On the Treatment of Uncertainties and Probabilities in Engineering Decision Analysis

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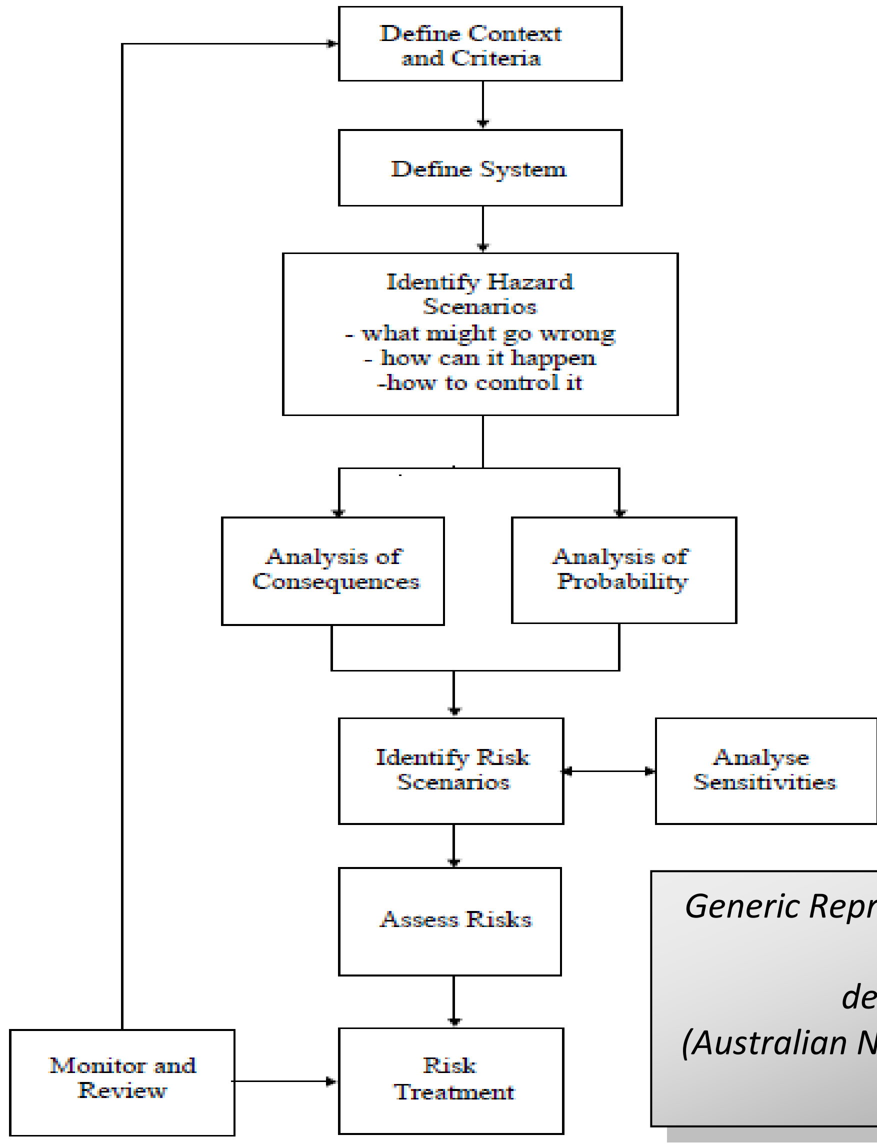
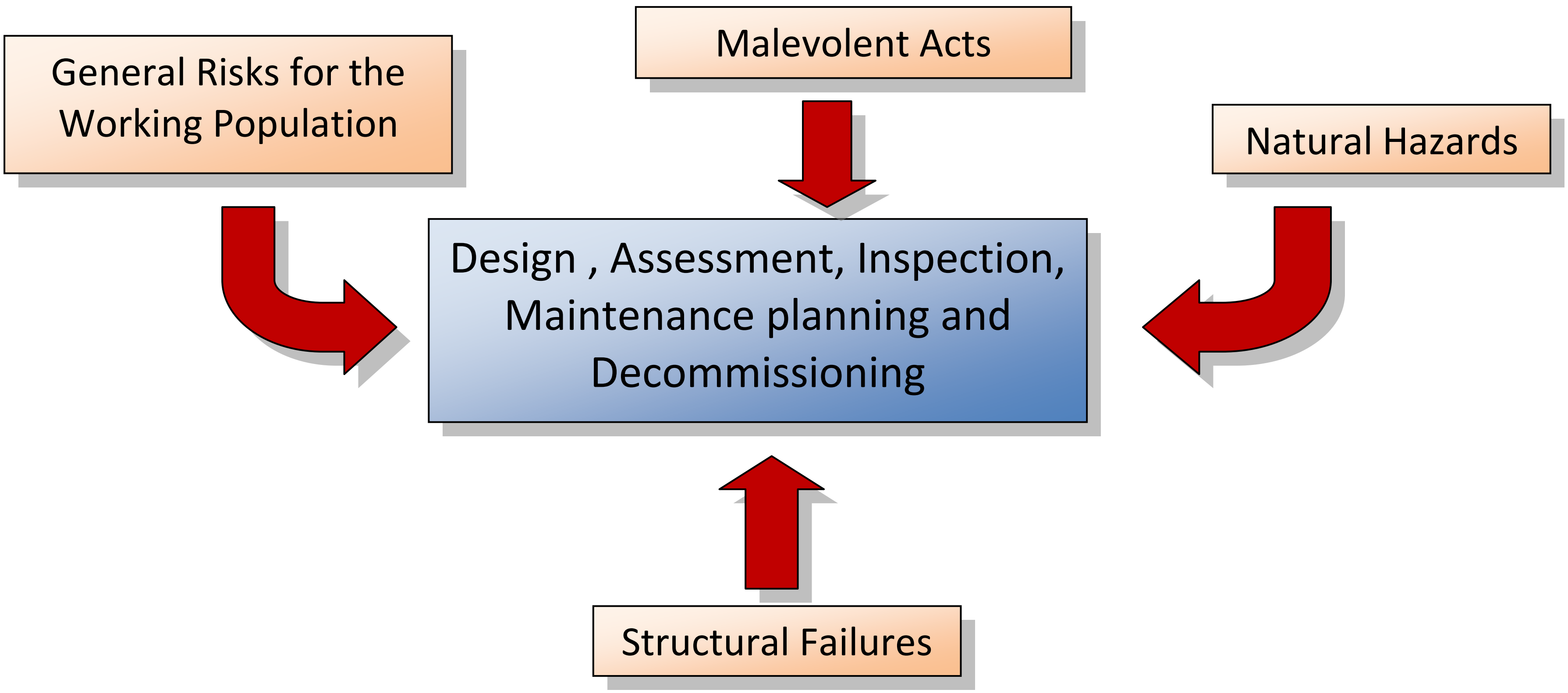
Uncertainties in Engineering Problems (1/2)



Generic Representation of the flow of risk-based decision analysis (Australian New Zealandic code 4369) (1995)

Uncertainties in Engineering Problems (2/2)



Types of Uncertainty

- Uncertainties due to inherent natural variability
- Modeling uncertainties
- Statistical uncertainties

Uncertainties due to Inherent Natural Variability

- Often denoted aleatory

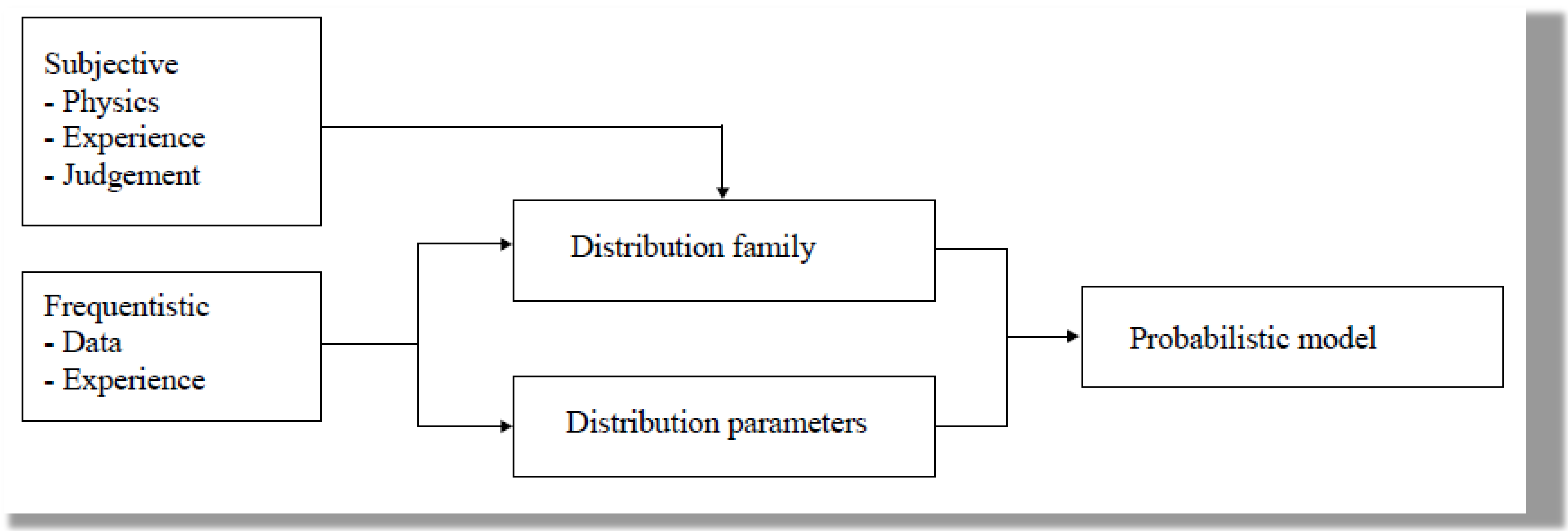
- Often understood as the uncertainty caused by the fact that the universe is not deterministic

- Regardless of the nature of the universe (deterministic or stochastic), this type of uncertainty may be interpreted as the uncertainty that cannot be reduced by means of the collection of additional information

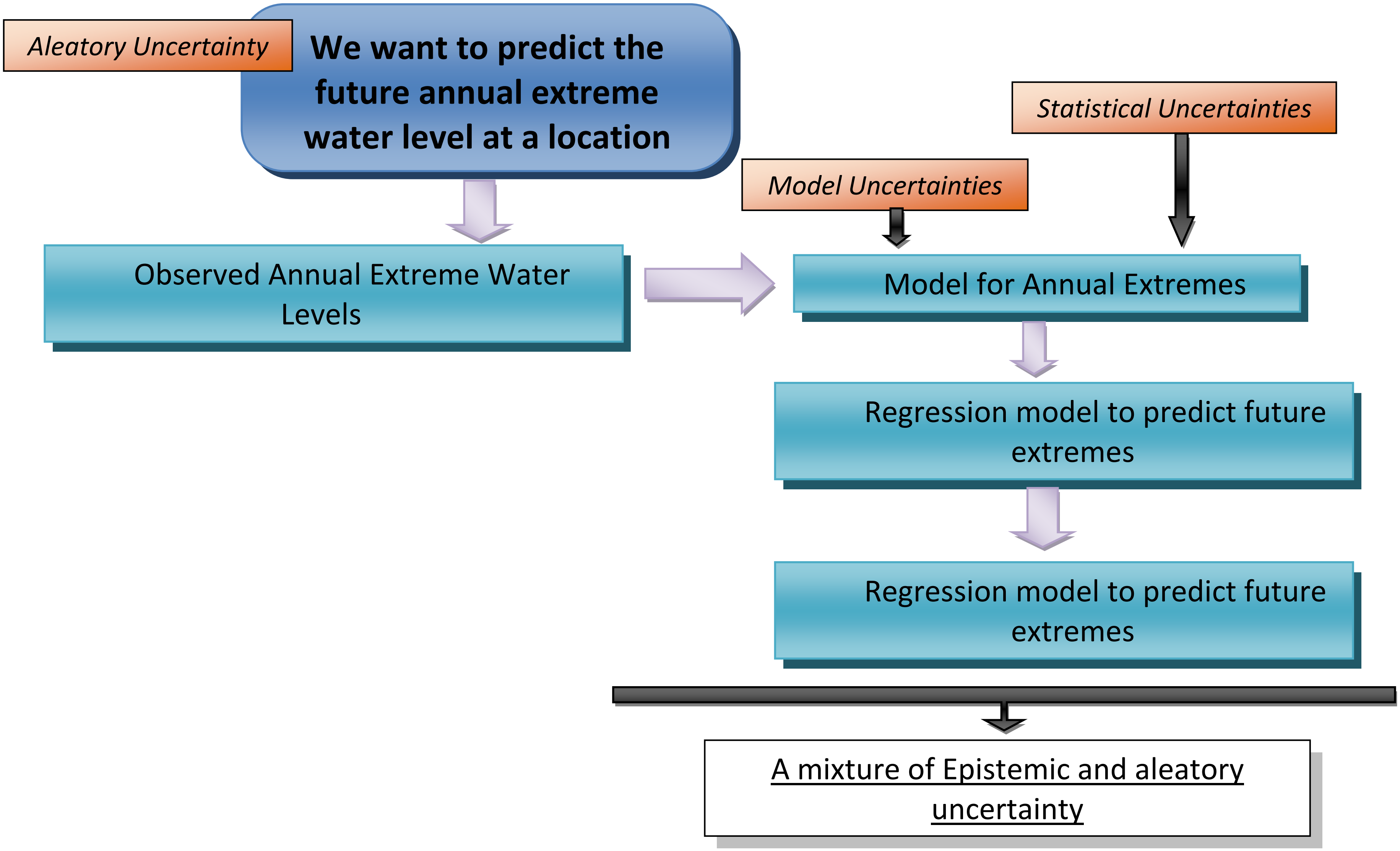
Modeling and Statistical Uncertainties

- Referred to as epistemic

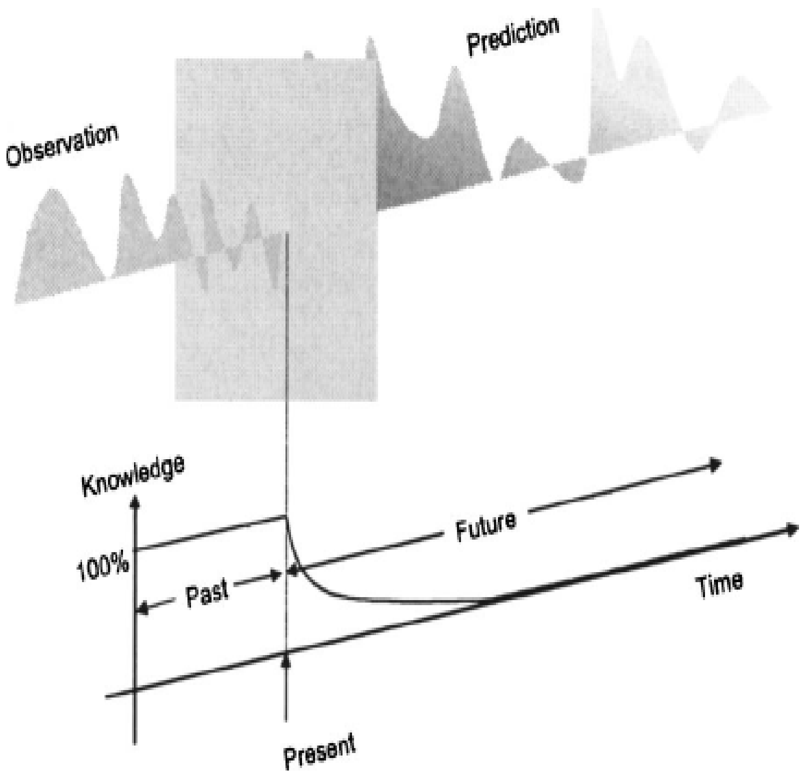
Principle for establishing a probabilistic model



Aleatory to Epistemic ? Are you Serious? (1/2)



Aleatory to Epistemic ? Are you Serious? (2/2)



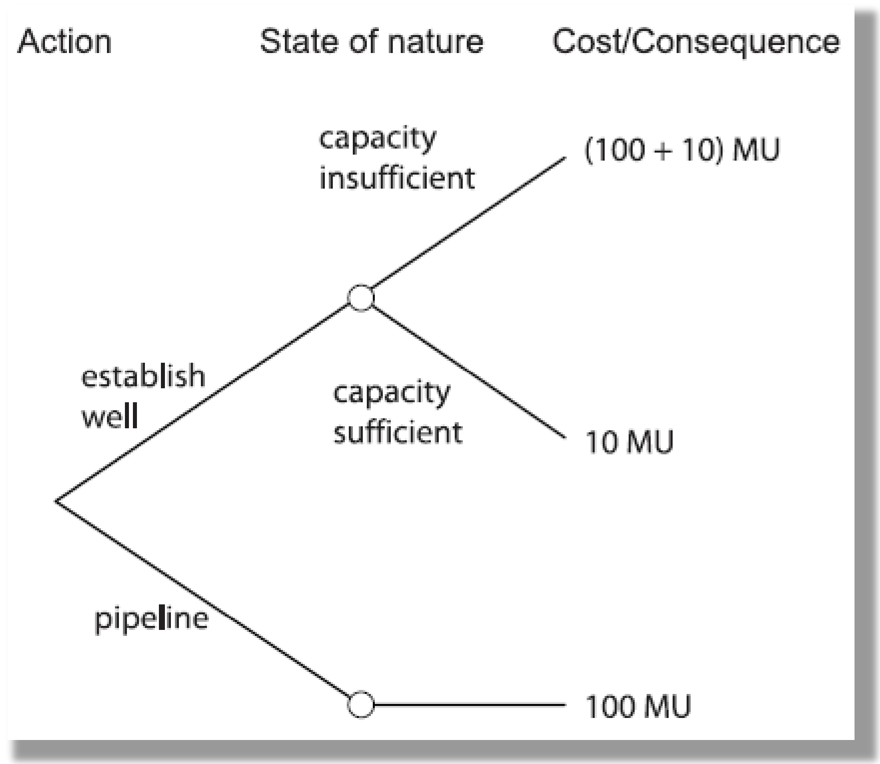
- If observation of a phenomenon is perfect (without any errors), the knowledge obtained is perfect.

- Model concerning the future is a mixture of aleatory and epistemic uncertainty

- But when the modeled phenomenon is observed, we have a purely epistemic uncertainty

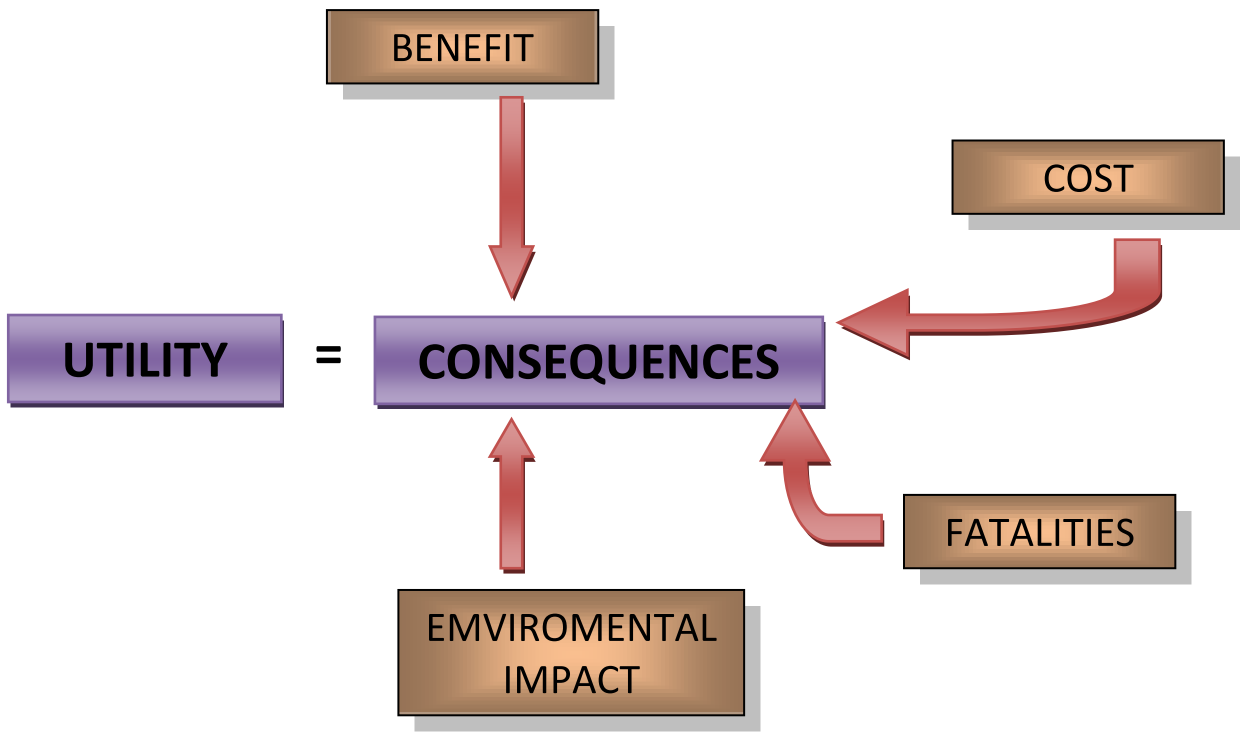
Decision / Event Tree

- In practical decision problems, the number of alternative actions can be extremely large
- A decision tree may conveniently represent the decision problem



Utility

- Optimal decisions are the ones resulting in the highest expected utility



Decision Analysis with Given Information - Prior Analysis (1/2)

- Risk (expected utility) is evaluated on the basis of statistical information and probabilistic modeling available prior to any decision and/or activity

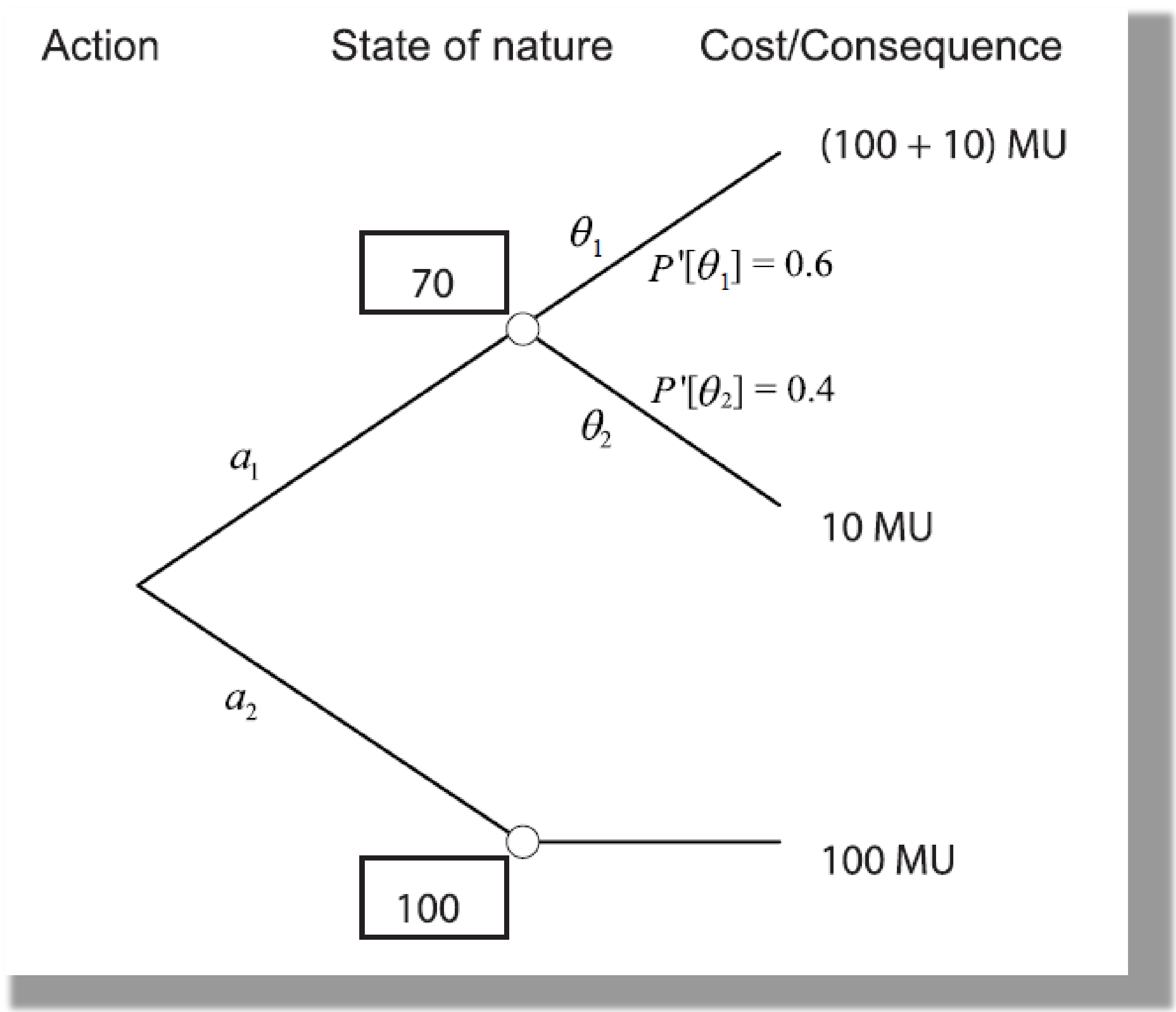
- It forms the basis for the simple comparison of risks associated with different activities

- The risk for each possible decision activity/option is evaluated as:

$$\mathbf{R} = \mathbf{E}[\mathbf{U}] = \sum_{\mathbf{i}=1}^{\mathbf{n}} \mathbf{P_i C_i}$$

R : *the risk*
U : *the utility (e.g. benefits, costs, environmental hazard)*
$\mathbf{P_i}$: *the probability of state (possible outcome) i*
$\mathbf{C_i}$: *the consequence of the event of branch i*

Decision Analysis with Given Information - Prior Analysis (2/2)



<u>Expected Cost</u>

E'[C} = min{C_{a1},C_{a2}}=70 MU

Decision Analysis with Additional Information - Posterior Analysis (1/3)

- In principle, of the same form as the prior decision analysis

- Changes in the branching probabilities and/or the consequences in the decision tree reflect that the considered problem has been changed as an effect of :

  - Risk reducing measures
  - Risk mitigating measures
  - collection of additional information

- Provides a means for updating in the decision analysis

- The most important in engineering applications

Decision Analysis with Additional Information - Posterior Analysis (2/3)

If X is a random variable with state space X_1 , X_2, and D_k is a piece of newly acquired data, then, Bayes' rule states that:

$$\mathbf{P[X_1|D_k] = \frac{P[D_k|X_1]P[X_1]}{P[D_k|X_1]P[X_1] + P[D_k|X_2]P[X_2]}}$$

$$\mathbf{P[X_2|D_k] = \frac{P[D_k|X_2]P[X_2]}{P[D_k|X_1]P[X_1] + P[D_k|X_2]P[X_2]}}$$

Decision Analysis with Additional Information - Posterior Analysis (3/3)

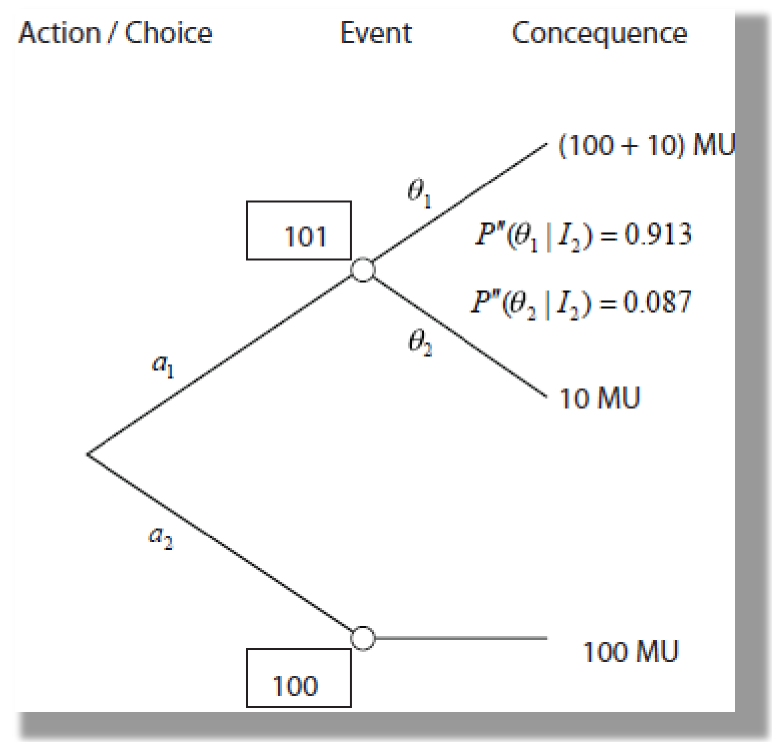
We establish a test well for the price of 1 MU. It's assumed that it can provide us with the following information:

	True capacity of the reservoir	
Indicators	θ_1 : Less than 100	θ_2 : Larger than 100
I_1 : Capacity >105	0.1	0.8
I_2 : Capacity < 95	0.7	0.1
I_3 : 95≤Capacity≤105	0.2	0.1

$P(I_2/\vartheta_1)$

If we assume that a trial pump test was conducted and the indication I_2 was given as a result - that is, the pump test indicated that the capacity of the well was less than 95 water units.

What is the optimal decision now?



$$E''[C \mid I_2] = \min\{P''[\theta_1 \mid I_2]\cdot(100+10) + P''[\theta_2 \mid I_2]\cdot 10;\ 100\}$$
$$= \min\{101.3; 100\} = 100\ MU$$

Considering the additional information (from the trial pump), the optimal decision has switched from a_1 (for the prior analysis), to a_2 !

Decision Analysis with 'Unknown' Information - Pre-posterior Analysis (1/4)

- Can be interpreted as a posterior decision analysis made before new information is actually collected

- Should be implemented when the decision maker has the possibility to "buy" additional information, through a conducted experiment.

- If the cost of the experiment (that is the cost of the provided information), is small in comparison to the potential value of the information (that is how much the acquired info contributes to the maximization of the corresponding utility), the decision maker should perform the experiment.

- If several types of information-sources (aka experiments) are available, the decision maker must choose the experiment yielding the overall largest utility

Decision Analysis with 'Unknown' Information - Pre-posterior Analysis (2/4)

Let's assume that the cost of the trial pump experiment is C_p. The likelihood of the true capacity of the reservoir given the trial pump results is given in the following table:

	True capacity of the reservoir	
Indicators	θ_1 : Less than 100	θ_2 : Larger than 100
I_1 : Capacity >105	0.1	0.8
I_2 : Capacity < 95	0.7	0.1
I_3 : 95≤Capacity≤105	0.2	0.1

<u>QUESTION</u>

To perform or not to perform?

That is the question.

Decision Analysis with 'Unknown' Information - Pre-posterior Analysis (3/4)

In this situation, the expected cost is given by:

$$
\begin{aligned}
E[C] &= E''[C \mid I_1]P'[I_1] + E''[C \mid I_2]P'[I_2] + E''[C \mid I_3]P'[I_3] \\
&= (25.8 + C_P)\cdot 0.38 + (100 + C_P)\cdot 0.46 + (85 + C_P)\cdot 0.16 \\
&= (69.4 + C_P)\, MU
\end{aligned}
$$

$$
\begin{aligned}
P'[I_1] &= P[I_1|\theta_1]P'[\theta_1] + P[I_1|\theta_2]P'[\theta_2] = 0.1\cdot 0.6 + 0.8\cdot 0.4 = 0.38 \\
P'[I_2] &= P[I_2|\theta_1]P'[\theta_1] + P[I_2|\theta_2]P'[\theta_2] = 0.7\cdot 0.6 + 0.1\cdot 0.4 = 0.46 \\
P'[I_3] &= P[I_3|\theta_1]P'[\theta_1] + P[I_3|\theta_2]P'[\theta_2] = 0.2\cdot 0.6 + 0.1\cdot 0.4 = 0.16
\end{aligned}
$$

$$
\begin{aligned}
E''[C \mid I_1] &= \min\{P''[\theta_1 \mid I_1]\cdot(100+10) + P''[\theta_2 \mid I_1]\cdot 10;\ 100\} \\
&= \min\{0.158\cdot 110 + 0.842\cdot 10;\ 100\} \\
&= \min\{25.8; 100\} = 25.8\ MU
\end{aligned}
$$

$$
\begin{aligned}
E''[C \mid I_2] &= \min\{P''[\theta_1 \mid I_2]\cdot(100+10) + P''[\theta_2 \mid I_2]\cdot 10;\ 100\} \\
&= \min\{0.913\cdot 110 + 0.087\cdot 10; 100\} \\
&= \min\{101.3; 100\} = 100\ MU
\end{aligned}
$$

$$
\begin{aligned}
E''[C \mid I_3] &= \min\{P''[\theta_1 \mid I_3]\cdot(100+10) + P''[\theta_2 \mid I_3]\cdot 10;\ 100\} \\
&= \min\{0.75\cdot(100+10) + 0.25\cdot 10; 100\} \\
&= \min\{85; 100\} = 85\ MU
\end{aligned}
$$

Decision Analysis with 'Unknown' Information - Pre-posterior Analysis (4/4)

The expected cost corresponding to the prior information was derived to be :

$$E'[C] = 70 \text{ MU}$$

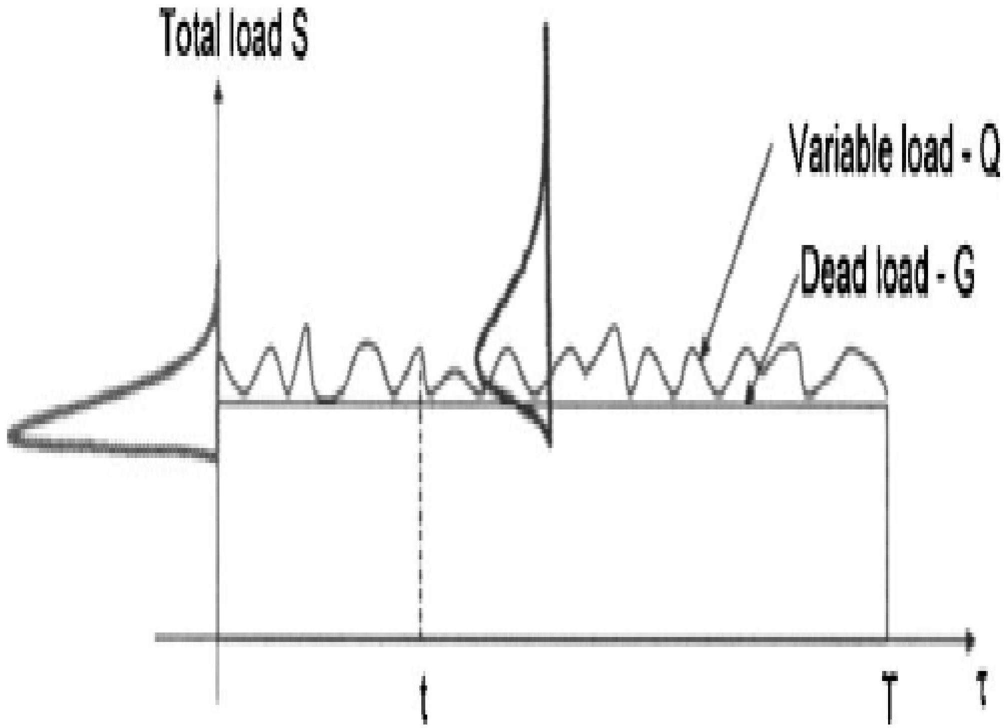
Comparing it with the expected cost corresponding to the pre-posterior analysis :

$$E'[C] - E[C] = 70 - (69.4+C_p) = 0.6 - C_p$$

Conclusion

If they ask you more than 0.6 MU for the trial pump , politely decline and spend the difference in beers, while establishing a new well

Probability Updating



- Load events in consecutive time intervals may be highly dependent due to, e.g., a dominating dead load component

- Before the dead load is realized, the loading in the future might be subject to aleatory uncertainty only

- As soon as the dead load component is realized, a large part of the uncertainty associated with the future loading becomes epistemic

- If in the representation of the adverse event and the updating event types of uncertainty and the temporal dependency is not consistently taken into account, the results may become erroneous and nonphysical