Temporal variability in corrosion modeling and reliability updating

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**Motivation**

- Corrosion models are applied in many industries, for both
  - Design
  - Inspection & maintenance planning
- To optimize inspection and maintenance activities, a probabilistic approach is required
- Most of the applied models are simplistic and/or deterministic
- They do not consider neither **spatial variability** nor **temporal variability**

1) What are the implications of using simplistic models in this context?

2) How can the temporal variability be accounted for?
Modelling of corrosion

- Typical models from the literature:
  
  A: Constant corrosion rate
  B: Constant corrosion rate with initiation period
  C: Corrosion loss follows a power law
  D: A model considering different driving mechanisms
Probabilistic modelling of corrosion

- Comparing two (simple) models:
  - Model A: constant corrosion rate
  - Model B: As A, with initiation period
- Model A can be considered as conservative compared to B
Probabilistic modelling of corrosion

- Influence of an inspection result on the reliability as evaluated with the two models
Example

\[ g = d_{cr} - d_{C}(t) \]
\[ g_m = d_m - d_{C}(t) \]

\[ d_{C}(t) = \begin{cases} 
0, & t < t_I \\
\exp(-r(t - t_I)), & t \geq t_I 
\end{cases} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dim.</th>
<th>(\mu)</th>
<th>(\sigma)</th>
<th>Dist.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrosion rate (r)</td>
<td>mm/yr</td>
<td>1</td>
<td>0.3</td>
<td>W</td>
</tr>
<tr>
<td>Initiation Model A</td>
<td>yr</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Model B</td>
<td>yr</td>
<td>5</td>
<td>2</td>
<td>LN</td>
</tr>
<tr>
<td>Critical depth (d_{cr})</td>
<td>mm</td>
<td>20</td>
<td>-</td>
<td>D</td>
</tr>
<tr>
<td>Insp. time (t_{insp})</td>
<td>yr</td>
<td>8</td>
<td>-</td>
<td>D</td>
</tr>
<tr>
<td>Corrosion measurement (d_m)</td>
<td>mm</td>
<td>6</td>
<td>1</td>
<td>N</td>
</tr>
</tbody>
</table>

\(\mu\): Mean value; \(\sigma\): standard deviation; W: Weibull distr.; LN: Lognormal distr.; D: deterministic; N: Normal distr.
Results example

- Without inspection: Model A is conservative
Results example

- With inspection: Model A is no longer conservative
Probabilistic modelling of corrosion

- The non-conservatism of model A can be illustrated by considering only the measurement event deterministically:

- This points out the importance of an appropriate phenomenological model of the corrosion deterioration.
Temporal variability in corrosion modeling

- So far no temporal variability in the model has been considered, i.e., all random variables were assumed constant with time.

- In the real world, influencing factors vary with time, e.g.,
  - Chemical composition of the environment
  - Temperature
  - Pressures

- What is the influence of these variation and how can it be considered?
Temporal variability in corrosion modeling

- CO₂ corrosion in a pipeline
- DeWaards-Miliams model:

\[ g = d_{cr} - d_{C}(t) \]
\[ d_{C}(t) = X_{M} r_{CO_2} t \]
\[ r_{CO_2} = 10^{(5.8 - 1710/T_o + 0.67 \cdot \log_{10} f_{CO_2})} \]
\[ f_{CO_2} = P_{CO_2} \cdot 10^{P_o (0.0031 - 1.4/T_o)} \]
\[ P_{CO_2} = n_{CO_2} P_o \]

- Parameters considered as stochastic processes:
  - Temperature
  - Pressure
Temporal variability in corrosion modeling

- Temperature & pressure are modelled by conditional Poisson square wave processes.
- The underlying Poisson processes are fully correlated.
- Additionally a correlation factor of 0.8 between the amplitudes of temperature & pressure at any point in time is assumed.
- For an ordinary Poisson square wave processes it is

\[
E[X(t)] = E[Y]
\]

\[
\text{Cov}[X(t_1), X(t_2)] = \text{Var}[Y] \exp[-\nu(t_2 - t_1)]
\]

- The mean values of the amplitude of T and P are uncertain themselves. Therefore it is:

\[
\text{Cov}[X(t_1), X(t_2)] = \text{Var}[Y] e^{-\nu(t_2 - t_1)} + \text{Var}[\mu_Y]\left(1 - e^{-\nu(t_2 - t_1)}\right)
\]
Temporal variability in corrosion modeling

- One realisation of the random processes:
Temporal variability in corrosion modeling

- The corrosion rate follows also a conditional Poisson square wave process

- For one realisation of the corrosion rate, an equivalent corrosion rate can be computed:

\[
r_{e}(t) = \frac{d_{C}(t)}{t} = \frac{1}{t} \int_{0}^{t} r_{CO_{2}}(t) \, dt
\]

- The moments of such an integration are obtained as

\[
E[r_{e}(t)] = \frac{1}{t} \int_{0}^{t} E[r_{CO_{2}}(t)] \, dt = E[Y_r]
\]

\[
\text{Var}[r_{e}(t)] = \frac{1}{t^2} \int_{0}^{t} \int_{0}^{t} \text{Cov}[r_{CO_{2}}(t_1), r_{CO_{2}}(t_2)] \, dt_1 \, dt_2
\]
Temporal variability in corrosion modeling

- The variance for the considered process is obtained as

\[
\text{Var}[r_e(t)] = \text{Var}[\mu_{Y_r}] + 2\left(\text{Var}[Y_r] - \text{Var}[\mu_{Y_r}]\right)\left(\frac{1}{\nu t} + \frac{(e^{-\nu t} - 1)}{(\nu t)^2}\right)
\]

- To calculate the moments of equivalent corrosion rate at any time, the following must be evaluated numerically:

- \(\text{E}[Y_r]\), Constant
- \(\text{Var}[Y_r]\), The random-point-in-time variability \(\sigma_{r_e}^2(\nu t = 0)\)
- \(\text{Var}[\mu_{Y_r}]\), The variability related to the time-invariant variability \(\sigma_{r_e}^2(\nu t = \infty)\)
Temporal variability in corrosion modeling

\[
\text{Var}[r_e(t)] = \text{Var}[\mu_{Y_r}] + 2\left(\text{Var}[Y_r] - \text{Var}[\mu_{Y_r}]\right)\left(\frac{1}{\nu t} + \frac{(e^{-\nu t} - 1)}{(\nu t)^2}\right)
\]
Temporal variability in corrosion modeling

- The equivalent corrosion rate $r_e(t)$ can now be calculated at any point in time.
- The time-variant analysis can be replaced by a time-invariant reliability analysis with $r_e(t)$.
- How to consider the temporal variability in reliability updating?
Temporal variability in corrosion reliability updating

- The application of equivalent values for the calculation of corrosion reliability may not be appropriate when considering reliability updating.
- This is investigated by calculating the correlation between the equivalent corrosion rate before and after the inspection:

\[
\text{Cov}\left[ r_e(t_{\text{insp}}), r_e(t) \right] = \frac{1}{t_{\text{insp}}} \int_0^{t_{\text{insp}}} dt_1 \int_0^t dt_2 \text{Cov}\left[ r_{CO_2}(t_1), r_{CO_2}(t_2) \right]
\]

\[
= \text{Var}[\mu_{Y_r}] + \left( \text{Var}[Y_r] - \text{Var}[\mu_{Y_r}] \right) \left( \frac{2}{\nu t} + \frac{1}{\nu t_{\text{insp}} \nu t} \left( -e^{-\nu(t-t_{\text{insp}})} + e^{-\nu t} + e^{-\nu t_{\text{insp}}} - 1 \right) \right)
\]
\[ \rho \left[ r_e (v_{t_{\text{Insp}}}), v_e (0) \right] \]

\( v_{t_{\text{Insp}}} = 1 \)

\( v_t \) (Expected number of realizations)

\[ \rho \left[ r_e (v_{t_{\text{Insp}}}), v_e (0) \right] \]

\( v_{t_{\text{Insp}}} = 5 \)

\( v_t \) (Expected number of realizations)

\[ \rho \left[ r_e (v_{t_{\text{Insp}}}), v_e (0) \right] \]

\( v_{t_{\text{Insp}}} = 20 \)

\( v_t \) (Expected number of realizations)

\[ \rho \left[ r_e (v_{t_{\text{Insp}}}), v_e (0) \right] \]

\( v_{t_{\text{Insp}}} = 100 \)

\( v_t \) (Expected number of realizations)
Discussion & conclusions

• The use of an appropriate corrosion model is crucial for inspection & maintenance planning

• Temporal variability can be considered through the use of an equivalent corrosion rate as proposed

• The equivalent corrosion rate is derived for one example, but can be extended to other corrosion models

• The equivalent corrosion rate principle is also valid for reliability updating

• The same considerations apply also for other deterioration mechanisms