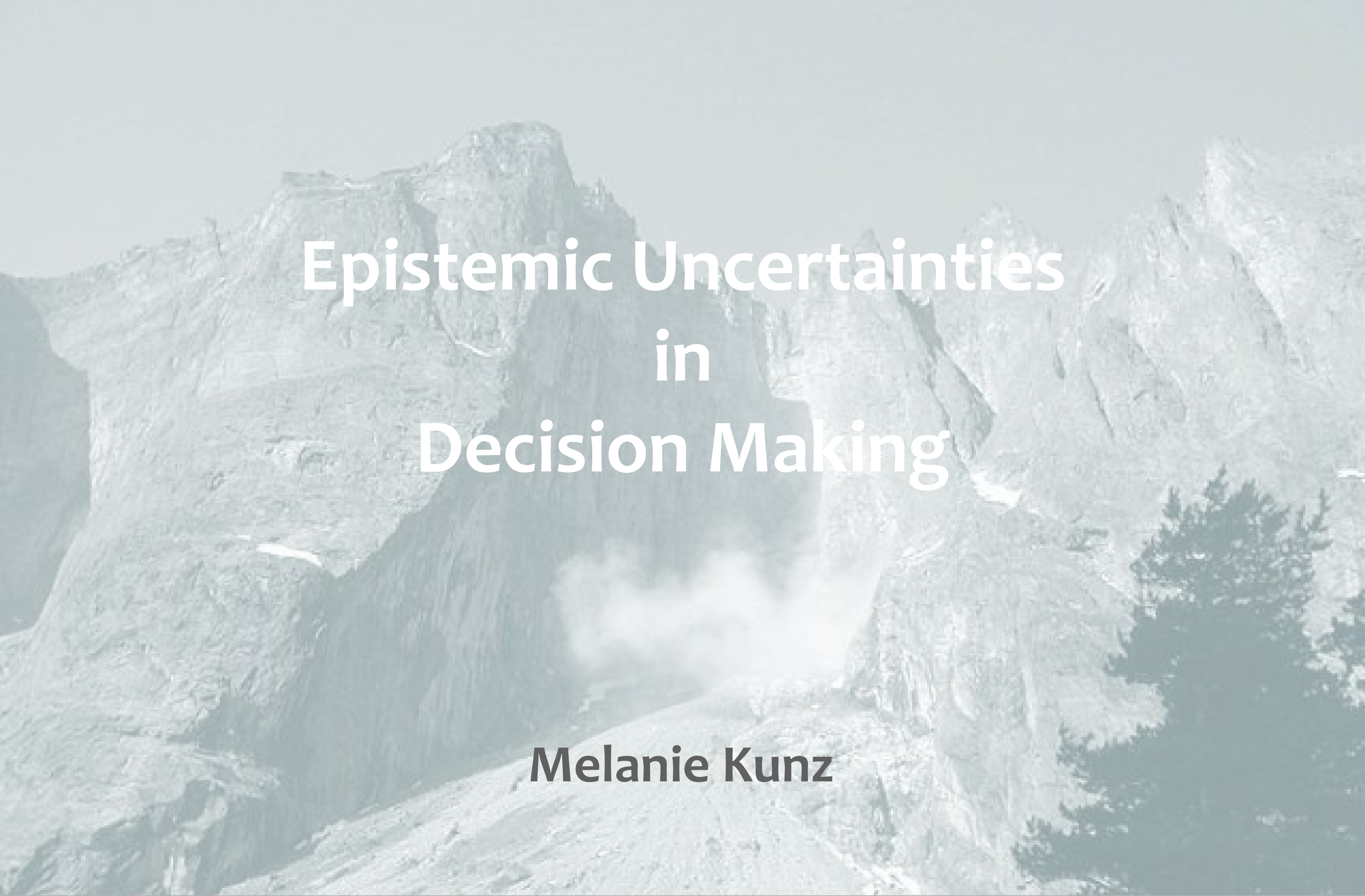

Epistemic Uncertainties in Decision Making

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PhD seminar „Probabilistic Approach to Natural Hazards Assessment“

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Uncertainties and Engineering Models

In Quantitative Risk Analysis (QRA) and
Structural Reliability Analysis (SRA):

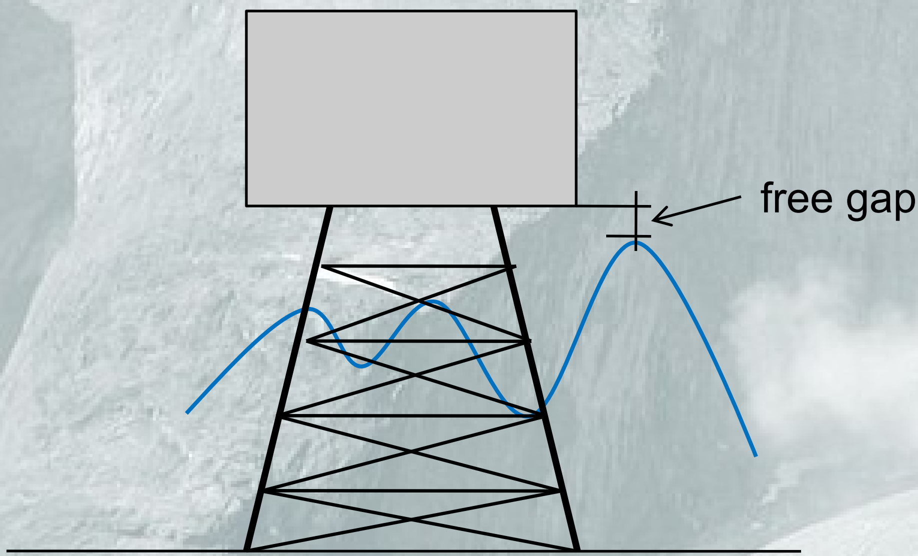
- Uncertainties included
- Uncertainties differentiated according to type and origin
    - Natural variability (aleatory, type 1)
    - Model uncertainties (epistemic, type 2)
    - Statistical uncertainties (epistemic, type 2)

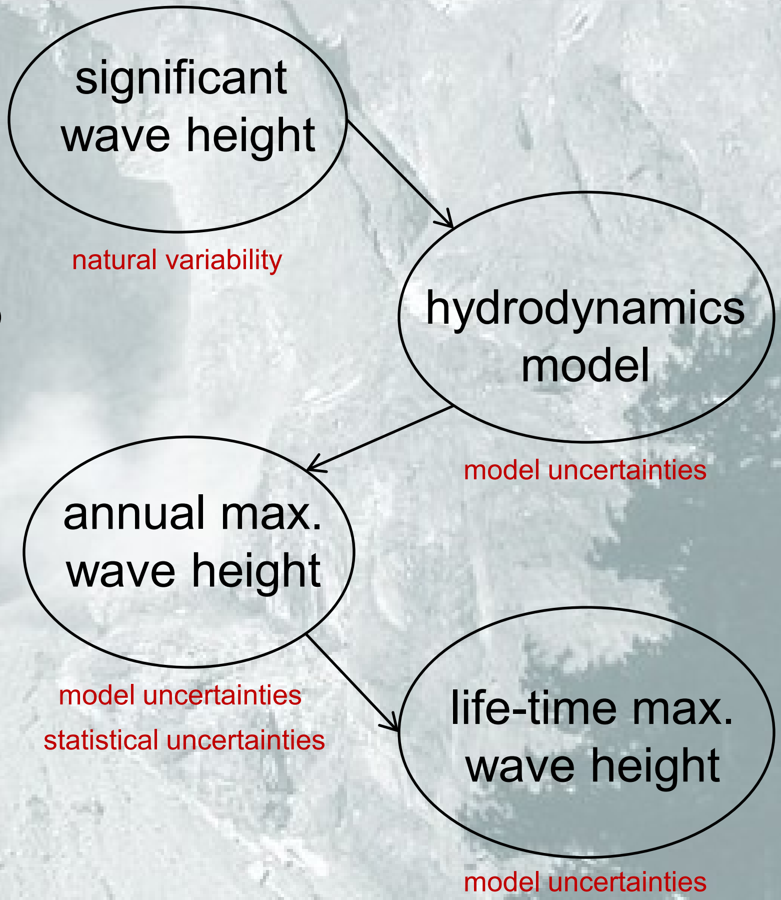
Uncertainty is scale and time dependent.

ALL UNCERTAINTIES HAVE TO BE TAKEN INTO ACCOUNT!

Uncertainties and Engineering Models

Example: Engineering model for free gap decision problem





Framework for Risk Based Decision Making

Risk = main ingredient for utility function

Engineering decision making = game

Goal of game = optimizing the benefits

Opponents = nature, people

Rules = success or acceptance criteria,

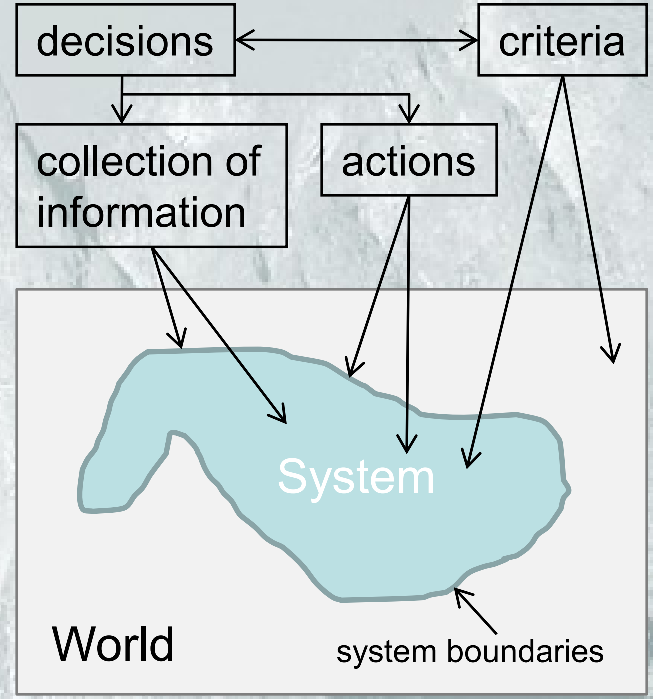
system, system boundaries,

possible consequences,

influences

Decisions: based on anticipated behavior of nature & people

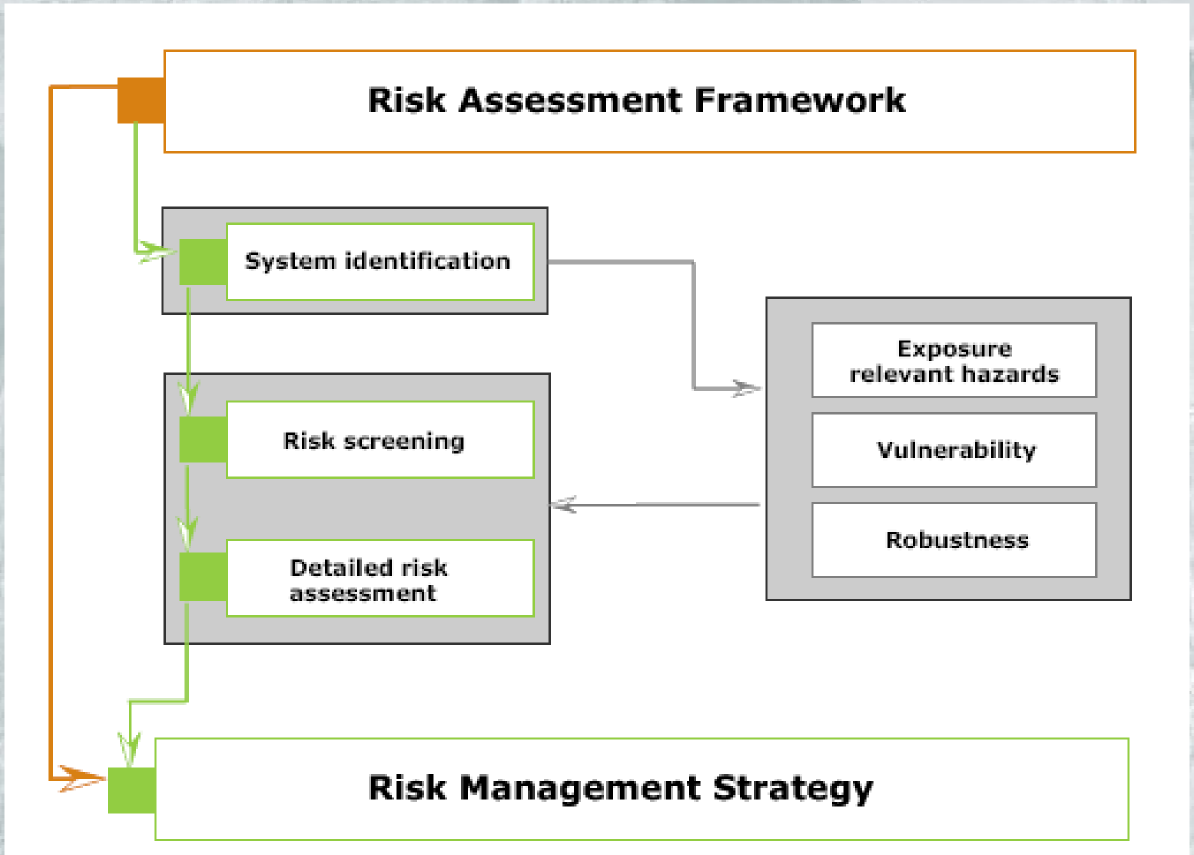
Playing: “buying” physical changes of system & knowledge



Constituents of engineering decision analysis, Faber (2002)

Framework for Risk Based Decision Making

Ingredients of risk assessment:



Framework for risk assessment, Faber (2005)

Decision Theoretical Basis

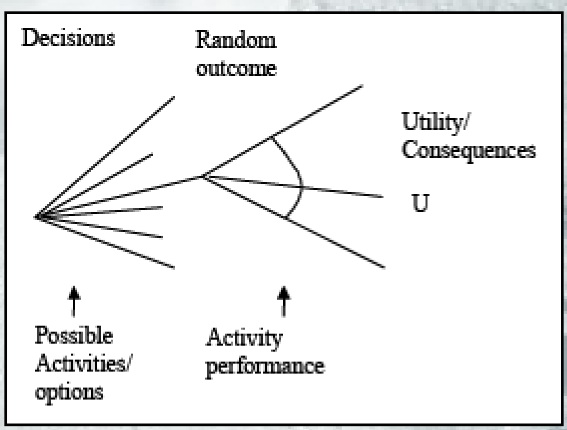
Basis: Optimal decisions = maximum expected utility

Prior decision analysis: statistical information, probabilistic modeling
Prior to any decision or activity. Simple comparison of utilities.

Posterior decision analysis: changes are introduced, additional information has been collected.
Basis for assessment of benefit of future risk mitigation measures or information gathering.

$$\max_{\mathbf{a}} \ U^*(a) = \max_{\mathbf{a}} E'_{\mathbf{X}}\left[U(a,\mathbf{X})\right]$$

U: utility
a: decision
X: vector of all random variables



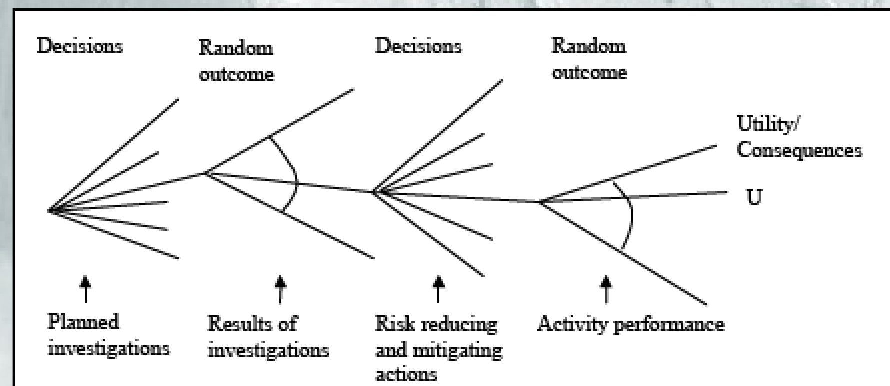
Decision/event tree for prior and posterior decision analysis, Faber (2005)

Decision Theoretical Basis

Pre-posterior decision analysis: basis for assessment of benefits of future risk mitigation measures or information gathering.
Decision rules specify future actions, based on outcomes of planned activities

$$\max_{a} U^{*}(a) = \max_{a} E'_{\mathbf{Z}}\left[E''_{\mathbf{X}|\mathbf{Z}}\left[U(a(\mathbf{Z}),\mathbf{X})\right]\right]$$

U: utility
a(.): set of possible different action
z: outcome of considered investigations
E[ ]: expected value operator
': events based on prior information
'': events based on posterior information



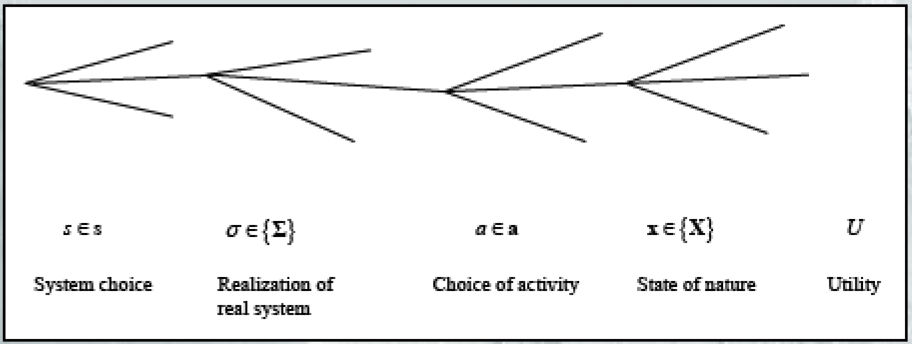
Decision tree for pre-posterior decision analysis, Faber (2005)

Informal decision analysis: simplified (e.g. not including all uncertainties)
Quality of informal analyses can be doubtful & hard to judge.

Optimal System Choice

Correct System?

s: possible systems
σ: actual system
a: course of action (dependent on system choice)
x: outcome (dependent on all previous choices)
U: utility



Decision/event tree for optimal choice of system in a prior analysis context, Faber (2005)

Yes (s = σ): Optimal action based on prior decision analysis

$$\max_{\mathrm{a}} U^*(a) = \max_{\mathrm{a}} E'_{\mathbf{X}|\sigma=s}\left[U(a,\mathbf{X})\right]$$

No (s ≠ σ): Optimal system choice s* and action a* subject to an expectation operation over the possible systems

$$\max_{\mathrm{s,a}} U^*(s,a) = \max_{\mathrm{s}}\left(\max_{\mathrm{a}} P(\Sigma=s)E'_{\mathbf{X}|s}\left[U(a,\mathbf{X})\right] + E'_{\Sigma\backslash s}\left[\underbrace{E'_{\mathbf{X}|\Sigma}\left[U(a^*,\mathbf{X})\right]}_{\text{utility term}}\right]\right)$$

utility term
(corresponding to real outcome of system)

Example: Optimal Design

Problem: Optimizing design variable a (out of a set of possible values)
Goal: achievement of a required reliability β of a structural component with material characteristics r subjected to loading l (both uncertain)

System known but parameters subject to uncertainty:
r and l modeled by R (aleatory and epistemic (μ_R)) and L (aleatory);
Limit state function: g(a, r, l) = a*r – l

Optimal choice for minimized expected costs:
$C(a) = C_f(a) + C_D(a)$

C_f: cost of failure, C_D: cost of design

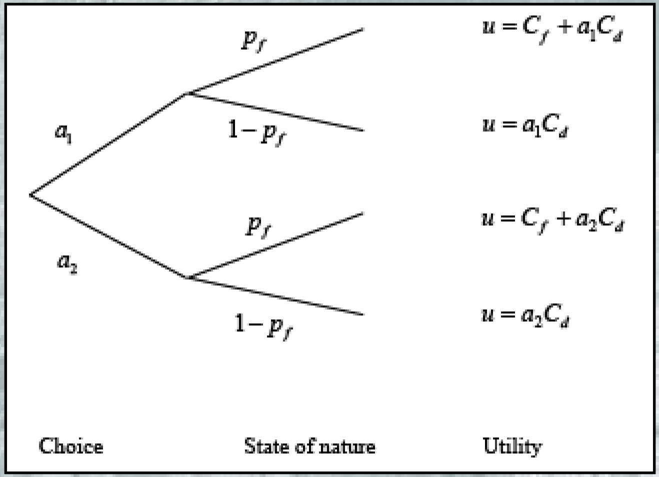
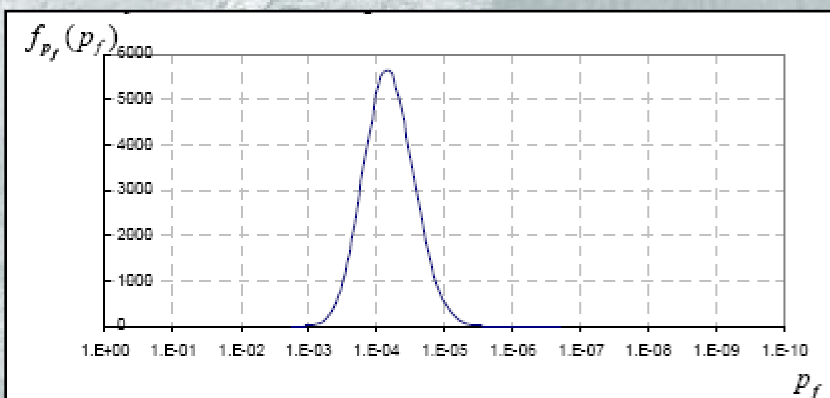


Illustration of optimal design decision/event tree, Faber (2005)

Example: Optimal Design

Informal prior decision analysis for design:

- Disregarding epistemic uncertainty (μ_R)
- Taking epistemic uncertainty into account indirectly (probability density function to identify probability of failure on a certain fractile value)



Probability density function for the probability of failure, Faber (2005)

Example: Optimal Design

Formal prior decision analysis for design:
Takes into account the epistemic uncertainty directly, together with aleatoric uncertainty (failure probability has to take into account μ_R which changes the possible choices for a)

Example: Optimal Design

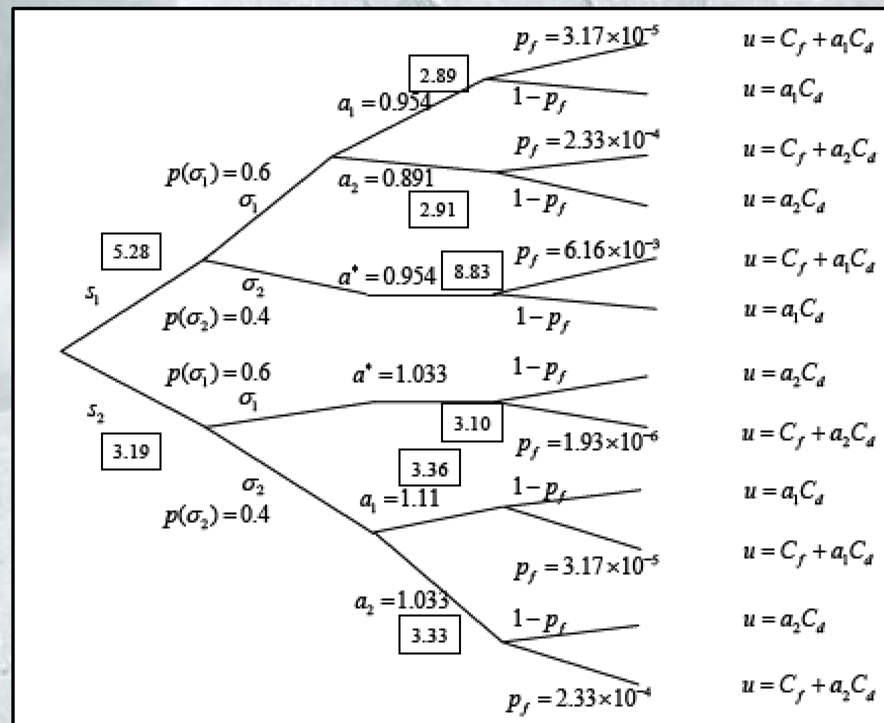
Formal prior decision analysis for system choice:

Problem: selection of a system out of:

system 1; s1=N(400,10), $p(\sigma_1)$=0.6

system 2; s2=N(350,10), $p(\sigma_2)$=0.4

$$\max_{s,a} U^*(s,a) =$$
$$\max_{s}\left(\max_{a} P(\Sigma = s) E'_{\mathbf{X}|s}\left[U(a,\mathbf{X})\right] + E'_{\Sigma\backslash s}\left[E'_{\mathbf{X}|\Sigma}\left[U(a^*,\mathbf{X})\right]\right]\right)$$



Decision/event tree for the optimal system choice decision problem
Faber (2005)

Example: Optimal Design

System assumption in repair decision problem:

Consultant A: system 1 (RSR = 1.4), repair will increase RSR to 2.0
Consultant B: system 2 (RSR =2.0), repair has no effect on RSR

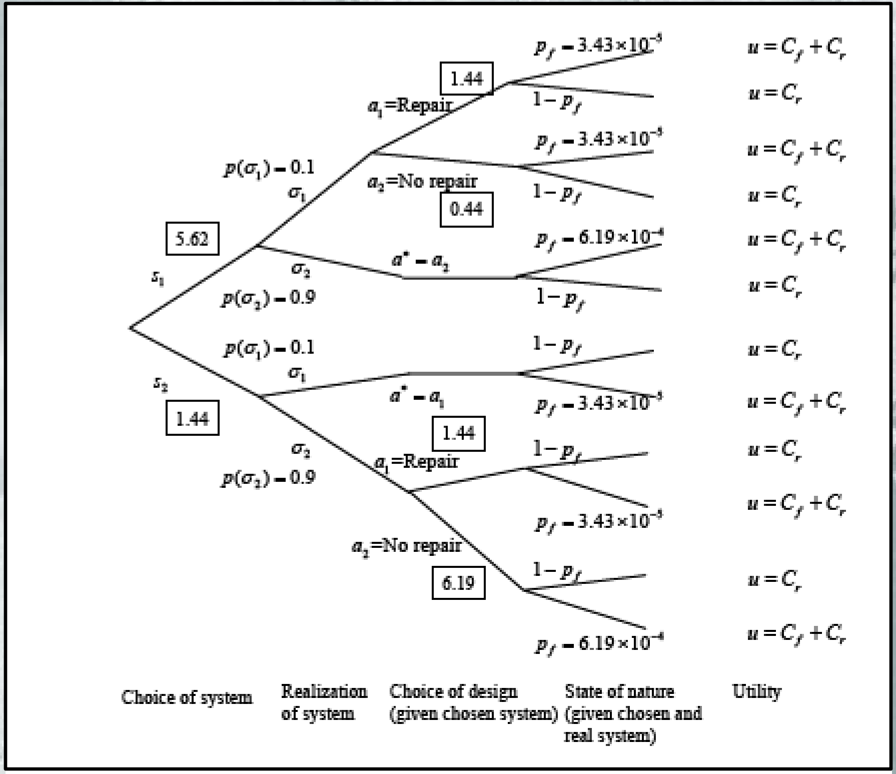
Which consultant to trust?
Repair or not?

Structural collapse: $g(x) = r - b*h^2$

r: resistance,
b: load scale parameter,
h: normalized wave height,
modeled by R, B, H

Collapse failure probability p_f can be found by FORM analysis, C_f and C_r have to be defined, and the degree of believe $P(\sigma_1)$ and $P(\sigma_2)$ assigned

$\Rightarrow$ Optimal System and action can be determined



Decision tree for the repair decision problem
Faber (2005)

Conclusions of the Paper

- Bayesian approaches allow for the integration of frequentistic and subjective information
- Uncertainties are time and space dependent
- Only epistemic component of uncertainty is subject to updating
- Simplification and omissions = informal decision analysis
- Quality of informal decision analyses is difficult to judge
- Understanding and adequate representation of system are prerequisite for the identification of rational decisions
- System representation also involves the choice of prior probability distributions
- Where different system representations are considered: Bayesian decision theory provides a solid basis for optimal choice

Conclusions of the Paper (cont.)

- It is important to consider all uncertainties in informal and formal decision analyses
- Results of informal decision analyses can differ significantly from formal analyses
- With the derived decision theoretical formulation the optimal system may be identified (even for different systems with different prior probabilistic models)

Rockfall Example

Rockfall Example

Road or railway in rockfall area:
What can happen (risk analysis)?
What is OK to happen (risk assessment)?

Swiss protection goals:

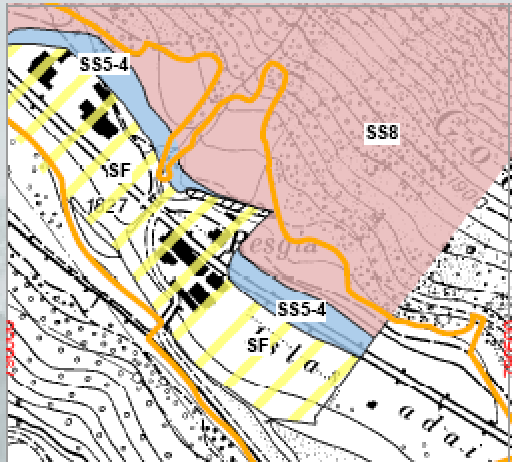
Intensitäten gemäss Bundesempfehlungen   0: Intensität Null   1: schwache Intensität   2: mittlere Intensität   3: starke Intensität

Objektkategorie				Schutzziele [max. zulässige Intensität]		
				Wiederkehrperiode [Jahre]		
Nr.	Sachwerte	Infrastrukturanlagen	Naturwerte	1 – 30	30 – 100	100 – 300
1	Standortsgebundene Bauten, exkl. Sonderobjekte	Skitouren-, Bergtourenrouten (gemäss Karten SAC u.a.)	Ödland, Naturlandschaften	3	3	3
2.1		Wanderwege und Loipen von kant. Bedeutung, Flurwege, Leitungen von kommunaler Bedeutung	Alpweiden	2	3	3
2.2	Unbewohnte Gebäude (Remisen, Weidescheunen u. ä.)	Verkehrswege von kommunaler Bedeutung, Leitungen von kantonaler Bedeutung	Wald mit Schutzfunktion (Waldbau B + C) landwirtschaftl. genutztes Land	2	2	3
2.3	Zeitweise oder dauernd bewohnte Einzelgebäude und Weiler, Ställe	Verkehrswege von kantonaler od. gr. kommunaler Bedeutung, Leitungen von nationaler Bedeutung, Bergbahnen, Zonen für Skiabfahrts- und -übungsgelände			1	2
3.1		Verkehrswege von nationaler od. grosser kantonaler Bedeutung, Ski- und Sessellifte			1	2
3.2	Geschl. Siedlungen, Gewerbe und Industrie, Bauzonen, Campingplätze, Freizeit- und Sportanlagen, allg. Nutzungen mit grossen Menschenansammlungen	Stationen diverser Beförderungsmittel			0	1
3.3	Sonderrisiken bez. besonderer Schadenanfälligkeit oder Sekundärschäden	Sonderrisiken bez. besonderer Schadenanfälligkeit oder Sekundärschäden		Festlegung fallweise		

Low intensity; E<30kJ

Medium intensity; 30<E<300kJ

Rockfall Example

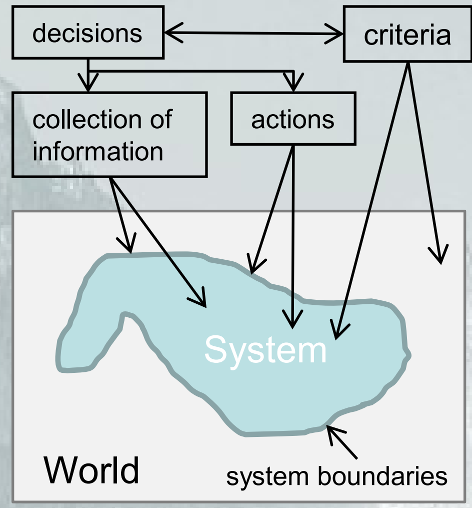


Hazard maps show the endangered areas

Where protection goal is not met:
Protection measures like nets or galleries

Length and strength of net?

Rockfall Example



Decision problem:

System/model definition

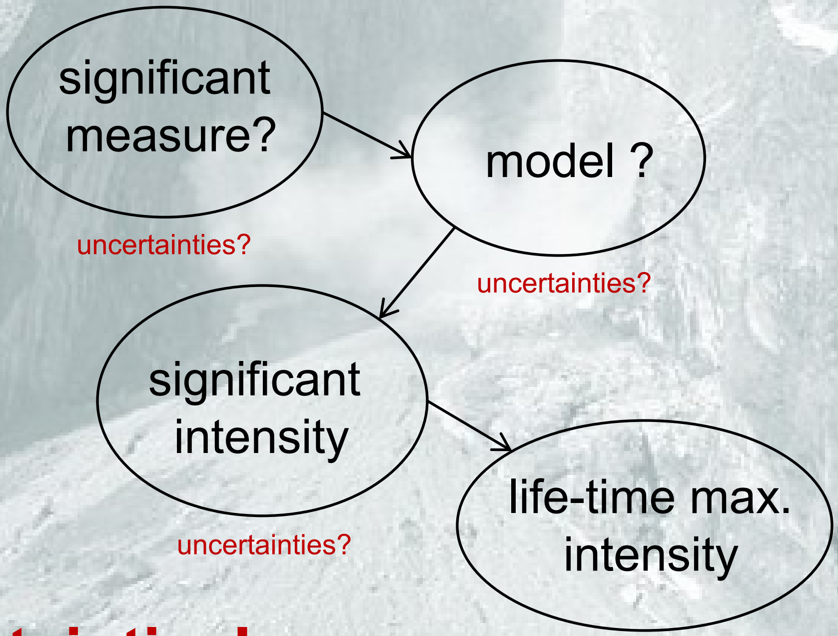
Scale?

Exposure?

Vulnerability?

Robustness?

Model definition



Include all uncertainties!