SYSTEM EFFECTS IN PORTFOLIO LOSS ESTIMATION

Matthias Schubert
ETH Zürich, Institute of Structural Engineering, Group Risk & Safety

Michael H. Faber
ETH Zürich, Institute of Structural Engineering, Group Risk & Safety
Overview

• Introduction

• Dependencies in portfolios

• Hierarchical model formulation

• System effects in portfolios

• Conclusions
Introduction

- Decision makers and stakeholders managing portfolios of assets (road authorities, rail companies, etc.) need an overview of their risks for strategic planning.

- The aggregation of risks and the loss estimation are crucial requirements for the management.

- Loss estimation is influenced by two factors: expected value and the variance of losses.

- In principle decision-makers are faced with three questions:
**Introduction**

- How large is the probability of a total loss of the portfolio?

- How large is the probability that a certain budget is exceeded?

- Are there dependencies and nonlinearities in the portfolio which lead to an increase of the risk?
Dependencies in Portfolios

- **Geographic locations** lead to common aleatoric effects:
  - Floods, earthquakes, strong winds, avalanches,...
  - common economic conditions.
Dependencies in Portfolios

- **Best practices** lead to common epistemic effects.

The set of available models is restricted; same models are used for the design for same failure modes. Design codes, standards, common procedures, ...

The models are imperfect – the same effect realizes in the same models.

→ All common effects introduce dependencies in the model and have to be considered explicitly in a portfolio model.
Dependencies in Portfolios

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- System effects
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[Diagram showing dependencies between assets and resistance, with epistemic and resistance labels]
Dependencies in Portfolios

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Common cause effects

Asset $i$

$R_i$

$X_r$

$G_i$

$X_g$

$Q_i$

$X_q$

$R_i^*$

$G_i^*$

$L_i^*$

$p_{F,i}$

$g_i(x) \leq 0$ Probability of failure
Hierarchical model formulation

Homogeneous portfolios:

- Number of assets.
- Identical failure probabilities.
- Uniform dependency structure.
Hierarchical model formulation

Inhomogeneous portfolios:

- Number of assets.
- Different external conditions.
- Nonuniform dependency structure.
Hierarchical model formulation

\[ p_N(n) = \binom{k}{n} \int_{g_1(x) \leq 0} \cdots \int_{g_n(x) \leq 0} \int_{g_{n+1}(x) > 0} \cdots \int_{g_k(x) > 0} f_X(x) \, dx \]

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Hierarchical model formulation

Advantages:

• Existing sub-models can be used; dependencies are modeled on a higher hierarchical level.

• Hierarchical approach utilize causal relations among components.

• Failure probabilities of assets can be assessed conditionally independent.
Hierarchical model formulation

- Loss distribution function $p_N(n)$ can be assessed *almost* independent from the size a homogeneous portfolio:

$$p_N(n) = \int_{\theta} \binom{k}{n} (p_F(F | \theta))^n (1 - p_F(F | \theta))^{k-n} f_\theta(\theta) \, d\theta$$

- Inhomogeneous portfolios can be divided into a set of homogeneous portfolios; each treated as a conditionally independent random variable:

$$p_N(n) = \Pr(N_1 + N_2 = n) = \int \sum_{i=0}^{n} \Pr(N_1 = i | \theta) \Pr(N_2 = n-i | \theta) f_\theta(\theta) \, d\theta$$
System Effects

- Homogeneous portfolio with 12 identical assets and subjected to an identical variant load.

- Distinct tail of the loss distribution. Expected loss is identical – nonlinearity of consequences will increase the expected losses.
Homogeneous portfolio

- Two effects are observable: systematic effects and unsystematic effects.

- Unsystematic effects vanish with the number of assets in the portfolio; systematic effects remain.
Homogeneous portfolio

- Two effects are observable: **systematic effects** and **unsystematic effects**.

- Unsystematic effects vanish with the number of assets in the portfolio; systematic effects remain.

- Increasing the number of assets does not decrease the probability for large losses.
Inhomogeneous portfolio

- Inhomogeneous portfolio; different geographical location, different variant loads.

- The probability that half of the portfolio is lost is decreased. Adding assets from the same population does not change significantly the probability of large losses.
Risk reduction measure

- Risk reduction: Reduction of epistemic uncertainties in a portfolio.

- Two positive effects: Expected number of failures is decreased; Dependency is decreased.

- Might become rational to improve the best practice.
Conclusions

- Hierarchical approach for the modeling of portfolio losses is presented.

- Allows using conditional independence among assets in the portfolio.

- Different sources of common causes (geographical, best practices) lead to large variance in the loss distribution function.

- Especially for the aggregation of risks of importance; neglecting such common causes lead to sub optimal decisions if consequences behave nonlinear.
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Thank you for your attention