Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Some practical considerations

Presentation in „Methods of finite element II“

Prof. Dr. M. H. Faber, Dr. N. Mojsilovic

9. November 2007

Daniel Caduff

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Topics

- General Approach to nonlinear Analysis
  - Transition from linear to nonlinear analysis
  - Principle
- Collapse and Buckling Analyses
  - Examples
  - Linearized buckling analysis
  - Results for the examples
  - General
- The Effects of Element Distorsions
- The Effects of Order of Numerical Integration
- Example 6.27 Bathe p. 638

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ibk

General Approach

Transition from linear to nonlinear analysis

- First assumption: linear analysis
- Indicate regions
  - Geometric nonlinearities and elastic limit of the material
- Choose appropriate nonlinear formulations and material models for this regions
- Different formulations can be used together in a mesh:
  - Linear analysis assumption
  - Materially- nonlinear- only formulation
  - TL and UL formulations

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ibk

General Approach

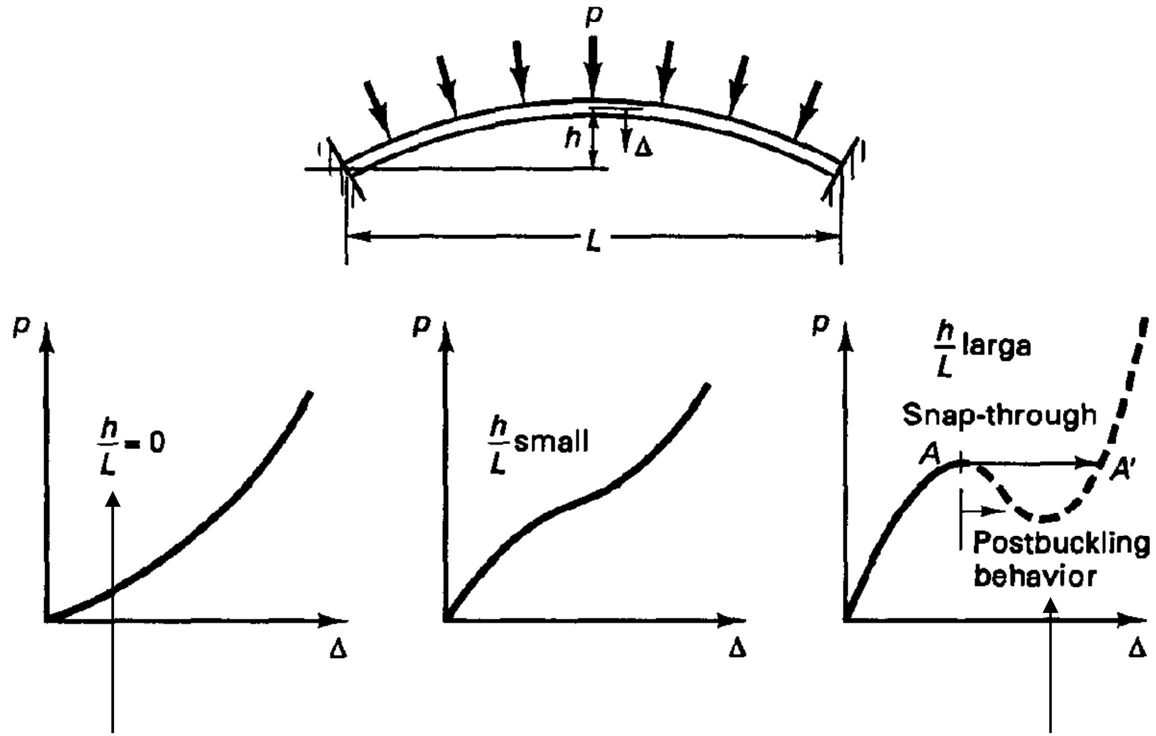
Principle

The complete process of analysis can be linkened to a series of laboratory experiments in whitch different assumptions are made in each experiments – in the finite element analysis these experimants are performed on the computer with a finite element program.

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Collapse and Buckling Analyses

Examples (1)



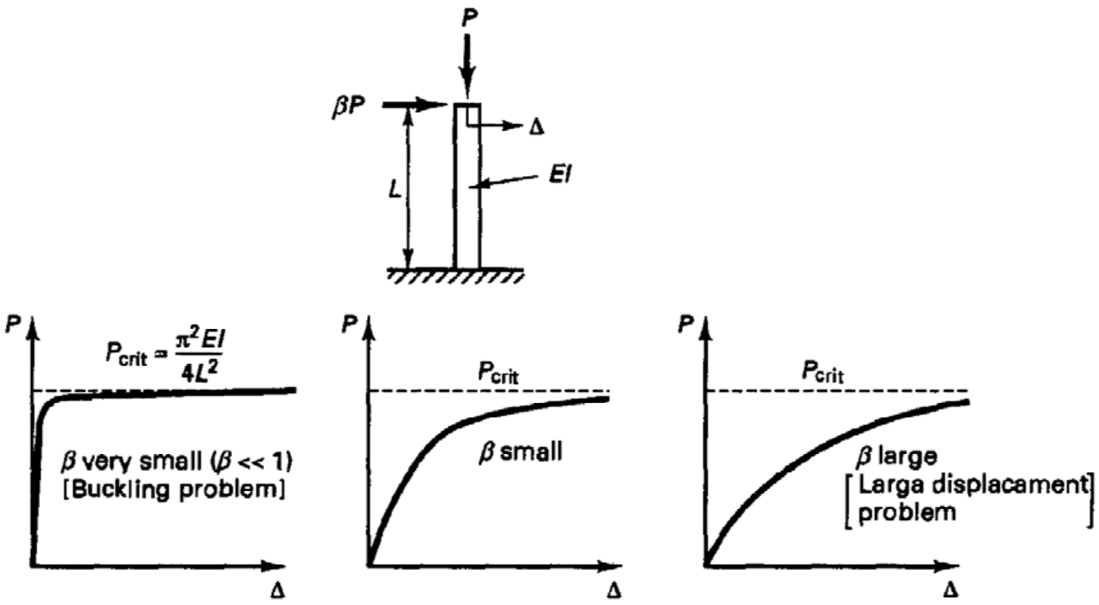
No collapse point

Dynamic response

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Collapse and Buckling Analyses

Examples (2)



–β: represent the imperfection in the geometry and material properties or the load application

ETH Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ibk

Collapse and Buckling Analyses

Linearized buckling analysis (1)

- Response calculated by an incremental analysis
- Definitions:

$^{\Delta t}\mathbf{R}$: Load distribution

$^{\tau}\beta$: Load multiplier

→ $^{\tau}\beta * {}^{\Delta t}\mathbf{R}$: Load at time τ

- Interest in calculating the response of the structures as τ increases
- See Section 8.4.3 for specific algorithm
- Complete incremental nonlinear solution can be expensive

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ibk

Collapse and Buckling Analyses

Linearized buckling analysis (2)

- Calculation of the lowest buckling load
- Lowest buckling modes are imposed on the structure to simulate imperfections

– Calculation:

$$ {}^{\tau}\mathbf{K} = {}^{t-\Delta t}\mathbf{K} + \lambda({}^{t}\mathbf{K} - {}^{t-\Delta t}\mathbf{K}) \qquad (1) $$

$$ {}^{\tau}\mathbf{R} = {}^{t-\Delta t}\mathbf{R} + \lambda({}^{t}\mathbf{R} - {}^{t-\Delta t}\mathbf{R}) \qquad (2) $$

- λ **:** scaling factor; when λ **> 1 ?**
- At collapse or buckling the tangent stiffness matrix is singular:

$$ \det {}^{\tau}\mathbf{K} = 0 $$

$$ {}^{\tau}\mathbf{K}\boldsymbol{\phi} = \mathbf{0} \qquad (3) $$

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Collapse and Buckling Analyses

Linearized buckling analysis (3)

– Substitution (1) on (3):

$$^{t-\Delta t}\mathbf{K}\boldsymbol{\phi} = \lambda(^{t-\Delta t}\mathbf{K} - {}^{t}\mathbf{K})\boldsymbol{\phi} \qquad (4)$$

– λ_i **, i=1, ..., n:** Eigenvalues give the buckling loads (using (2))

– ϕ_i **, i=1, ..., n:** Eigenvectors give the corresponding buckling modes

– Smallest positive eigenvalue:

$$^{t}\mathbf{K}\boldsymbol{\phi} = \gamma\, {}^{t-\Delta t}\mathbf{K}\boldsymbol{\phi}$$

$$\gamma = \frac{\lambda - 1}{\lambda}$$

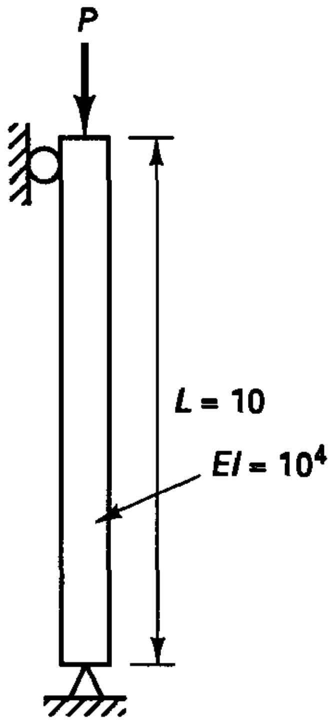
– γ_i **, i=1, ..., n:** Eigenvalues, all positive

→ $$\mathbf{R}_{\text{buckling}} = {}^{t-\Delta t}\mathbf{R} + \lambda_1({}^{t}\mathbf{R} - {}^{t-\Delta t}\mathbf{R})$$

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Collapse and Buckling Analyses

Results for the examples (1)



P_{cr} of mathematical model (analytical solution) = 986.96
P_{cr} of finite element model = 986.212 (for ${}^{\Delta t}P$ = 1, 10, and 100)

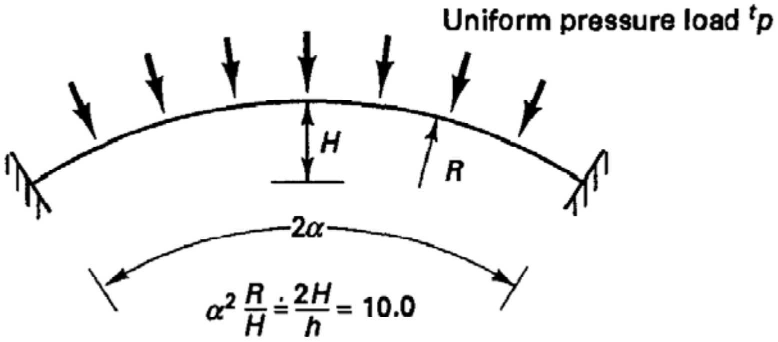
– Good results if the precollapsse displacements are relatively small

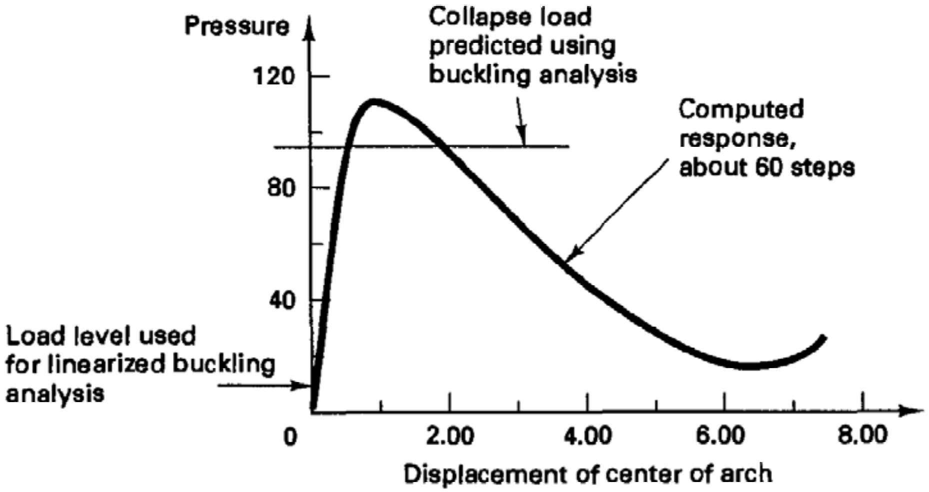
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ibk

Collapse and Buckling Analyses

Results for the examples (2)





- Calculated Response using a load-displacement-constraint method
- Precollapse displacements are large → analysis very much overestimate the collapse load

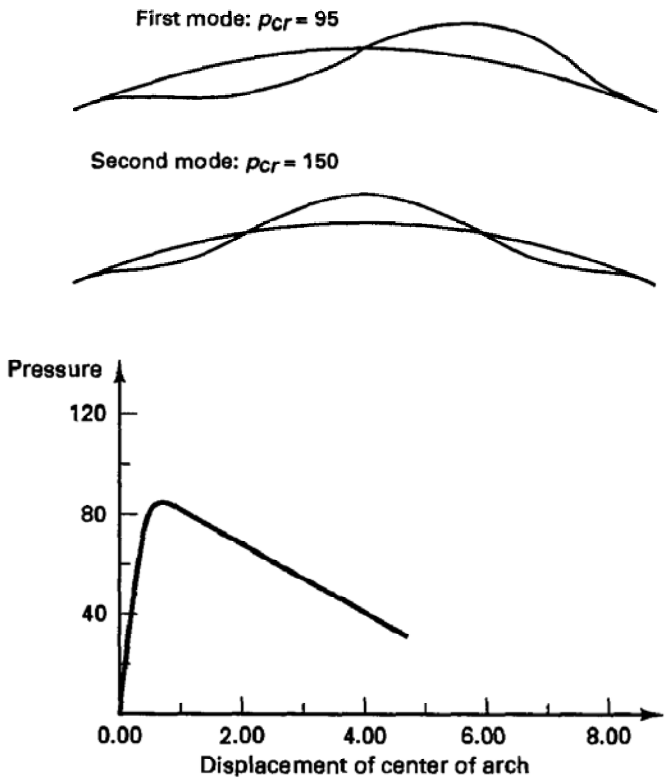
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ibk

Collapse and Buckling Analyses

Results for the examples (3)

– Impose the buckling mode as an imperfection on the structural model:



– Calculated Response using a load-displacement-constraint method

→ Collapse load smaller with structural imperfections

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ibk

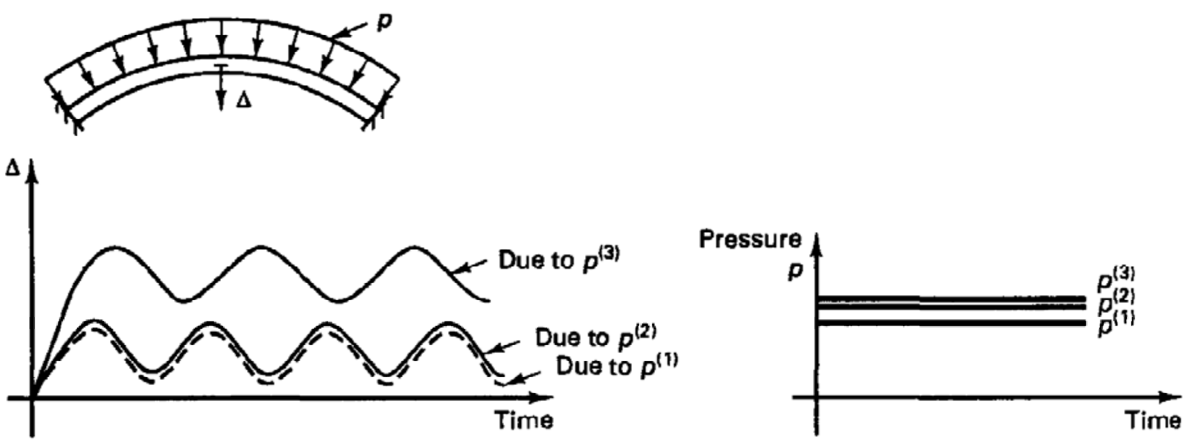
Collapse and Buckling Analyses

General

– Incremental load analysis should include:
  – Material nonlinearities
  – Imperfections (1nd, 2nd, ... buckling modes)
  – Perturbations in the material properties

➡ Find smallest load-carrying capacity for the structure

Dynamic solutions also may be considered!



ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

The Effects of Element Distorsions

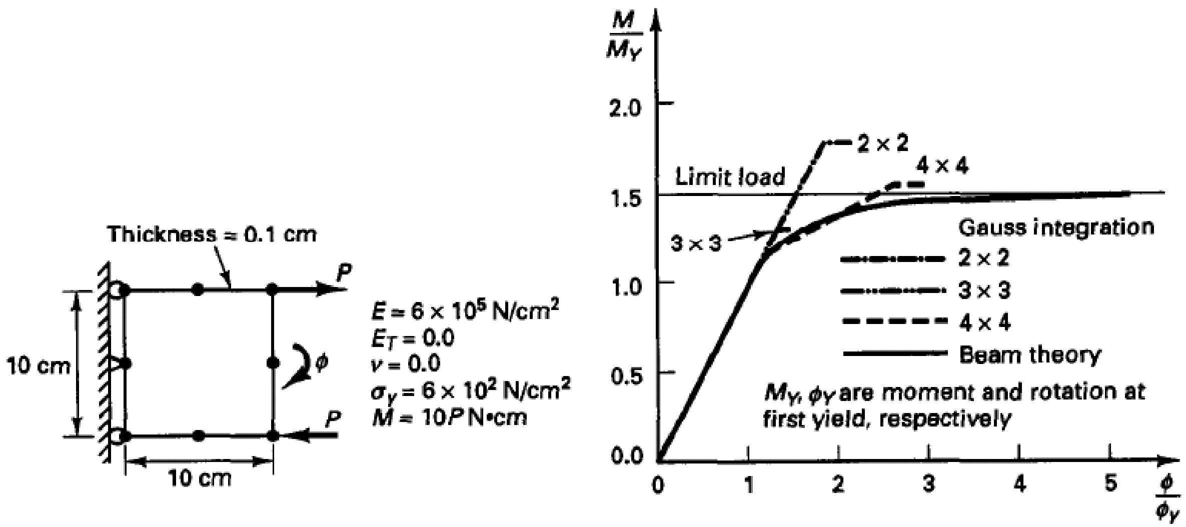
- Elements must be of general straight-sided shapes with angular distorsion in order to provide mesh gradings and to mesh complex geometries effectively → Lagrangian elements are effevtive
- Using the lagre displacement formulations, the principle of virtual displacements is applied to each individual element corresponding to the current configuration → monitor the changing shapes of of each element
- If element distorsions adversely affect the response prediction, a different and finer mesh may be required for the geometrically nonlinear analysis

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ibk

The Effects of Order of Numerical Integration

- A higher integration order than that used in linear analysis may be required in the analysis of materially nonlinear response in order for the analysis to capture the onset and spread of the materially nonlinear conditions accurately enought.
- Newton-Cotes methods effective because the integration points for stiffness and stress evaluations are on the boundaries of the elements



ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Example 6.27 Bathe p. 638

EXAMPLE 6.27: Consider element 2 in Example 4.5 and assume that in an elastoplastic analysis the stresses at time t in the element are such that the tangent moduli of the material are equal to $E/100$ for $0 \leq x \leq 40$ and equal to E for $40 < x \leq 80$ as illustrated in Fig. E6.27. Evaluate the tangent stiffness matrix ${}^t\mathbf{K}$ using one-, two-, three-, and four-point Gauss integration and compare these results with the exact stiffness matrix. Consider only material nonlinearities.

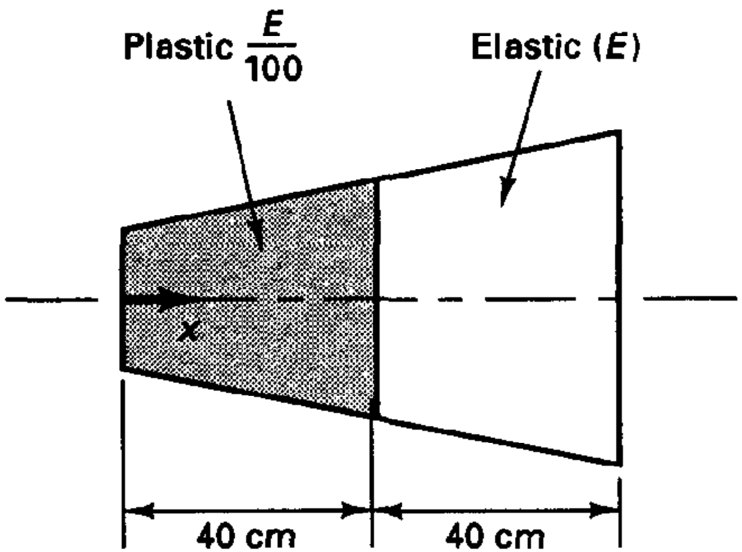


Figure E6.27 Element 2 of Example 4.5 in elastic-plastic conditions

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ibk

Example 6.27 Bathe p. 638

TABLE 5.6 *Sampling points and weights in Gauss-Legendre numerical integration (interval −1 to +1)*

n	r_i			α_i		
1	0.	(15 zeros)		2.	(15 zeros)	
2	±0.57735	02691	89626	1.00000	00000	00000
3	±0.77459	66692	41483	0.55555	55555	55556
	0.00000	00000	00000	0.88888	88888	88889
4	±0.86113	63115	94053	0.34785	48451	37454
	±0.33998	10435	84856	0.65214	51548	62546
5	±0.90617	98459	38664	0.23692	68850	56189
	±0.53846	93101	05683	0.47862	86704	99366
	0.00000	00000	00000	0.56888	88888	88889
6	±0.93246	95142	03152	0.17132	44923	79170
	±0.66120	93864	66265	0.36076	15730	48139
	±0.23861	91860	83197	0.46791	39345	72691