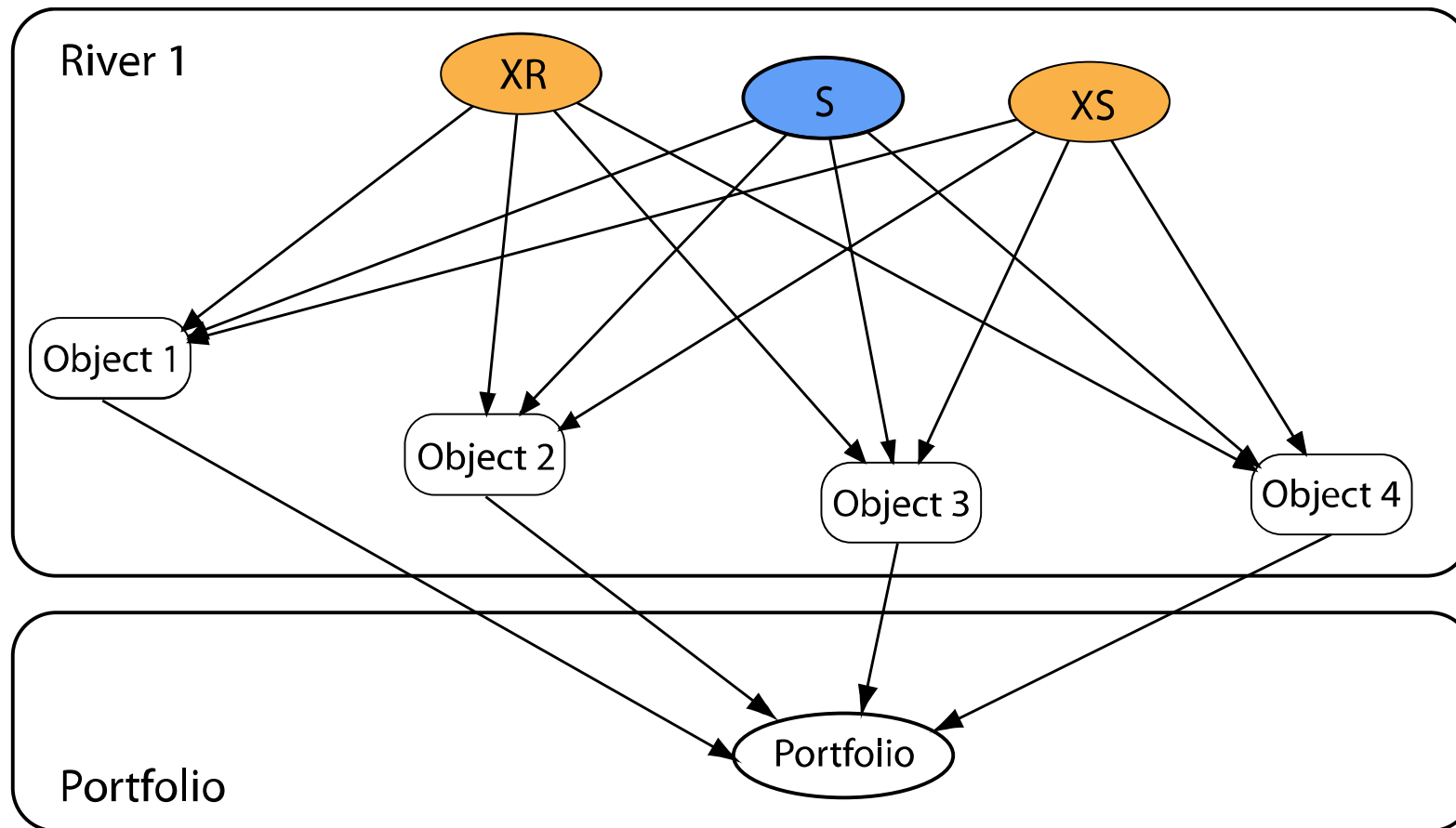
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# example for Hierarchical BPN (Schubert 2007)

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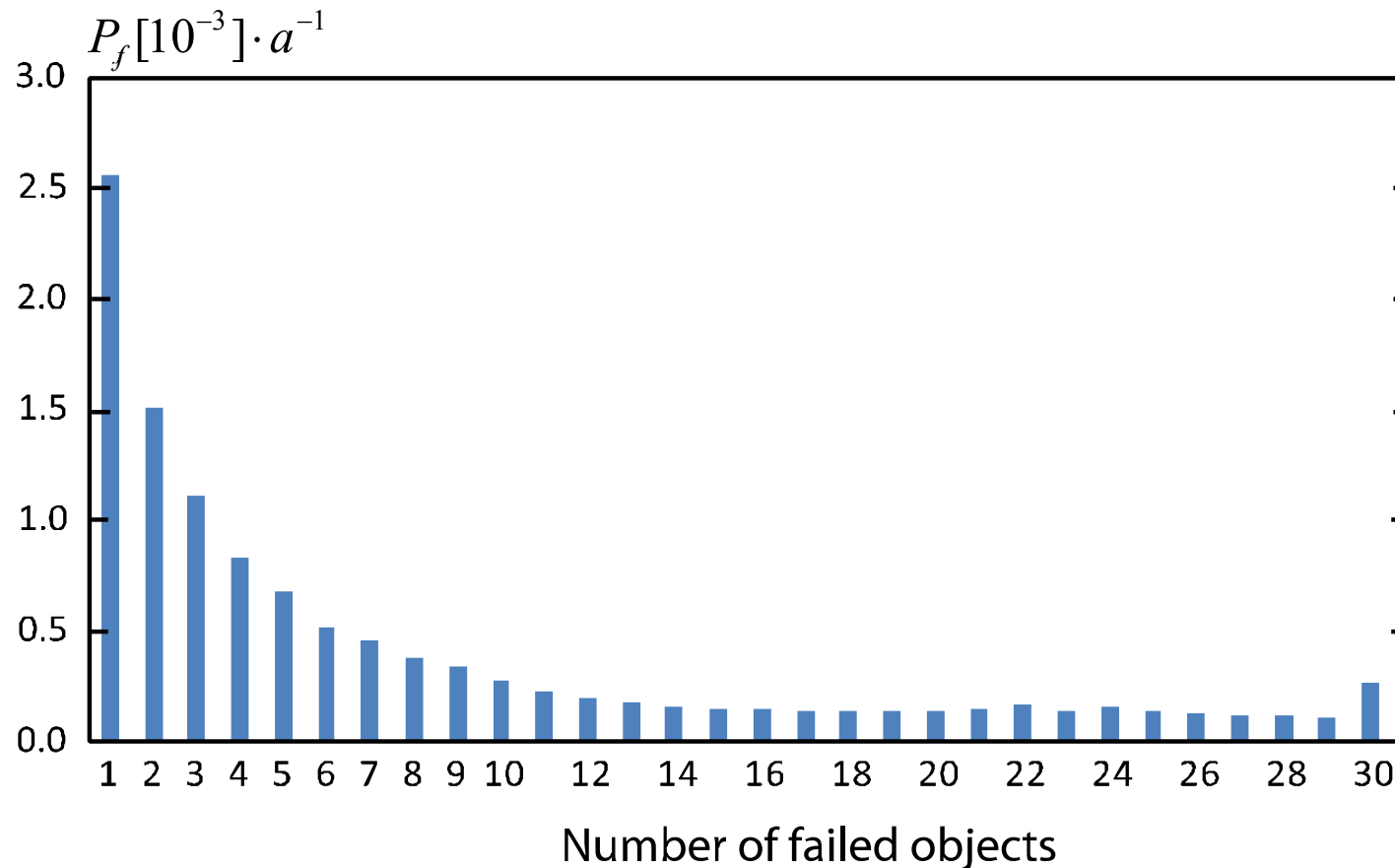
## network for four underpasses under one river



# example for Hierarchical BPN (Schubert 2007)

## probability of failure for 30 objects under one river

common model uncertainties for all objects



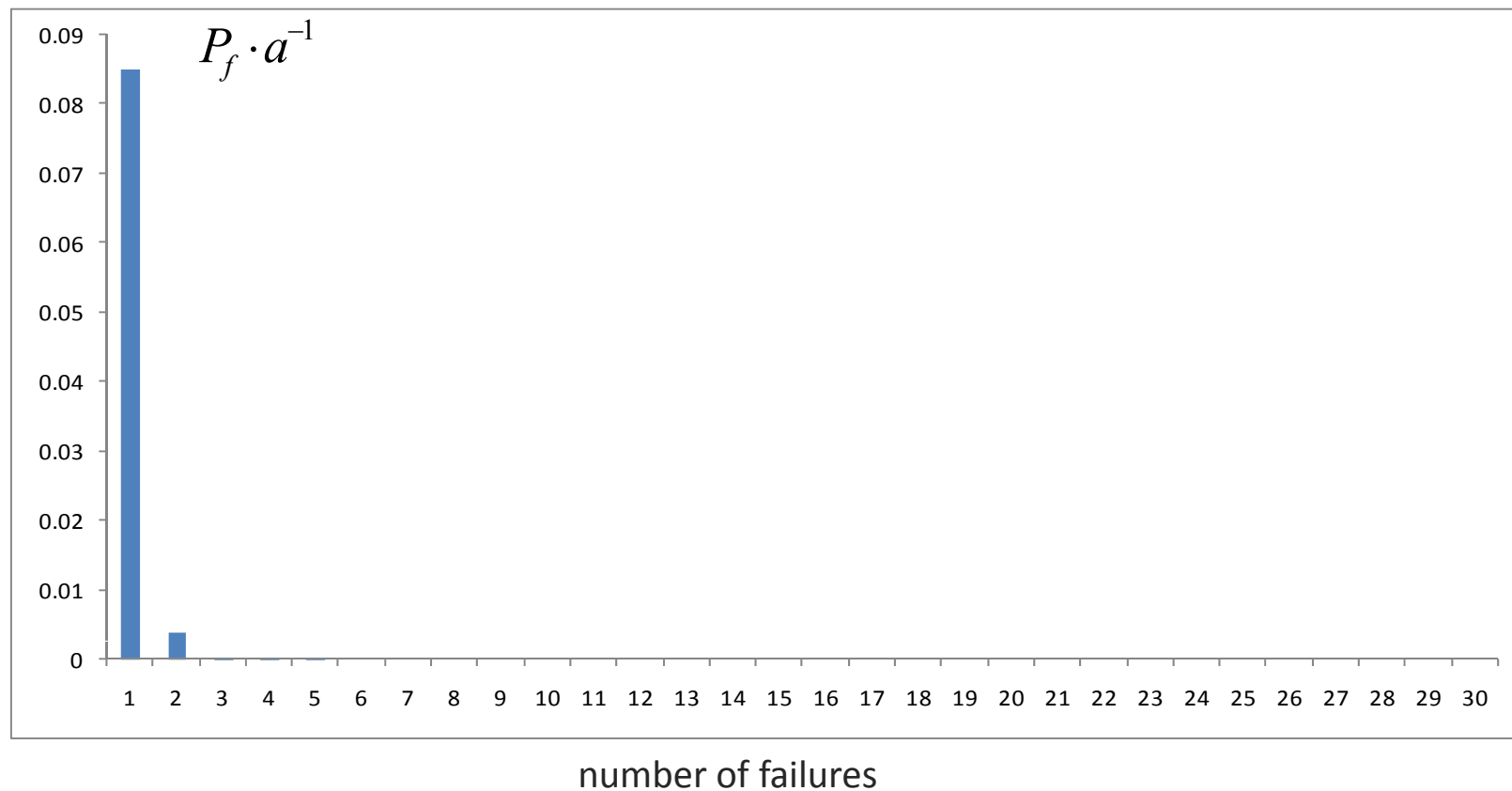


# example for Hierarchical BPN (Schubert 2007)

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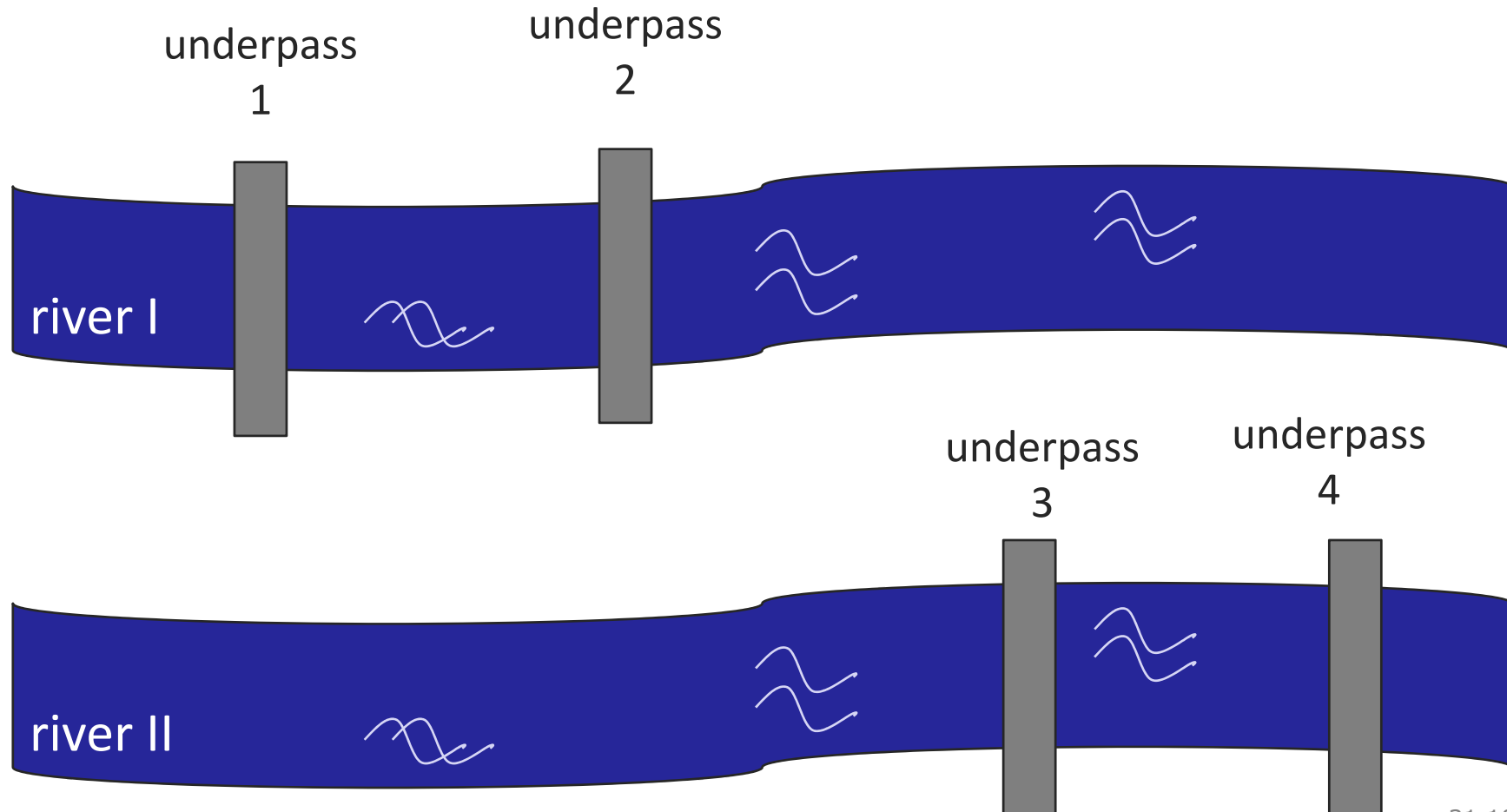
## probability of failure for 30 objects under one river

object related model uncertainties



# example for Hierarchical BPN (Schubert 2007)

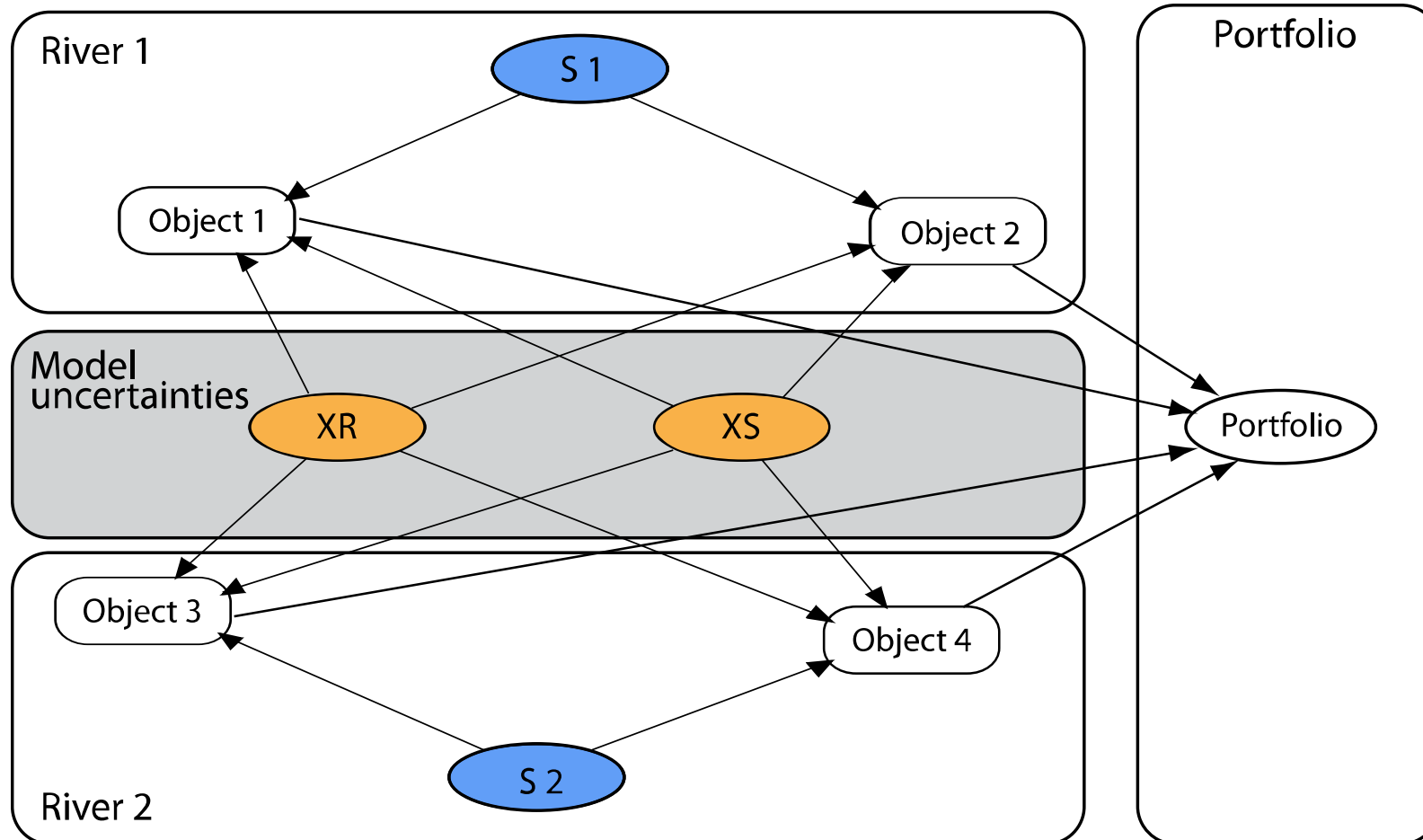
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# example for Hierarchical BPN (Schubert 2007)

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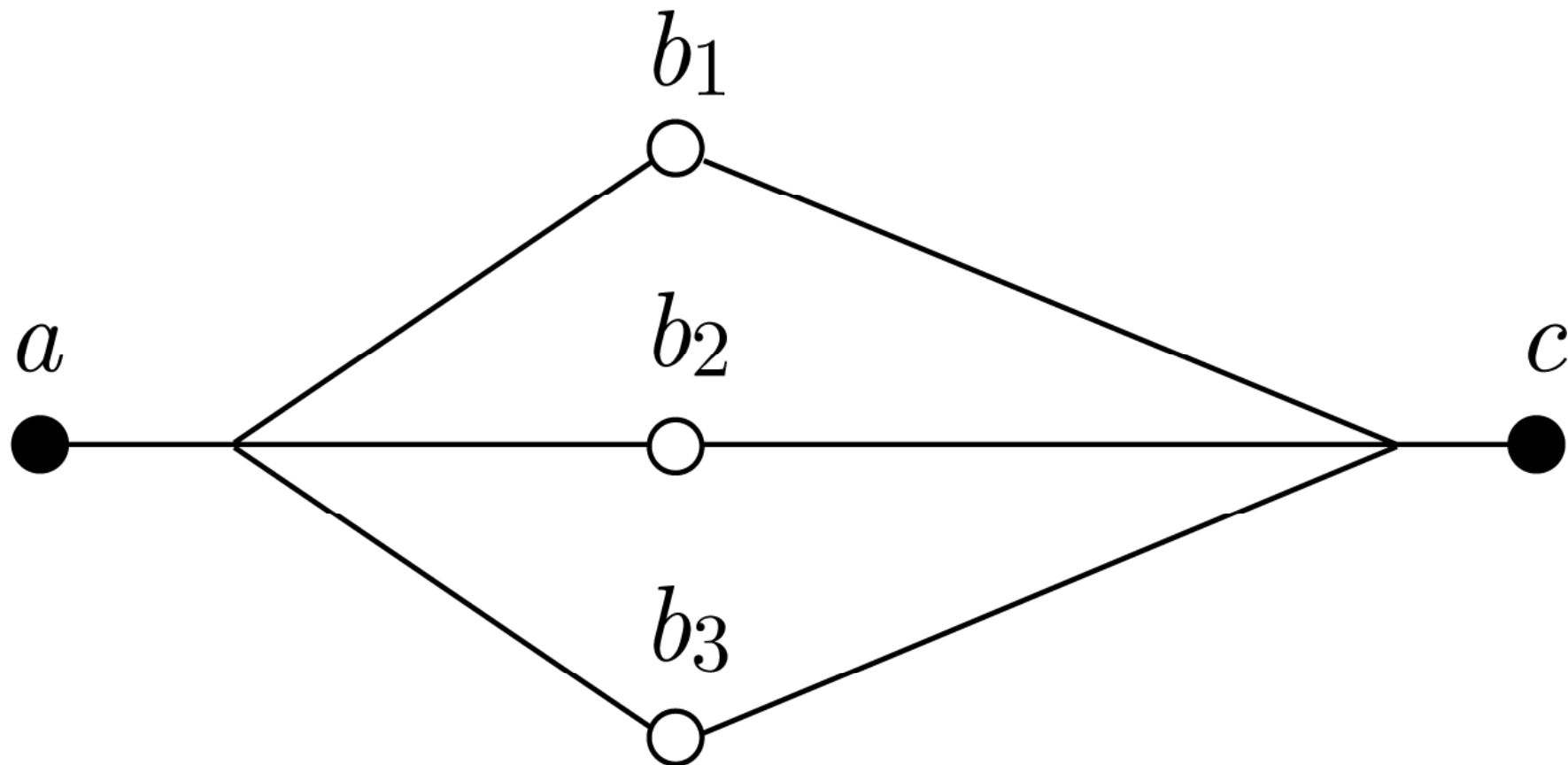
## network for four underpasses under two different rivers



# example for Hierarchical BPN (Nishijima et al. 2007)

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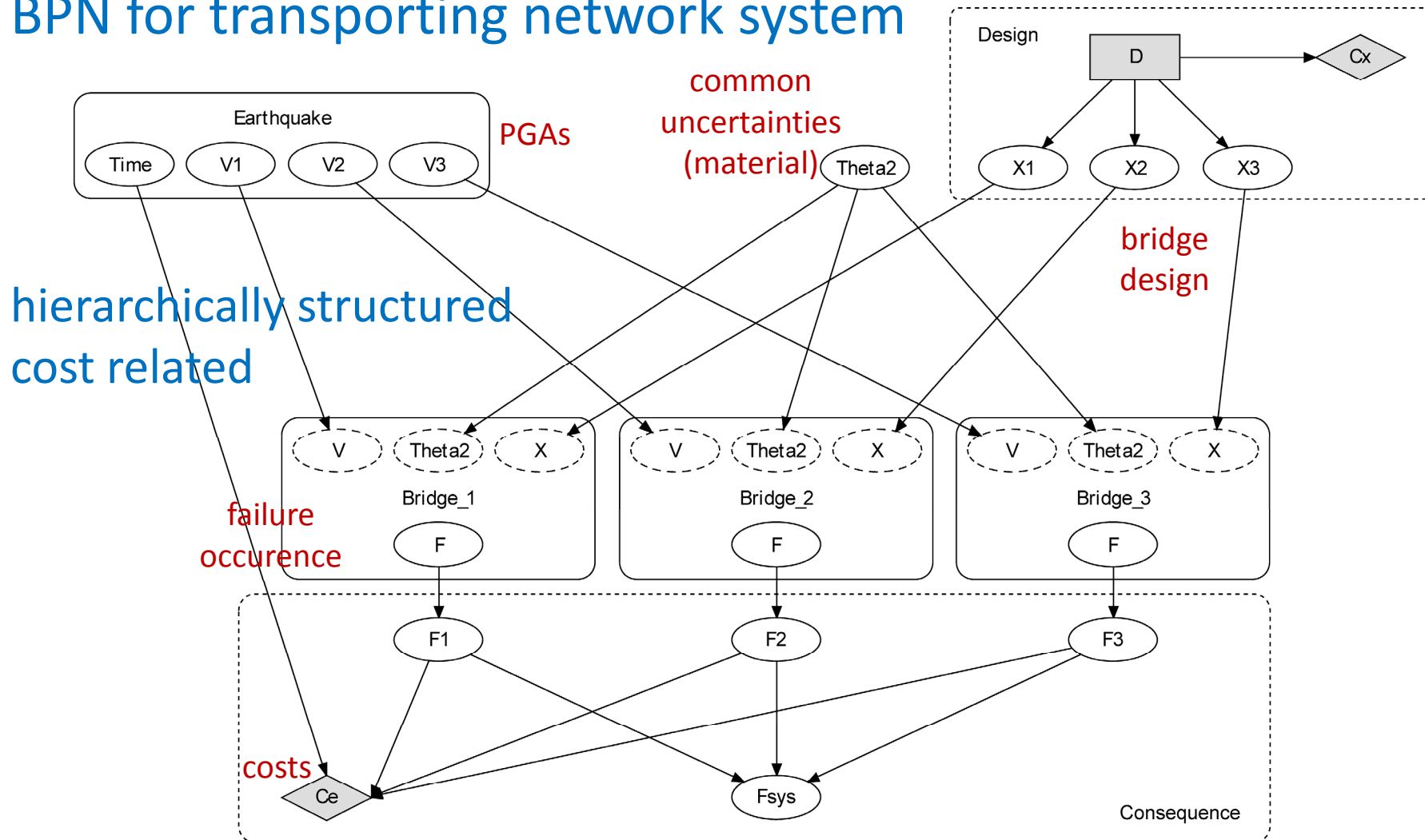
## example II: transporting network system



# example for Hierarchical BPN (Nishijima et al. 2007)

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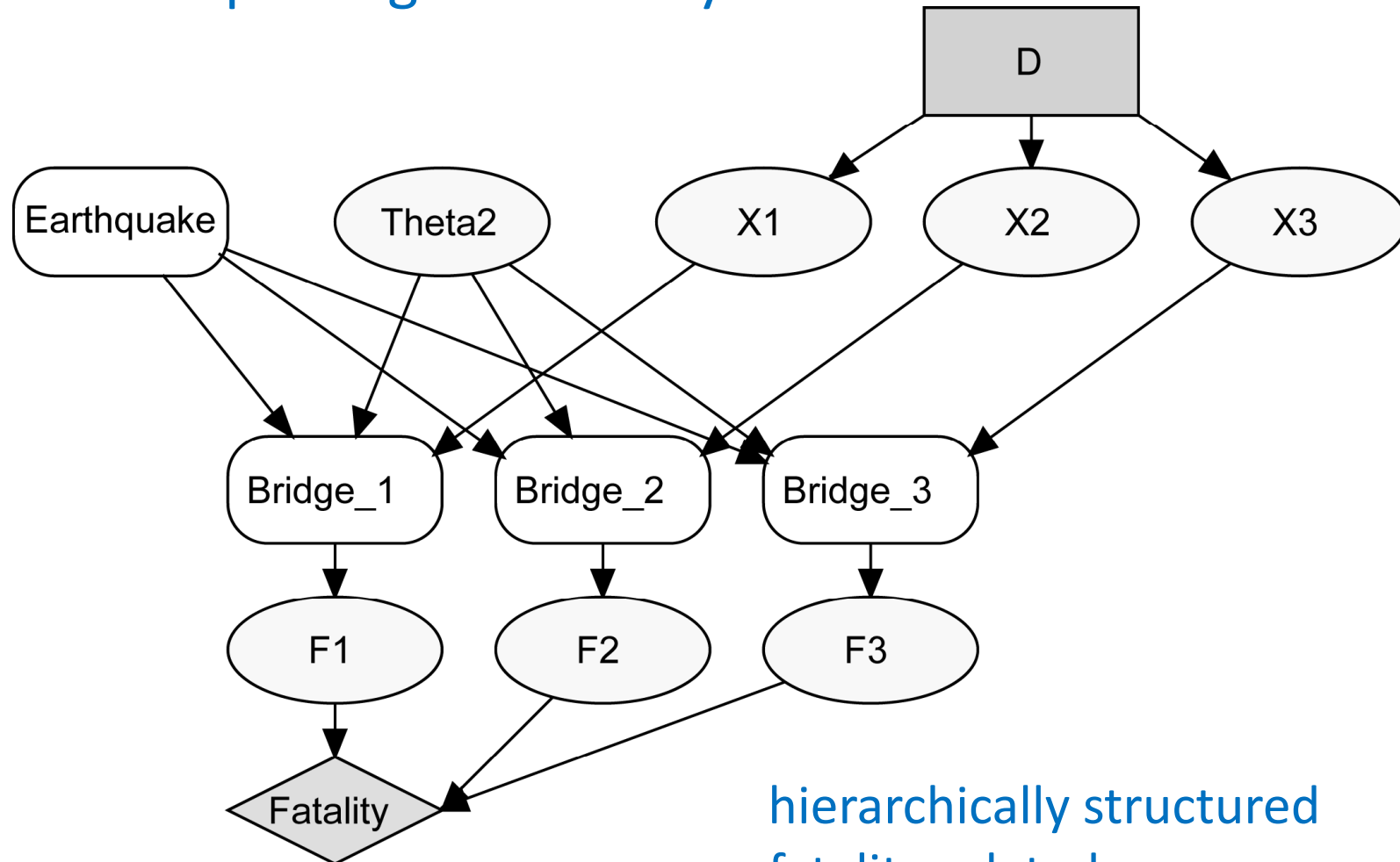
## BPN for transporting network system



# example for Hierarchical BPN (Nishijima et al. 2007)

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## BPN for transporting network system



hierarchically structured  
fatality related

# conclusions

## conclusions

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- BPN is a compact representation of the full joint probability of the random variables in the DAG.
- Probabilistic inference mechanisms of standard BPNs can generally be extended for Hierarchical BPNs as well.
- The hierarchical BPN provides a clear perspective of how the whole system should be built up using the modules representative of different levels of analyses.
- Expressive power due to the possibility that one node may correspond to an aggregation of simpler types of variables.
- Helpful tool for illustration and communication, since causal and mind mapping representations of the system characteristics and functionalities are provided.



any questions ?