Hierarchical Modeling of Pipeline Defect Growth Subject to ILI Uncertainty

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background

deterioration of pipelines as a result of internal and external corrosion processes
-> subject to uncertainties:

- material uncertainties
- uncertainties in external influences (loading, environment)
- uncertainties in operating conditions
- various spatial and temporal uncertainties
- inspection uncertainties
- modeling uncertainties

NY-times online; largest U.S. oilfield in Alaska

www.exponent.com

background

challenges in life-time inspection

1. imprecise inspection data to estimate the progress of deterioration
2. heterogeneity in the degradation paths of distinct corrosion features
3. many types of corrosion can occur simultaneously such as internal pitting corrosion and micro-bacterial corrosion -> use of flexible stochastic processes.

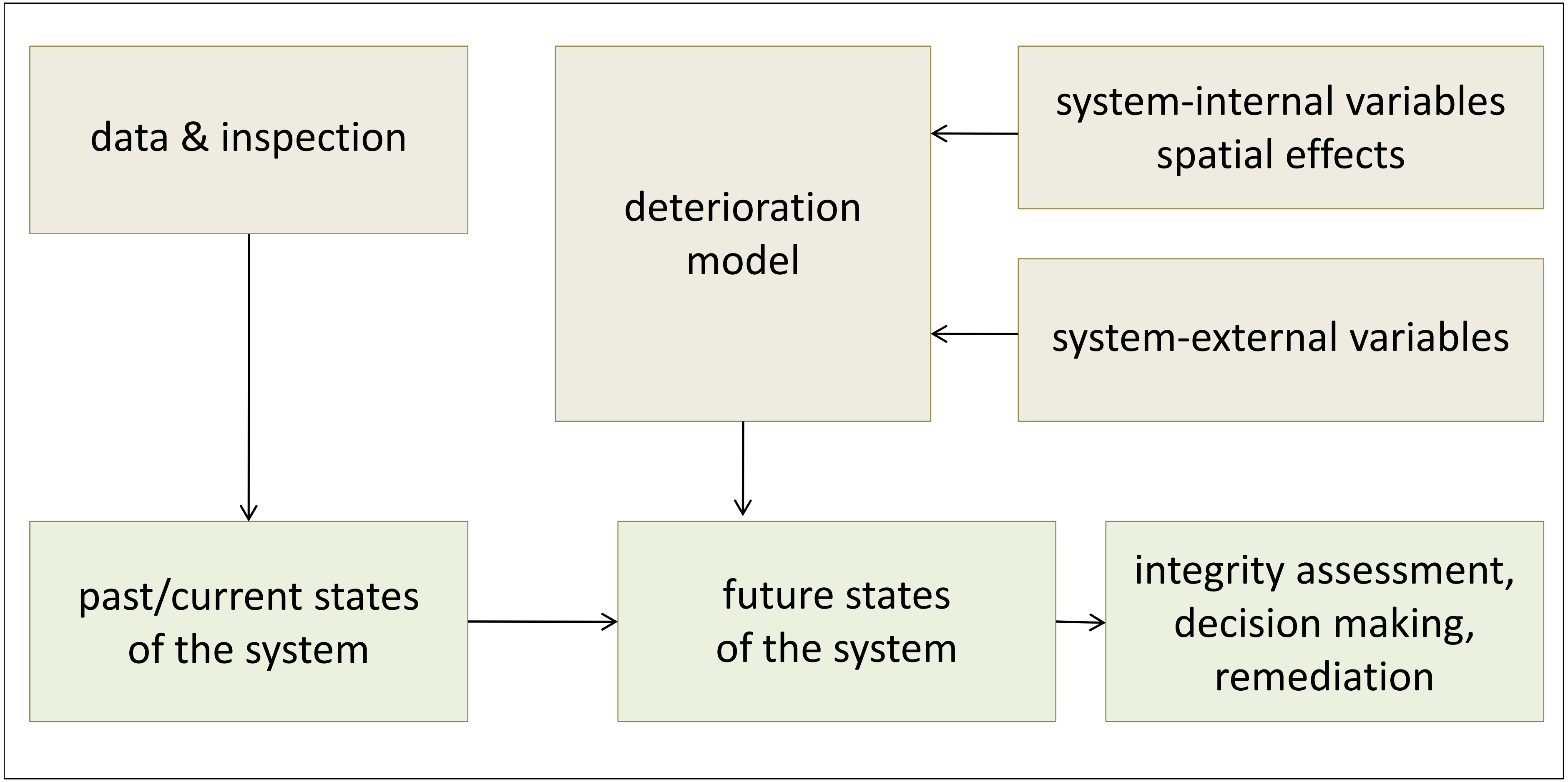
inline inspection using an intelligent pig can only be performed infrequently and suffers from various uncertainties

www.ceconet.com

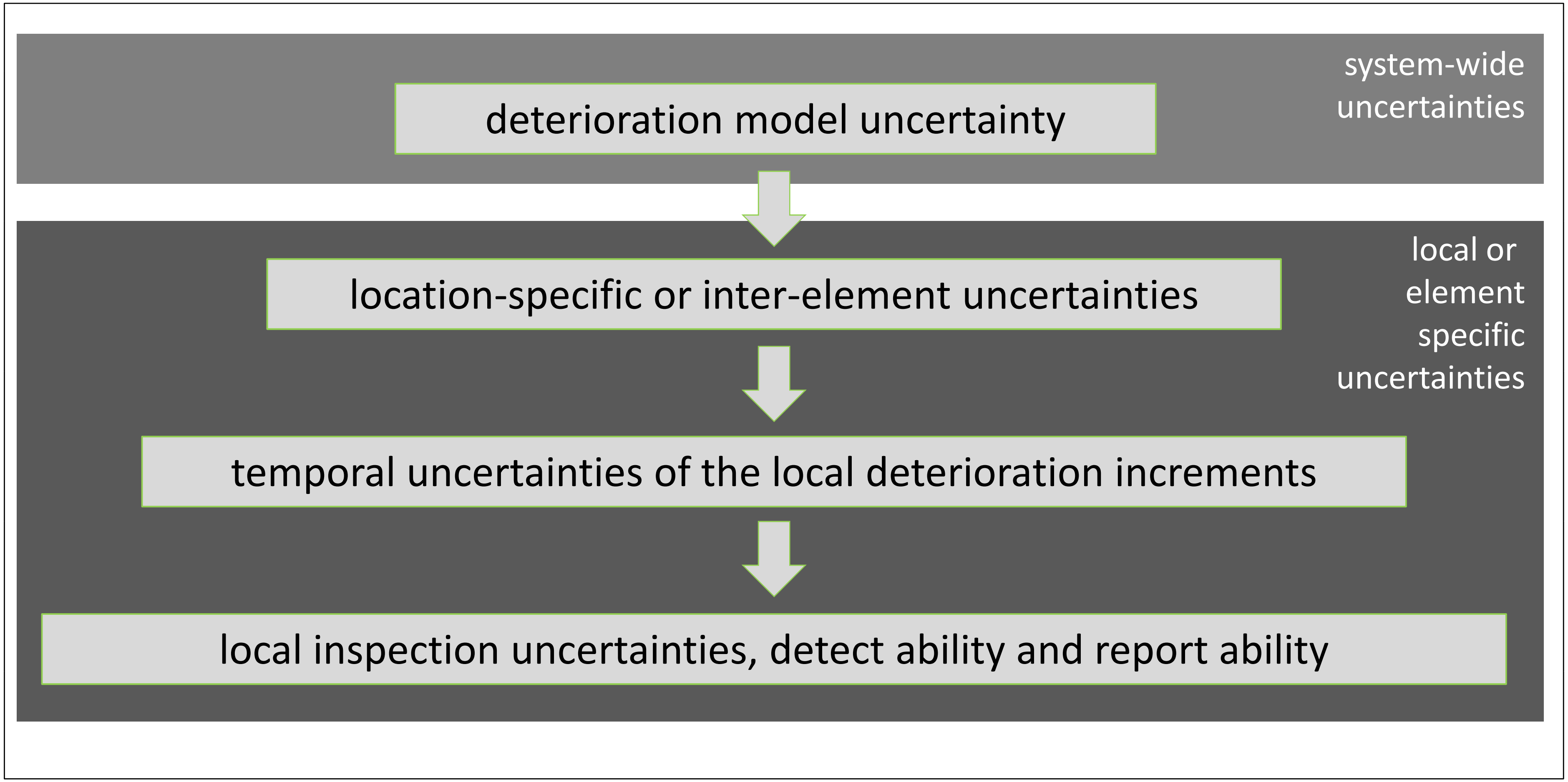
www-robot.mes.titech.ac.jp

challenges in lifetime-decision making

"Given imprecise past and current data, how can we estimate the future integrity and lifetime of the system in response to variables internal and external to the system?"



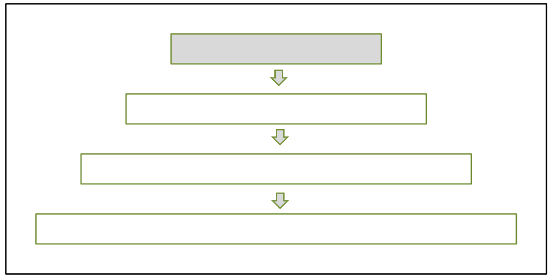
hierarchical representation of uncertainties



Top level represents system-wide uncertainties.
Three inferior levels apply to each local point considered in the analysis.

top level:

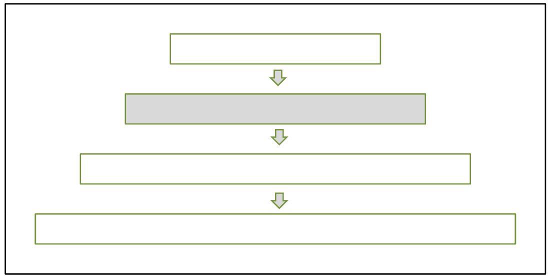
deterioration model uncertainties



- captures all model uncertainties
  - relating to corrosion model
  - uncertain mechanistic or phenomenological aspects
  - are common to all points within a pipeline segment
  - epistemic uncertainties -> reflect assumed or postulated behaviour

second level:

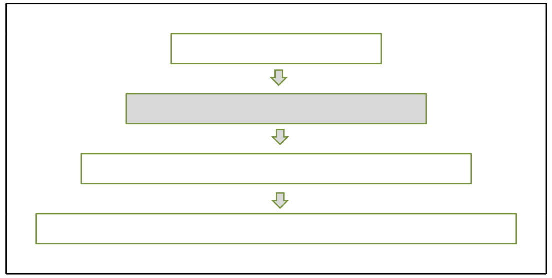
location-specific or inter-element uncertainties



- some deterministic -> explained by local variations of internal or external covariates (temperature, exposure, stress, material susceptibility)
- cause and effect relationships also subject to model uncertainties

second level:

location-specific or inter-element uncertainties



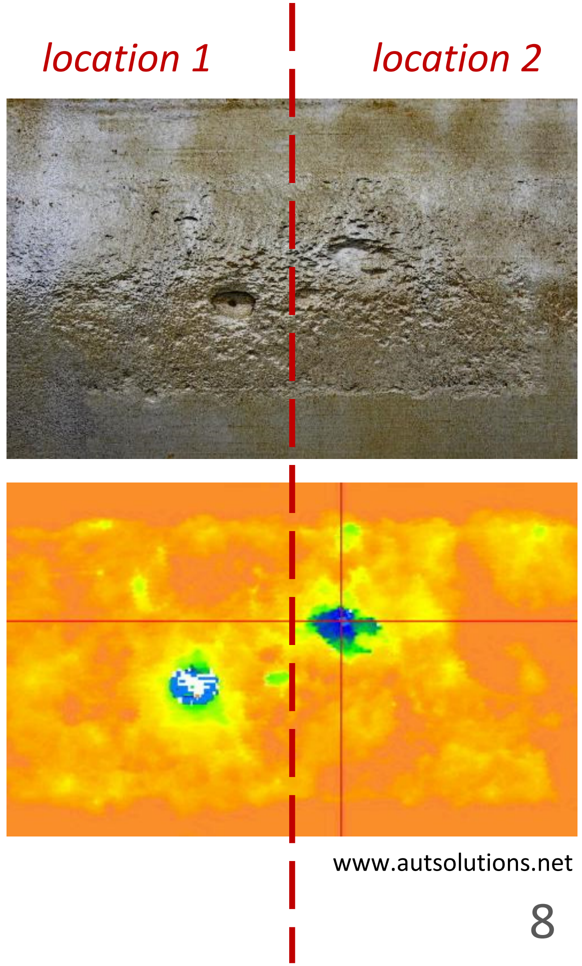
spatial variation

stochastic corrosion model
-> 2 steps of classifying “local” variation:

- explained by variation of the other local parameters
= “COVARIATES” (include material property fluctuations, exposure, operating conditions)

+

- additional local aleatory effects which cannot be explained by 1.



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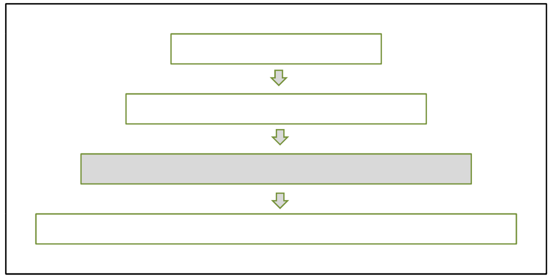
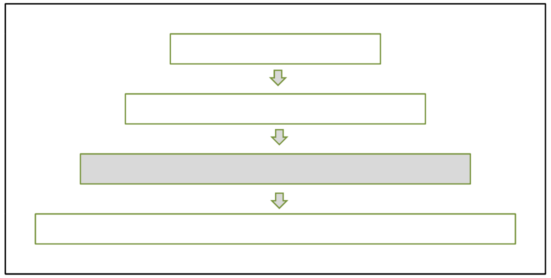
third level:

temporal uncertainties of the local deterioration increments



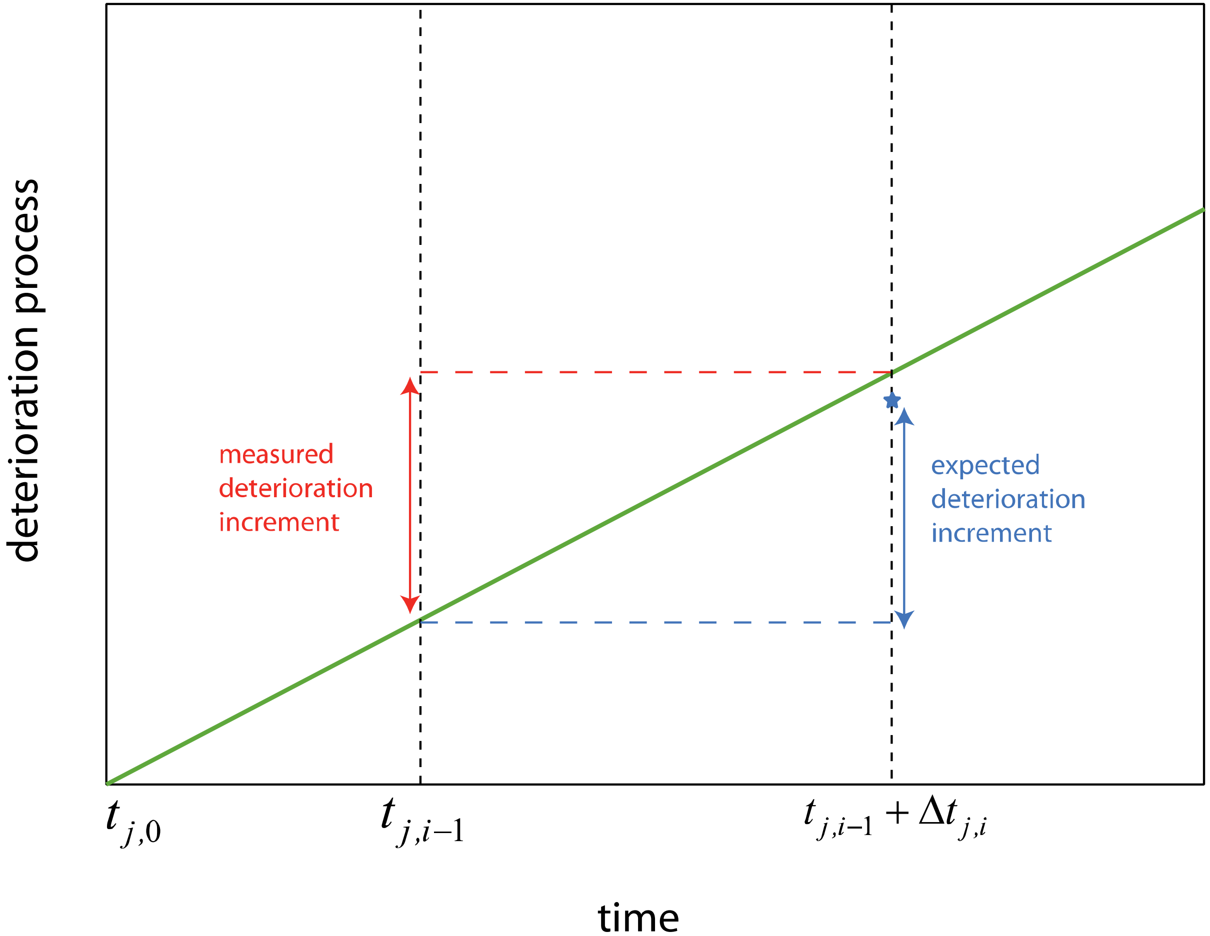
Given the uncertainties of the two levels above, ...

- ... any prediction of the degree of deterioration $X(t+\Delta t)$ given its state $X(t)$ at time t, is still subject to temporal uncertainty.
- ...the quantity $X(t+\Delta t)-X(t)$ is a random variable.
- -> serial correlation (typical for any random process)
  -> refute of the traditional use of simple regression models



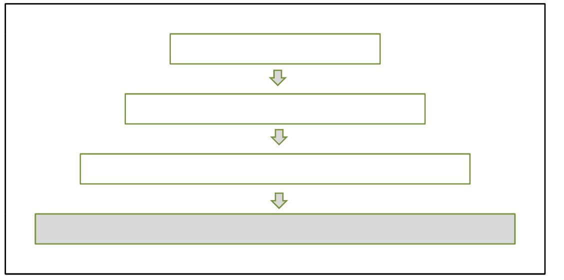
temporal uncertainties of the local deterioration increments

at one particular location j

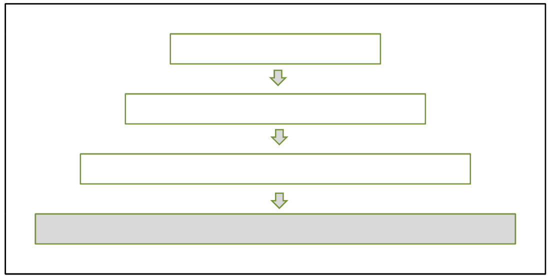


lowest level:

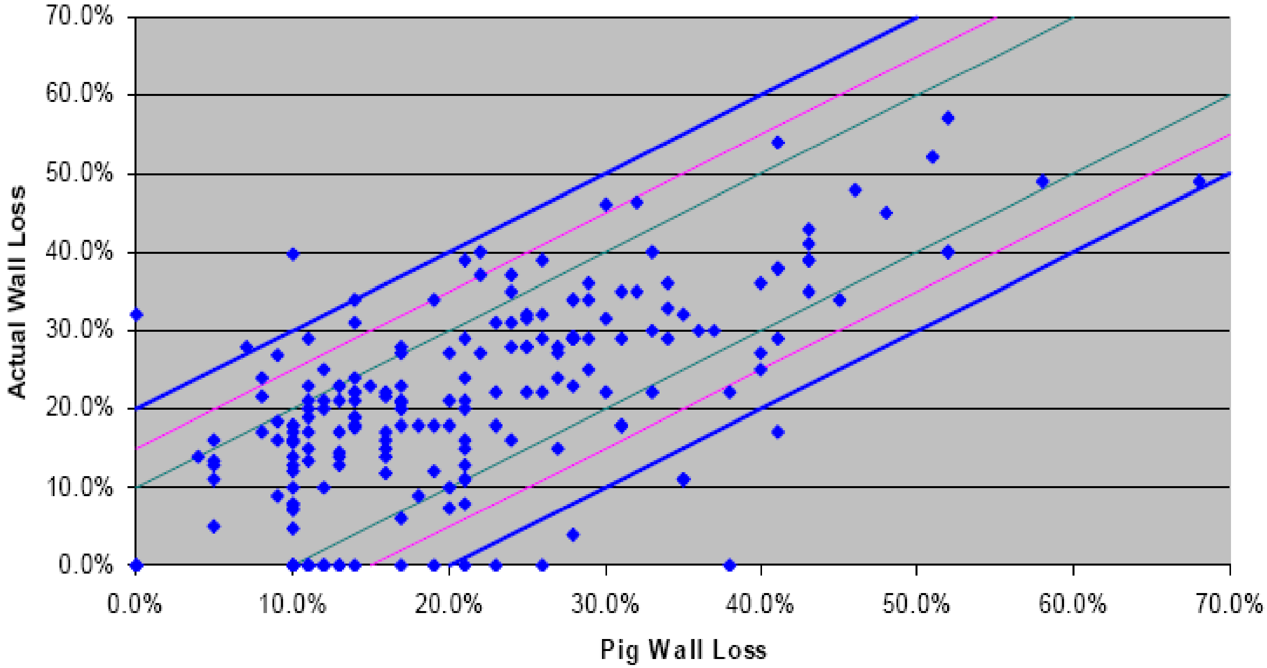
local inspection uncertainties, detect ability and report ability

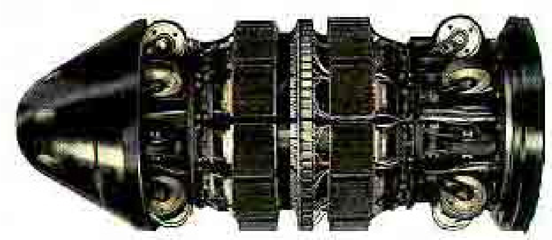


- represents observational uncertainties
- imprecise measurement and inspection
- -> $X'(t)$ is observed instead of the true but unobservable $X(t)$, both bivariate stochastic processes
- values of $X'(t)$ at discrete points in time $t_1, t_2, \ldots, t_n$
- conditional distribution of $X'(t_i)$ given $X(t_i)$ for all $i = 1, \ldots, n$
- $X'(t)$ accounts for detection, interpretation and reporting defects.



Actual % pipeline wall loss due to internal corrosion versus measured % wall loss as observed by high resolution Magnetic Flux Leakage (MFL) in-line-inspection pig

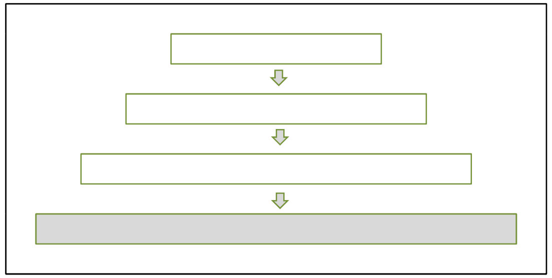




www.ndt-ed.org

-> considerable uncertainties are associated with in situ in-line-inspection defect reports

lowest level:



local inspection uncertainties, detect ability and report ability

3 effects errors:

a) sizing b) detect ability c) report ability

inspection devices measure an inaccurate defect size: $X_{M,j,i}$

actual true deterioration: $X_{j,i}$ at one of m specific locations $j\left(i=1,\ldots,n_j\right)$ and at one of n_j inspections $i\left(i=1,\ldots,n_j\right)$ in time $t_{j,i}$.

$$X_{M,j,i} = X_{j,i} + \varepsilon_{j,i}$$ → *measurement error due to inaccurately observed defect size*

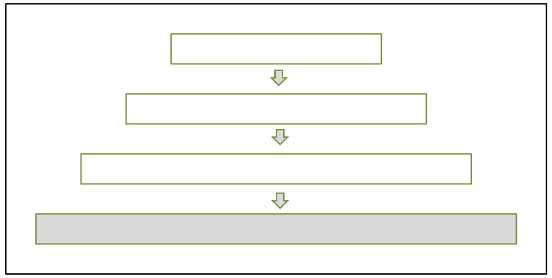
$$\varepsilon_{j,i} \middle| \mathbf{C}_{\varepsilon,j}, j \sim multi - normal\left(0, \mathbf{C}_{\varepsilon,j}\right)$$

↓ *known variance-covariance matrix*

Errors at given locations are usually correlated due to device bias and algorithmic interpretation.

lowest level:

local inspection uncertainties, detect ability and report ability



3 effects errors:

a) sizing b) detect ability c) report ability

Probability of detection: PD .

Depending on size of deterioration, defects may not be properly detected.

The larger the defect the higher the probability that the defect will be detected.

undetected defect: $1-PD\left(X_{j,i}\right)$

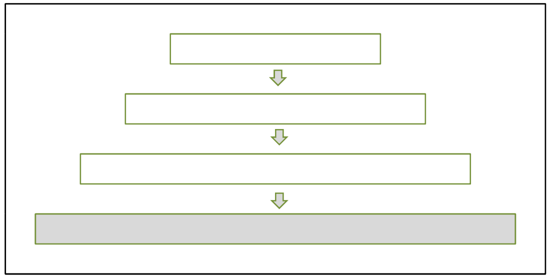
detection indicator variable: $D_{j,i}\left|PD\left(X_{j,i}\right) \sim bernoulli\left(PD\left(X_{j,i}\right)\right)\right.$

successful detection: $D=1$

observable deterioration: $X_{o,j,i}=D_{j,i}\cdot X_{M,j,i}$

detection threshold: x_T

lowest level:



local inspection uncertainties, detect ability and report ability

3 effects errors:

a) sizing b) detect ability c) report ability

The deterioration reported by the inspection device as the actually observed deterioration corresponds to:

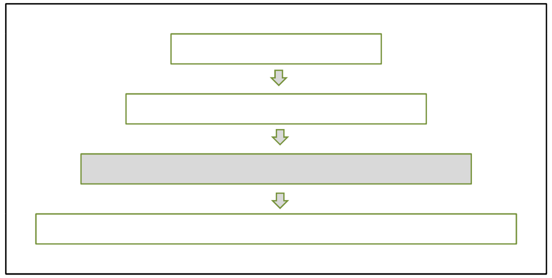
$$X_{R,j,i} = R_{j,i} \cdot X_{o,j,i}$$

$$R_{j,i}\left(X_{o,j,i}\right) = \begin{cases} 0 \; if \; X_{o,j,i} < x_T \\ 1 \; if \; X_{o,j,i} \geq x_T \end{cases}$$

observable deterioration (cp. b) *detect ability)*

detection threshold: x_T

third level:



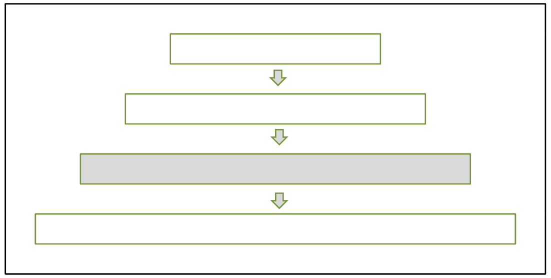
temporal uncertainties of the local deterioration increments

assumptions:

- actual deterioration is sum of previous inspection deterioration and increment between last two inspection times.
- at the beginning of process deterioration is 0 with probability 1
- deterioration increments are independent
- increments positive
- -> deterioration modeled as gamma process

$$\Delta X_{j,i} \left| \Delta\alpha_{j,i}, \beta_j \sim gamma\left(\Delta\alpha_{j,i}, \beta_j\right)\right.$$

third level:



temporal uncertainties of the local deterioration increments

$$\Delta X_{j,i} \left| \Delta\alpha_{j,i}, \beta_j \sim gamma\left(\Delta\alpha_{j,i}, \beta_j\right)\right.$$

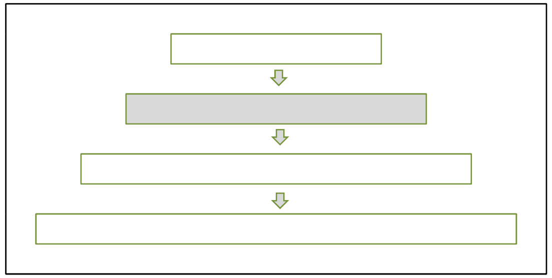
$\Delta X_{j,i}$ = deterioration increment = $X_{j,i} - X_{j,i-1}$

difference of actual deterioration and deterioration at previous inspection

$\Delta\alpha_{j,i}$ = shape param. of gamma distr. = $\theta_1 \left\{ \left(t_{j,i-1} + \Delta t_{j,i-1}\right)^{\theta_2} - \left(t_{j,i-1}\right)^{\theta_2} \right\}$

reflects increase of deterioration as a function of time depending on the physics of the process; θ_1 and θ_2 are system-wide deterioration model epistemic uncertainties -> top level; exponent $\theta_2 > 1$ denotes an accelerating process, $\theta_2 < 1$ implies a decelerating process over time;

second level:



location-specific or inter-element uncertainties

$$\Delta X_{j,i} \left| \Delta\alpha_{j,i}, \beta_j \sim gamma\left(\Delta\alpha_{j,i}, \beta_j\right)\right.$$

gamma distribution is parameterized by: $f\left(x|\alpha,\beta\right)=\left(\frac{1}{\beta}\right)^{\alpha}\cdot x^{\alpha-1}\cdot e^{-x/\beta}/\Gamma\left(\alpha\right)$

β_j is the positive location specific parameter representing the heterogeneity between the locations.

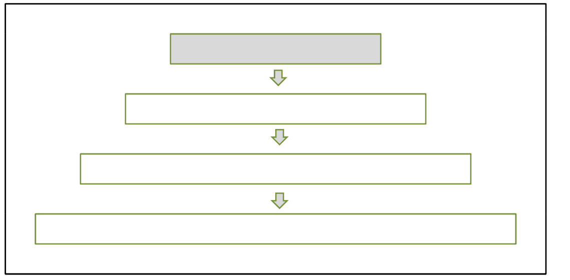
$$\beta_j = \exp\left\{\mathbf{z}_j^{\ T}\cdot\boldsymbol{\kappa}+\xi_j\right\}$$

$\mathbf{z}_j = \left\{1, z_{j,1}, z_{j,2}, \ldots, z_{j,q}\right\}^T$ *(q+1)-vector of local covariates aiming to explain the heterogeneity between locations*

$\boldsymbol{\kappa}_j = \left\{\kappa_0, \kappa_1, \ldots, \kappa_q\right\}^T$ *(q+1)-vector of cause-effect regression coefficients associated with* $\mathbf{z}_j$

$\xi_j \left| \sigma_\xi^2 \sim normal\left(0, \sigma_\xi^2\right)\right.$ *local random effect representing local aleatory effects which cannot be explained by the covariates of* $\mathbf{z}_j$

top level:



system-wide deterioration model uncertainties

- time and shape variables θ_1 and θ_2
- random effects variance σ_ξ^2
- cause effect variable vector $\boldsymbol{\kappa}$

prior distributions can be implemented

equivalent Log-Likelihood method (ELL)

Objective:

Parameter estimation by maximizing a *pseudo log-likelihood function* l_j

= simplified version of the actual HB log-likelihood function
-> equivalent log-likelihood

Gamma distributed independent deterioration increments $\Delta X_{j,i}$ at location j at inspection i are replaced by normal independent increments having the same first and second moments as the gamma increments:

$$\Delta X_{j,i} \left| \Delta\alpha_{j,i}, \beta_j \sim normal\left(\Delta\alpha_{j,i}\beta_j, \Delta\alpha_{j,i}\beta_j^2\right)\right.$$

equivalent Log-Likelihood method (ELL)

the observed increments $\Delta\mathbf{X}_{\mathbf{M,j}}$ at any location j are multi-normally distributed with a mean vector $\Delta\mathbf{X}_{\mathbf{M,j}} = \left\{\Delta X_{j,1}, \Delta X_{j,2}, \ldots, \Delta X_{j,nj}\right\}^T$

The measurement errors $\varepsilon_{j,i}$ and the corresponding temporal increments $\Delta\varepsilon_{j,i} = \varepsilon_{j,i} - \varepsilon_{j,i-1}$ have zero means and a location-specific variance-covariance matrix of the measurement error increments $\mathbf{C}_{\boldsymbol{\Delta\varepsilon},\mathbf{j}}$.

$$\boldsymbol{\Delta}\mathbf{X}_{\mathbf{M,j}} \left| \boldsymbol{\Delta}\mathbf{X}_{\mathbf{j}}, \mathbf{C}_{\boldsymbol{\Delta\varepsilon},\mathbf{j}} \sim multi-normal\left(\boldsymbol{\Delta}\mathbf{X}_{\mathbf{j}}, \mathbf{C}_{\boldsymbol{\Delta\varepsilon},\mathbf{j}}\right)\right.$$

↓

covariance matrix of the measurement error increments

equivalent Log-Likelihood method (ELL)

the observed increments $\Delta\mathbf{X}_{\mathbf{M,j}}$ have an n_j-mean-vector $\boldsymbol{\Delta\alpha}_{\mathbf{j}}\beta_j$ with $\boldsymbol{\Delta\alpha}_{\mathbf{j}} = \left\{\Delta\alpha_{j,1},\ldots,\Delta\alpha_{j,nj}\right\}^T$ and a variance-covariance matrix $\mathbf{C}_{\mathbf{\Delta X,M,j}}$.

shape and scale parameters of deterioration increments

↑

$$\mathbf{\Delta X}_{\mathbf{M,j}} \left| \mathbf{\Delta}\alpha_{\mathbf{j}}, \beta_j, \mathbf{C}_{\mathbf{\Delta X,M,j}} \right. \sim multi-normal\left(\mathbf{\Delta}\alpha_{\mathbf{j}}\beta_j, \mathbf{C}_{\mathbf{\Delta X,M,j}}\right)$$

$\mathbf{C}_{\mathbf{\Delta X,M,j}}$ is the sum of $\mathbf{C}_{\mathbf{\Delta X,j}}$ and $\mathbf{C}_{\mathbf{\Delta \varepsilon,j}}$.

↓ *deterioration increment*

↓ *measurement error increment*

equivalent Log-Likelihood method (ELL)

as $\boldsymbol{\Delta\alpha_j}, \beta_j$ and $\mathbf{C_{\Delta X,M,j}}$ are functions of the unknown hyperparameters θ_1 , θ_2 , $\boldsymbol{\kappa}$ and the random effect ξ_j , the corresponding log-likelihood function l_j **at one location** j is given by:

$$l_j\left(\theta_1,\theta_2,\boldsymbol{\kappa},\xi_j \middle| \boldsymbol{\Delta X}_{M,j}\right) \propto$$

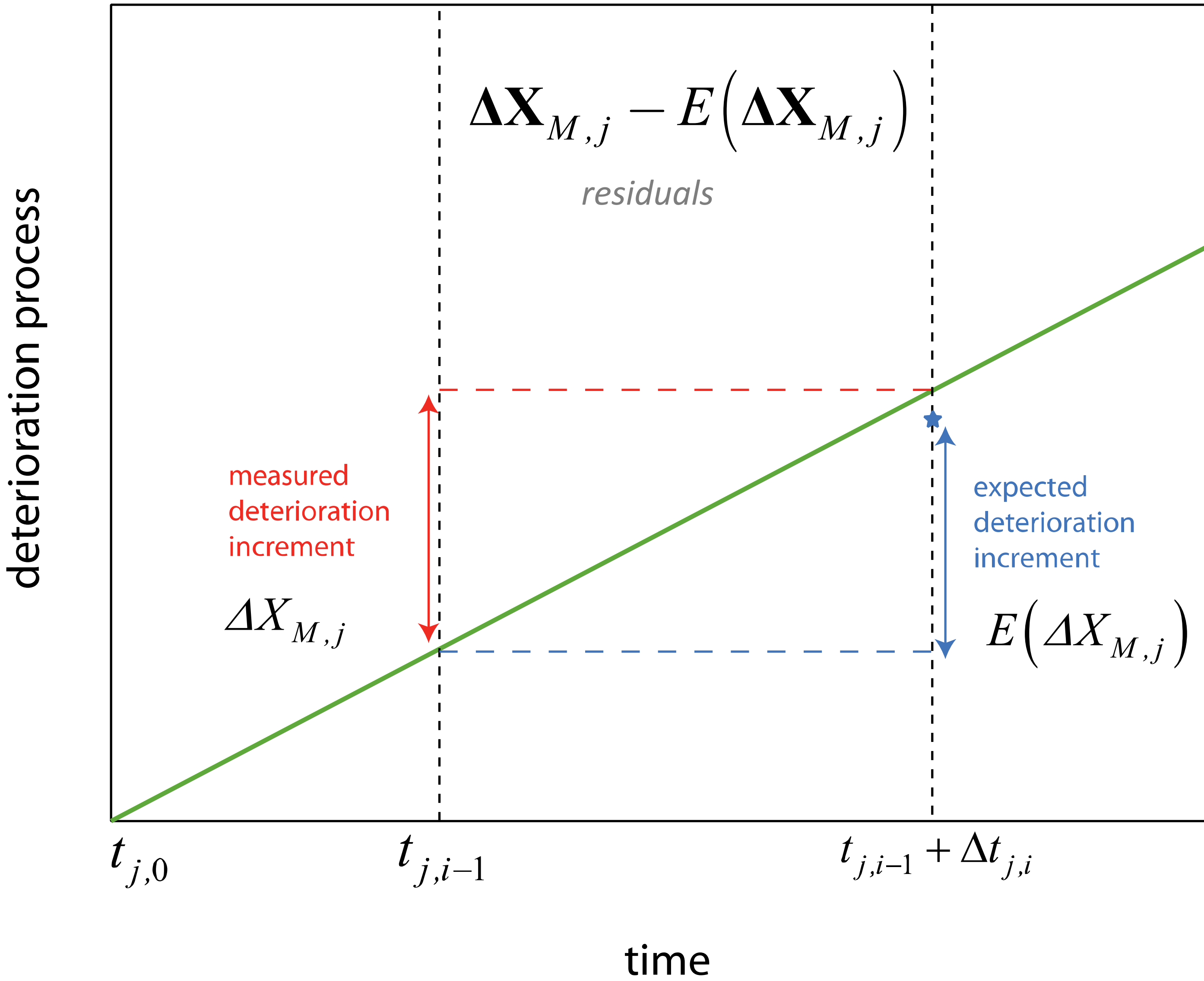
$$-\frac{1}{2}\left\{log\left|\mathbf{C_j}\right| + \left[\boldsymbol{\Delta X}_{M,j} - E\left(\boldsymbol{\Delta X}_{M,j}\right)\right]^T \cdot \mathbf{C_j}^{-1} \cdot \left[\boldsymbol{\Delta X}_{M,j} - E\left(\boldsymbol{\Delta X}_{M,j}\right)\right]\right\}$$

↓ *variance-covariance matrix of observed deterioration increments*

where

$$E\left(\boldsymbol{\Delta X}_{M,j}\right) = \underbrace{\theta_1\left[\left(t_{j,i-1}+\Delta t_{j,i}\right)^{\theta_2} - \left(t_{j,i-1}\right)^{\theta_2}\right]}_{\Delta\alpha} \underbrace{exp\left(\mathbf{z}_j^T\boldsymbol{\kappa}+\xi_j\right)}_{\beta}$$

equivalent Log-Likelihood method (ELL)



equivalent Log-Likelihood method (ELL)

extension **for all locations** is given by:

$$l\left(\theta_1, \theta_2, \boldsymbol{\kappa}, \xi_j \middle| \boldsymbol{\Delta X}_{M,j}\right) \propto$$

$$-\frac{1}{2}\sum_{j=1}^{m}\left\{log\left|\mathbf{C_j}\right| + \left[\boldsymbol{\Delta X}_{M,j} - E\left(\boldsymbol{\Delta X}_{M,j}\right)\right]^T \cdot \mathbf{C_j}^{-1} \cdot \left[\boldsymbol{\Delta X}_{M,j} - E\left(\boldsymbol{\Delta X}_{M,j}\right)\right]\right\}$$

estimation of parameters now by maximization of l .

parameter estimation in ELL

random effects are assumed to have zero mean value

-> maximization of l by setting random effects $\xi = 0$

$$l\left(\theta_1, \theta_2, \boldsymbol{\kappa} \middle| \boldsymbol{\Delta X}_{M,j}, \xi = 0\right) \propto$$

$$-\frac{1}{2}\sum_{j=1}^{m}\left\{log\left|\mathbf{C_j}\right| + \left[\boldsymbol{\Delta X}_{M,j} - E\left(\boldsymbol{\Delta X}_{M,j}\right)\right]^T \cdot \mathbf{C_j}^{-1} \cdot \left[\boldsymbol{\Delta X}_{M,j} - E\left(\boldsymbol{\Delta X}_{M,j}\right)\right]\right\}$$

if covariates are neglected, hyperparameters are: θ_1, θ_2 and $\boldsymbol{\kappa}_0$

given the point estimates of the hyperparameters, the remaining unknown random effects are estimated for each location separately by maximizing l :

$$l_j\left(\xi_j \middle| \boldsymbol{\Delta X}_{M,j}, \hat{\theta}_1, \hat{\theta}_2, \hat{\kappa} = 0\right) \propto$$

$$-\frac{1}{2}\sum_{j=1}^{m}\left\{log\left|\mathbf{C_j}\right| + \left[\boldsymbol{\Delta X}_{M,j} - E\left(\boldsymbol{\Delta X}_{M,j}\right)\right]^T \cdot \mathbf{C_j}^{-1} \cdot \left[\boldsymbol{\Delta X}_{M,j} - E\left(\boldsymbol{\Delta X}_{M,j}\right)\right]\right\}$$

parameter estimation in ELL

finally, a point estimate of the standard-deviation $\hat{\sigma}_\xi$ can be obtained as the sample standard deviation of the estimated random effects $\hat{\xi}_j \left(j = 1,\ldots,m \right)$

statistical uncertainties of the estimated parameters can be established using standard maximum likelihood techniques and the first and second derivatives of the equivalent log-likelihood function

the variance-covariance matrix for the hyper-parameters θ_1, θ_2 and $\boldsymbol{\kappa}_0$ of the simplified procedure is determined as the inverse of their information matrix $\mathbf{I}$

the standard deviation of $\hat{\sigma}_\xi$ can be estimated based on the chi-squared sampling distribution of the variance of the random effects

comparative example – approach

deterioration process, inspections at $m = 5$ defect locations, each having $n = 4$ inspections at times $\mathbf{t} = \{4,6,8,10\}$ years.

Table 1: Observed deterioration $x_{M,j,i}$ in location j at inspection i.

Inspection i	1	2	3	4
Time t_i [years]	4	6	8	10
Location j	Observed Deterioration $x_{M,j,i}$ [% of metal loss]			
1	8.9	19.6	26.9	30.5
2	12.7	14.1	16.8	20.5
3	8.9	14.7	23.3	29.8
4	14.1	21.0	25.0	30.2
5	7.2	10.7	13.8	22.3

assumptions:

- All locations are subject to the same operational and material conditions -> covariates are not considered in this analysis.
- standard deviation σ_ε and correlation coefficient ρ_ε of the measurement device are given to be 2.0 and 0.20, respectively.
- $R = D = 1$

comparative example – results

Table 2: Posterior hyper-parameters, random effects ξ_j and location specific parameters β_i for HB and ELL.

	HB		ELL	
	median	sd	point. est.	sd
θ_1	4.363	3.103	3.971	2.386
θ_2	1.025	0.120	1.023	0.120
κ_0	-0.541	0.569	-0.452	0.524
σ_ξ	0.120	0.133	0.209	0.099
ξ_1	0.048	0.137	0.163	0.143
ξ_2	-0.061	0.144	-0.253	0.217
ξ_3	0.028	0.133	0.101	0.152
ξ_4	0.039	0.137	0.140	0.147
ξ_5	-0.061	0.147	-0.236	0.214
β_1	0.620	0.423	0.749	0.508
β_2	0.538	0.400	0.494	0.357
β_3	0.606	0.422	0.704	0.481
β_4	0.615	0.415	0.732	0.498
β_5	0.537	0.399	0.502	0.362

comparative example – results

assessment of posterior distribution of deterioration $f_{X_j(t)}\left(x,t \middle| \mathbf{x_M}\right)$ at a future time t

based on HB conditionally independent deterioration increments between 0 and t for any specific location j are added

$$X_j\left(t \middle| \mathbf{x_M}\right) = \sum_{i=1}^{n_j} \Delta X_{j,i}\left(t_i, t_{i-1} \middle| \mathbf{x_M}\right) + \Delta X_j\left(t, t_{n_j} \middle| \mathbf{x_M}\right)$$

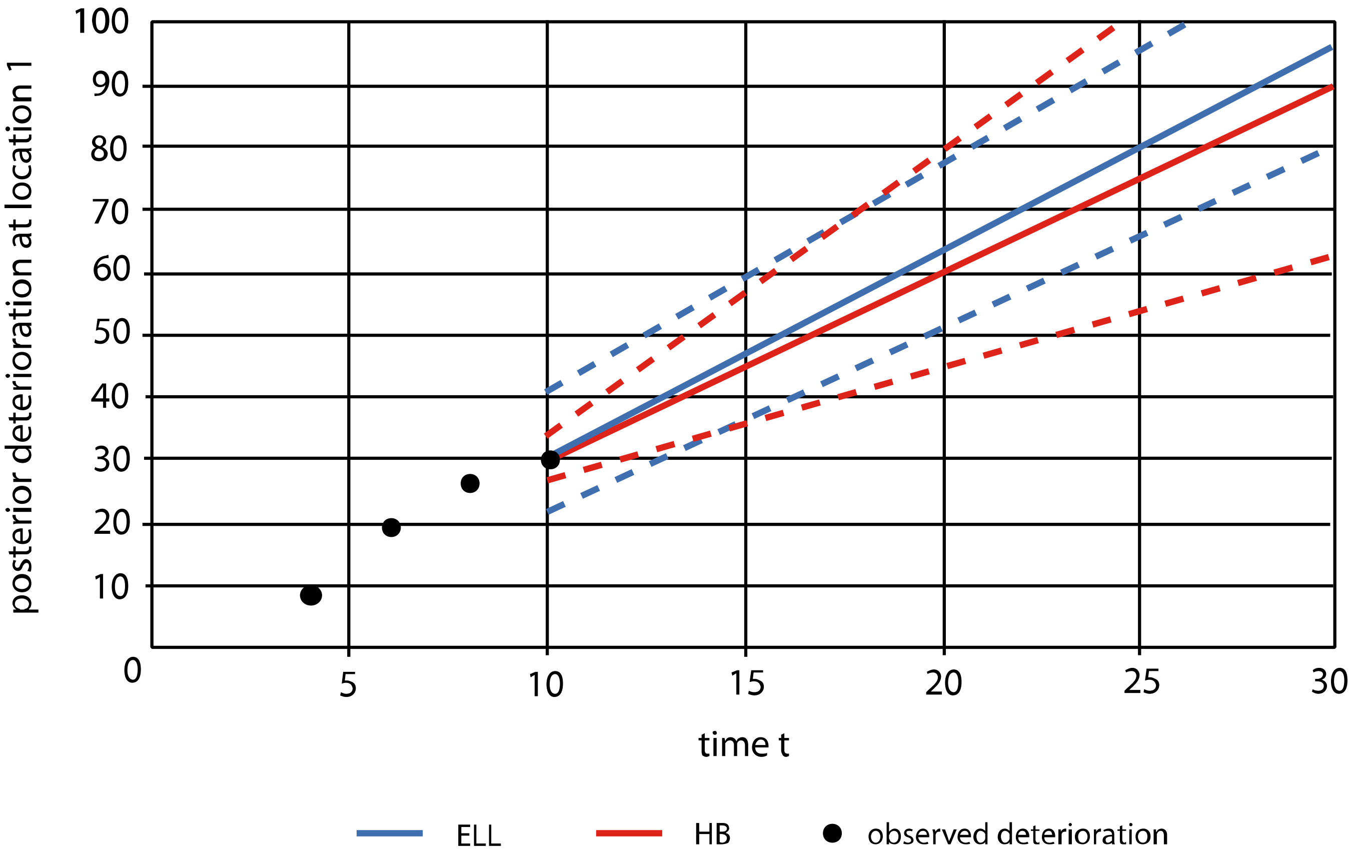
ELL only allows approximation, epistemic uncertainties are not taken into account. estimate of the actual deterioration at time t by ELL:

$$X_j\left(t \middle| \mathbf{x_M}\right) \cong X_j\left(t \middle| \hat{\theta}_1, \hat{\theta}_2, \hat{\boldsymbol{\kappa}}_0, \hat{\xi}_j\right) \sim gamma\left(\hat{\theta}_1 \cdot t^{\hat{\theta}_2}, exp\left\{\hat{\boldsymbol{\kappa}}_0 + \hat{\xi}_j\right\}\right)$$

comparative example – results

posterior deterioration medians (full lines)

2.5%- and 97.5%-quantiles (dashed lines)



in general good agreement, but uncertainty bands differ between the HB model and the ELL approach due to the simplified treatment of statistical uncertainty in ELL.

comparative example – results

assessment of posterior cumulative lifetime distribution $F_{L_j}(t)$ at location j

lifetime is defined as the time required to achieve a critical metal loss equal to a threshold value x^*.

$$P\left[L_j < t \middle| \mathbf{x_M}\right] = P\left[X_j(t) > x^* \middle| \mathbf{x_M}\right] = 1 - \int_0^{x^*} f_{X_j(t)}\left(x_j, t \middle| \mathbf{x_M}\right) dx_j$$

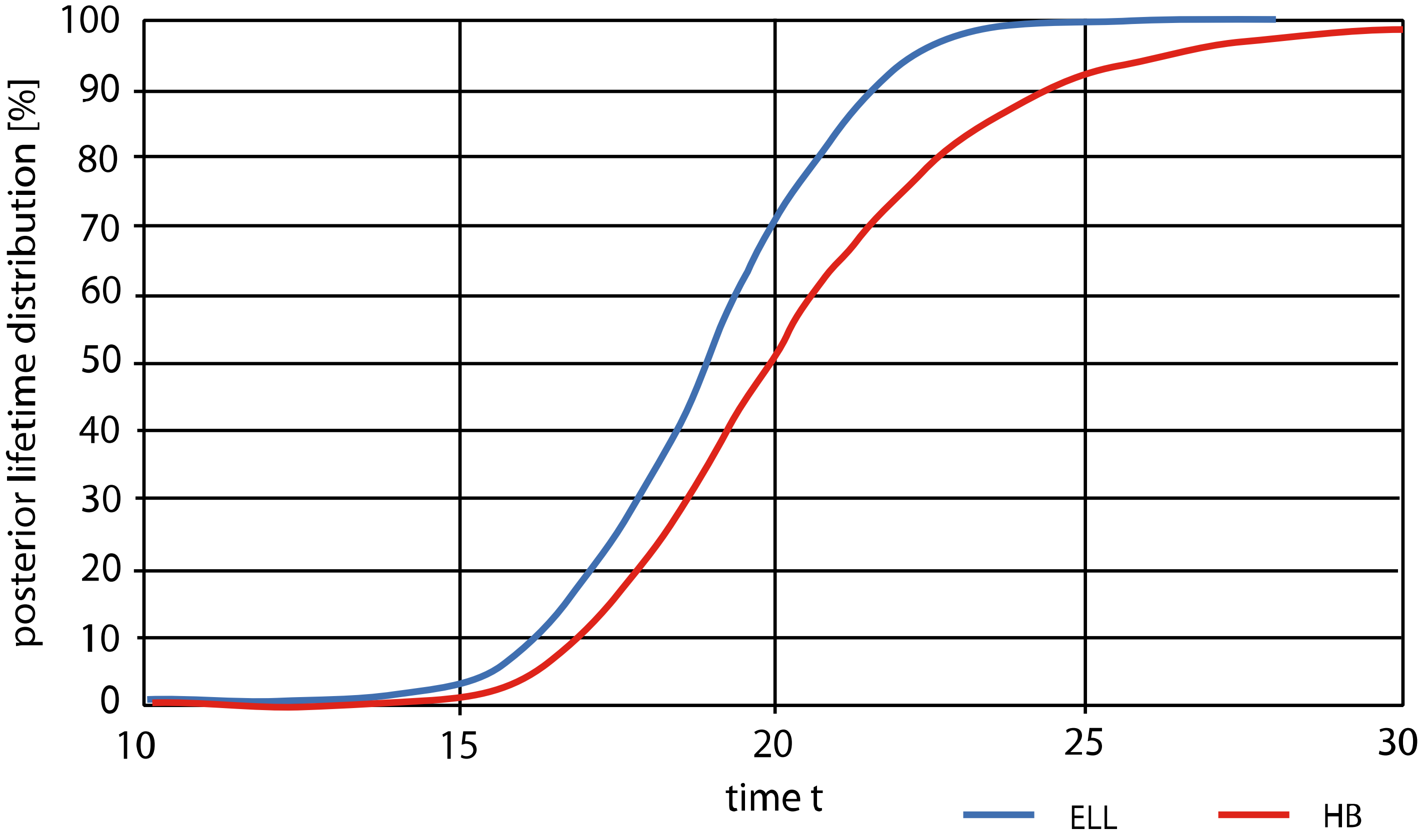
lifetime calculation by simulation using MCMC for HB and approximation of integration for ELL

comparative example – results

posterior cumulative lifetime distribution at location 1

threshold $x^* = 60$

both approaches (HB and ELL) are in good agreement



example: integrity assessment by HB

Scenario:

- gas pipeline is subject to metal loss due to internal corrosion
- inspections at $m = 24$ locations
- 11 locations inspected in a first ILI at time $t = 2$ years
- two additional ILI inspections for all locations at time $\mathbf{t} = \{4, 6\}$ years

example: integrity assessment by HB

results of inspection – part I

Table 3: Observed percentage of metal loss at the 24 defect locations as well as the observed defect widths $w_{j,obs}$ and defect lengths $L_{j,obs}$ at t=6years.

	Time t [years]	2	4	6	Defect width $w_{j,obs}$ [m]	Defect length $L_{j,obs}$ [m]
	Location j	Observed defect depth $x_{M,j,i}$ [%]				
	1	9.1	10.9	9.3	0.401	0.147
	2	3.9	3.7	7.9	0.107	0.625
	3	1.8	3.4	5.5	0.508	0.236
	4	1.5	5.8	8.6	0.584	0.455
	5	7.7	7.1	9.2	0.071	0.800
	6	0.4	9.9	8.0	0.231	0.386
	7	5.3	4.2	8.2	0.417	0.221
wide & extensive	8	0.5	3.2	3.2	0.539	1.171
local & deep	9	7.6	18.4	21.9	0.031	0.038
	10	5.2	5.4	10.4	0.262	0.422
	11	1.8	3.0	15.0	0.216	0.107

example: integrity assessment by HB

Time t [years] / Location j	2	4	6	Defect width $w_{j,obs}$ [m]	Defect length $L_{j,obs}$ [m]
	Observed defect depth $x_{M,j,i}$ [%]				
12	n/a	3.9	4.8	0.155	0.272
13	n/a	4.3	10.5	0.323	0.617
14	n/a	5.1	5.7	0.061	0.081
15	n/a	2.8	8.9	0.523	0.723
16	n/a	1.5	6.6	0.185	0.419
17	n/a	1.9	7.8	0.246	0.480
18	n/a	11.7	13.7	0.170	0.305
19	n/a	5.5	2.4	0.231	0.721
20	n/a	3.1	3.7	0.384	0.922
21	n/a	7.8	12.1	0.046	0.051
22	n/a	12.1	13.6	0.307	0.218
23	n/a	3.9	14.2	0.155	0.348
24	n/a	1.7	6.0	0.292	0.688

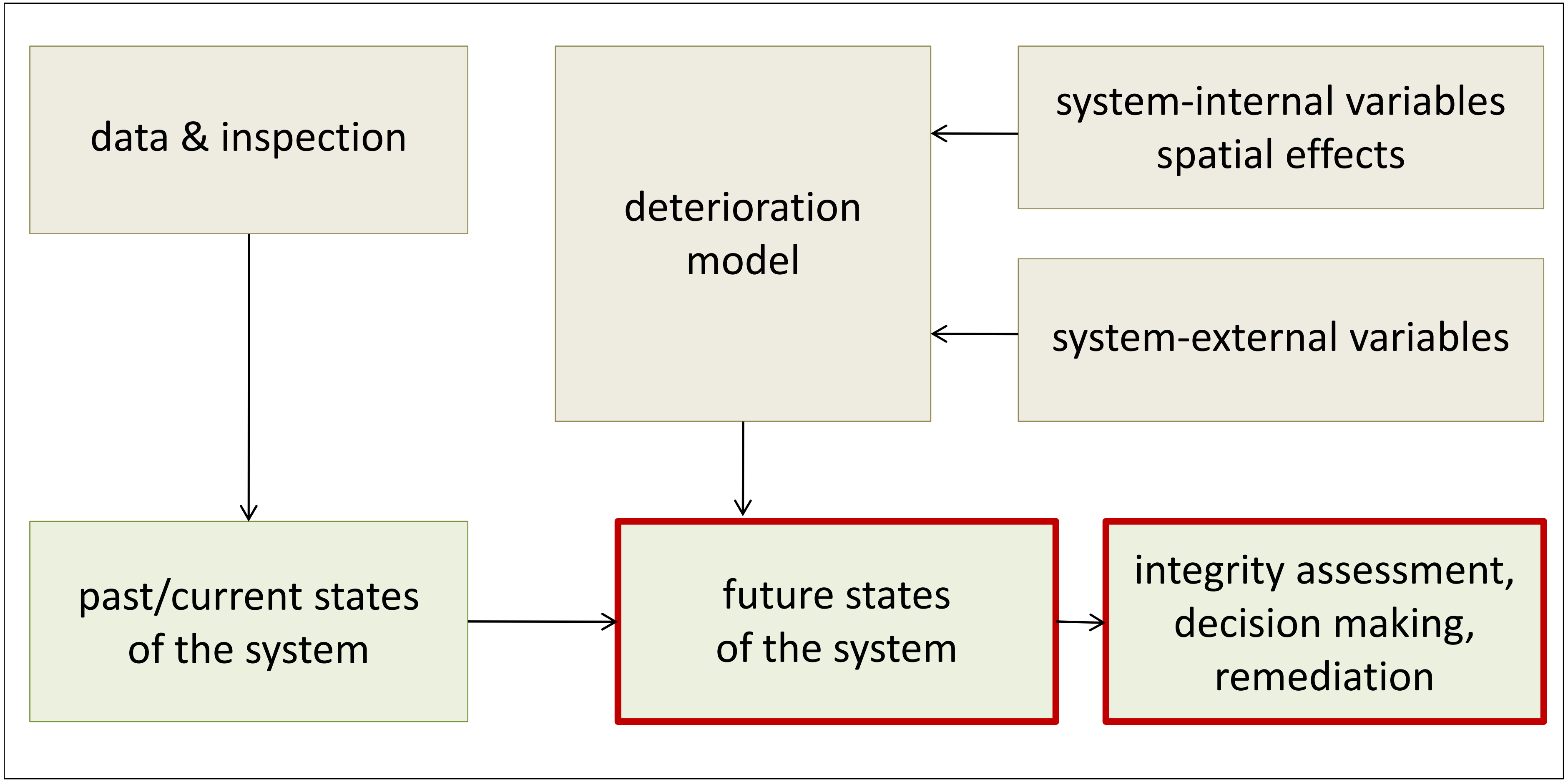
example: integrity assessment by HB

2 steps of analysis

1. Hierarchical Bayes analysis
   using observed percentage of wall loss for estimation of posterior distribution of future deterioration at all locations

2. Reliability analysis
   estimation of the posterior pobability of failure as a function of time using a defect/burst limit state

example: integrity assessment by HB



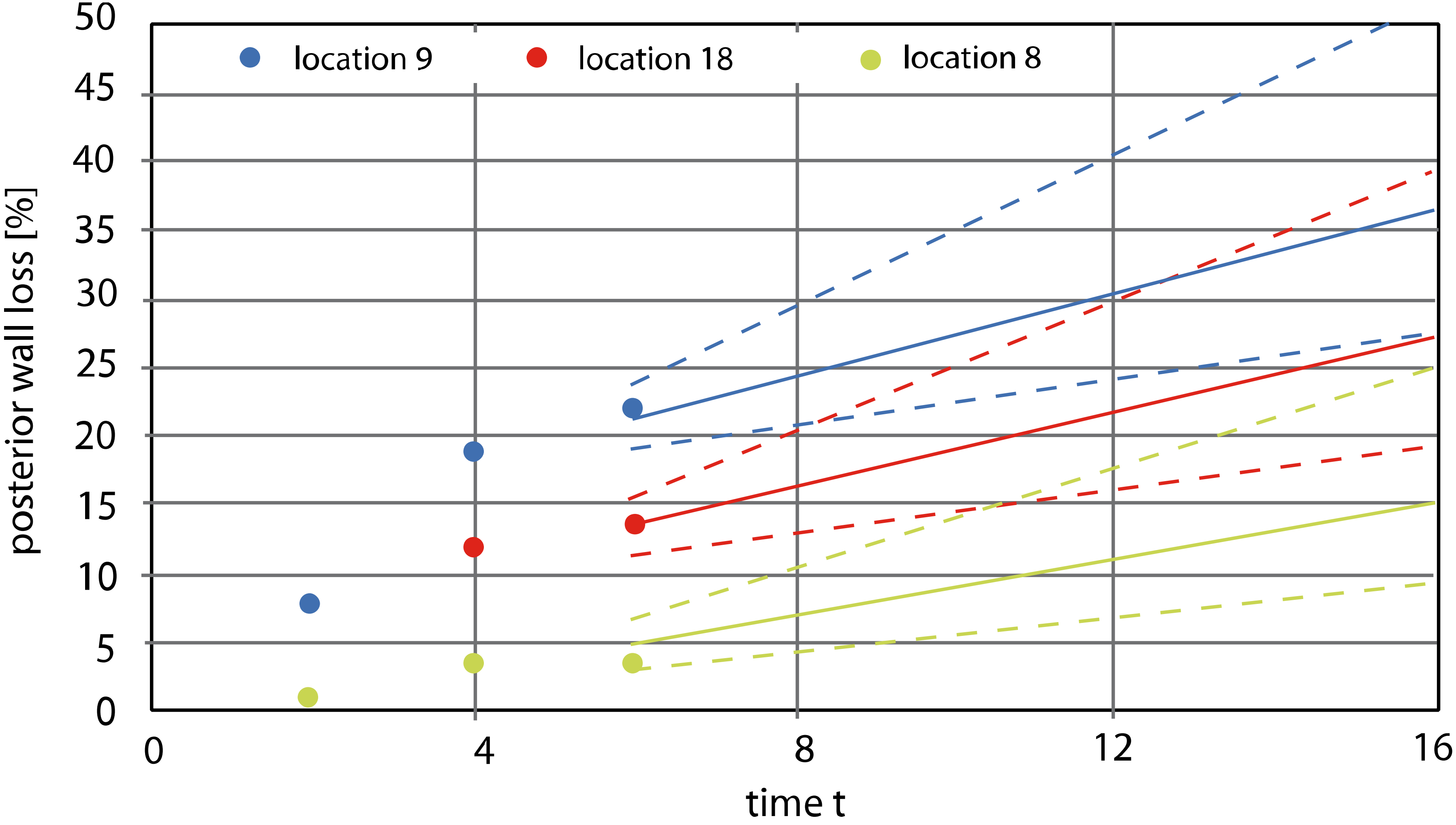
example: integrity assessment by HB

results of parameter assessment

Table 4: Posterior hyper-parameters and posterior scale-parameter β_j for location 8, 9 and 18 given by their mean values $E[.|\boldsymbol{x_M}]$, standard deviations $SD[.|\boldsymbol{x_M}]$ as well as 10%-quantile $q_{0.1}|\boldsymbol{x_M}$ and 90%-quantile $q_{0.9}|\boldsymbol{x_M}$.

	$E[\,.\,\vert\boldsymbol{x}_M]$	$SD[\,.\,\vert\boldsymbol{x}_M]$	$q_{0.1}\vert\boldsymbol{x}_M$	$q_{0.9}\vert\boldsymbol{x}_M$
θ_1	1.525	0.986	0.644	2.751
θ_2	0.907	0.176	0.689	1.137
κ_0	0.287	0.533	-0.428	0.891
σ_ξ	0.200	0.135	0.051	0.390
β_8	1.403	0.783	0.464	2.402
β_9	1.875	0.808	1.024	2.873
β_{18}	1.672	0.771	0.815	2.644

results of posterior distribution of the deterioration at time t after the last inspection



example: integrity assessment by HB

input for reliability analysis

Table 5: Fixed parameters used in the reliability analysis

Parameter	Value	Unit
Outer pipeline diameter D	0.508	m
Maximum annual operating pressure p_{MAOP}	32.04	MPa
Correlation coefficient between σ_{UTS} and n_L	0.73	-

Table 6: Random variables used in the reliability analysis

Random variable	Distribution	Mean value	CoV / St.Dev
X70 tensile strength σ_{UTS}	log-normal	638.45 MPa	5.5 %
X70 strain hardening index n_L	log-normal	0.08	20.0 %
Yield-to-tensile ratio q	normal	0.83	2.0 %
Original wall thickness t_S	normal	0.0181 m	3.0 %
Defect width w_j	log-normal	$w_{j,obs}$	0.0254 m
Defect length L_j	log-normal	$L_{j,obs}$	0.0254 m
Additive model error C_1	normal	0.0	6.68 MPa
Multiplicative model error C_2	log-normal	1.0	5.0 %
Annual extreme pressure differential Δp	gumbel	$1.07 p_{MAOP}$	2.0 %

maximum annual operating pressure

example: integrity assessment by HB

time-dependent reliability analysis -> assessment of increasing annual probability of failure at each defect

limit state: fully plastic, strain-hardening, large-strain burst of a tubular section containing a defect of % depth x_j

w_j = *defect width;* L_j = *defect length;* j = *location;* t = *time;*

$$g_L = \frac{0.935 \cdot C_2}{\sqrt{3}} \cdot \frac{4t_S}{D - t_S} \left[\left(1 - \frac{1 - f_j \left(1 - x_{j,thres} \right)}{x_{j,thres}} X_j(t) \right) \left(1 - r_j(t) \right) + r_j(t) \right] \times \left(\frac{\sigma_{UTS}}{3^{n_L/2}} + \frac{C_1}{exp(n_L)} \right) - \Delta p$$

for $X_j(t) \leq x_{j,thres}$

σ_{UTS} = *ultimate tensile strength of the steel;*

n_L = *Ludwik law strain-hardening index;*

t_S = *original wall thickness;*

D = *original outer steel diameter;*

C_1 and C_2 = *model uncertainties axxiciated with the burst prediction failure;*

Δp = *annual extreme pressure differential;*

example: integrity assessment by HB

time-dependent reliability analysis -> assessment of increasing annual probability of failure at each defect

limit state: fully plastic, strain-hardening, large-strain burst of a tubular section containing a defect of % depth x_j

w_j = *defect width;* L_j = *defect length;* j = *location;* t = *time;*

$$g_L = \frac{0.935 \cdot C_2}{\sqrt{3}} \cdot \frac{4t_S}{D - t_S} f_j \left[\left(1 - X_j(t)\right)\left(1 - r_j(t)\right) + r_j(t) \right] \times \left(\frac{\sigma_{UTS}}{3^{n_L/2}} + \frac{C_1}{exp(n_L)} \right) - \Delta p$$

$$\text{for } X_j(t) > x_{j,thres}$$

σ_{UTS} = *ultimate tensile strength of the steel;*

n_L = *Ludwik law strain-hardening index;*

t_S = *original wall thickness;*

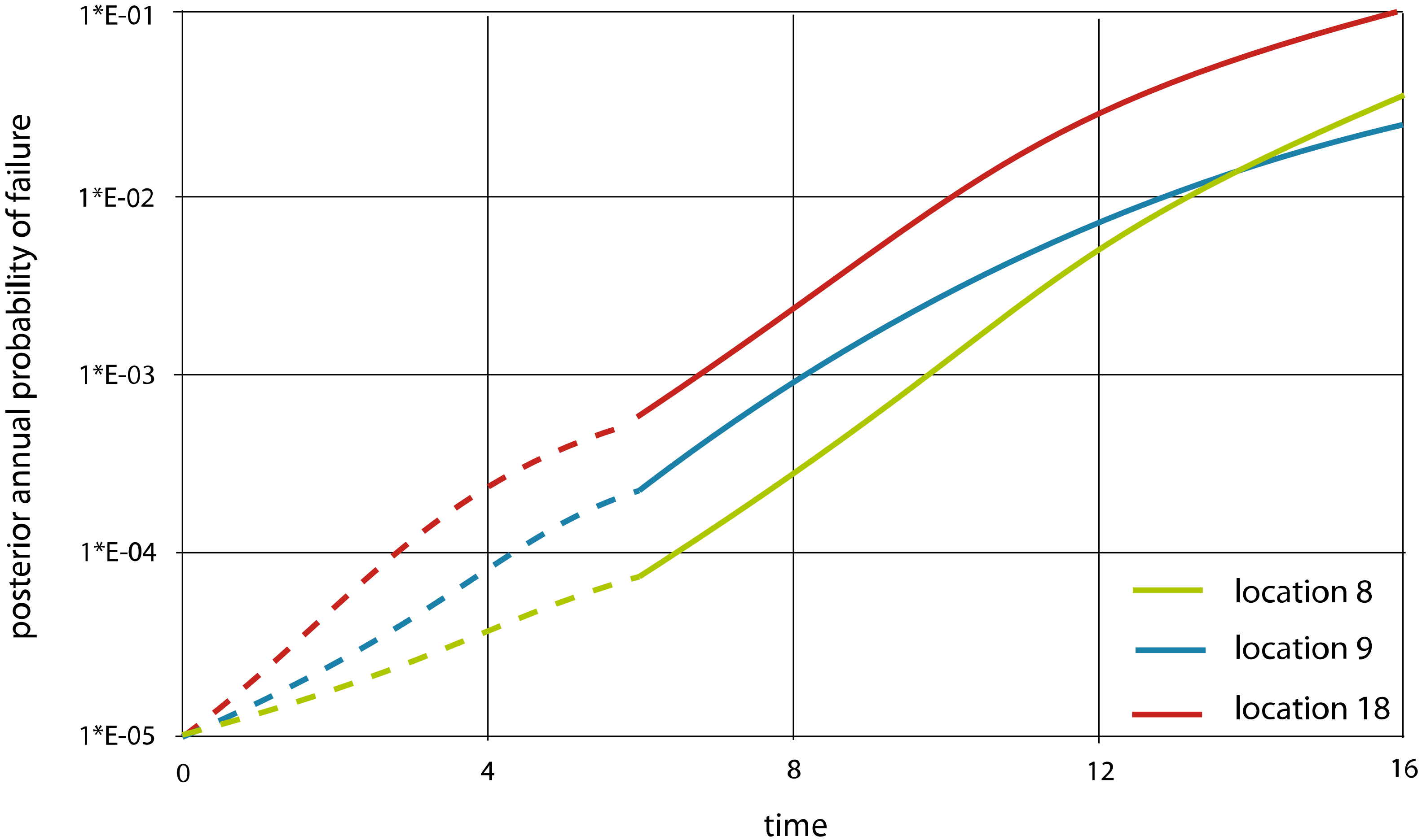
D = *original outer steel diameter;*

C_1 and C_2 = *model uncertainties axxiciated with the burst prediction failure;*

Δp = *annual extreme pressure differential;*

example: integrity assessment by HB

posterior annual probability of failure $P_{F,j}$ for locations 8, 9 and 18 as a function of time after the last inspection



dashed and solid lines indicate the annual probability of failure during the inspection period and after the last inspection, respectively

conclusions

Hierarchical Bayes

- mirrors 4-level uncertainty structure
- stochastic deterioration model is able to process and assimilate new inspection results
- facilitates a conceptual decomposition of the deterioration process into local, simple conditional relationships and sub-models
- provides satisfactory synthesis of current and future behaviour of complex systems

conclusions

Equivalent Log-Likelihood

- can not take into account some model and epistemic uncertainties properly
- simplification treatment of large inspection data sets
- provides reasonably accurate lifetime estimates
- ELL should be used to screen large amounts of measured deterioration data
- a full HB approach should be run on defects/elements which critically affect system integrity

conclusions

general

- deterioration affects reliability and safety of all types of structures and infrastructure
- informed lifetime integrity forecasts for a pipeline segment depends on progress of deterioration in all defects or critical locations
- spatial variation is observable from point-to-point or unit-to-unit
- local variation due to locally varying co-variances and aleatory effects
- valid stochastic deterioration model must contain all four levels of the hierarchy